ESTIGATING SCHOOL MATHEMATICS

TEACHERS' EDITION

STIGATION DISCUSSION UTILIZATION

Topic		One Units a, b, c, and d	Book T Units e, f, g, a
Sets, Logical Reasoning, and Patterns	Concept of sets: P13–14 Comparisons of sizes: P1–8, P11–12, P15 Similarities and differences: P17–32 Patterns: P9–10, P16 Maze: P32	Comparing sets: a1–9 Equivalent sets: a5–8 Cardinal number of a set: a13–14 Empty set: a19 Union of sets and addition: b1–13, b25–32, b41 Sets and subtraction: b15–23, b33–37, b45 Sets of ten: c1–4 Skip counting: d54, d62 Logic: a59–60, d42 Patterns: c14	Cardinal n[illegible]er of a set: e Maze: e40 [illegible]2 Sets and a[illegible]tion: e43 Sets and s[illegible]raction: e47 Counting s[illegible]ences: e15, 62; f36; g6, 11, 1[illegible], [illegible]38 Informal lo[illegible]: e52, 57; f55; g12, 34, [illegible]7; h20, 50 Patterns: f[illegible] Attribute pieces: g23 Puzzle pr[illegible]ems: g34, g62, h8 Multiplica[illegible]nd sets: g50–52, g55, g57
Numeration and Place Value	More than and less than concept: P33–37, P43–47 Matching, one-to-one correspondence: P37–42 Numerals and numbers 0 to 10: R1–7, R11–12, R17–32 Recognition of number of a set: R8–10, R13–16, R37–38	Introducing ten: c1–4 Grouping by tens: c5–13 Counting to 99: c15–26, d44 One more than: a9, a27, 29, 31, 33, 35 Greater than and less than: a41–49, d55–61 Numeral writing: a15–23, a27–36	Numeral writing: e3 2- and 3-digit numerals: e7–11, f13–18, g1- Counting to 99: e13–17, h62 Greater than and less than: e17–23, g12 Roman numerals: e32–39 Expanded notation: f49, g16–17 Regrouping: h33, h41–44, h52–56 Fraction numerals: h21–29
Addition and Subtraction of Whole Numbers	Addition combinations 1 to 10 (intuitive): R32–36, R39–44 Subtraction related to sums of 10 or less (intuitive): R45–46 Order of numbers 1 to 10: R48	Addition and subtraction facts, sums through 9: b1–51, c35–45 Zero principle for addition: b9 Inverse relation (+ and −): c57–60, d49–52 Missing addends: c53–56, d45–53 Sums of ten: d1–11 Addition and subtraction facts, sums greater than 10: d13–23, d35–43 Commutative (order) principle: d41, d51	Addition and subtraction facts, sums of 10 or less: e41–51, e59 Missing addends: e53–61 Inverse relation (+ and −): e57–58, h13–1 Basic principles: f1–9 Addition and subtraction facts, sums to 18: f11–35, g35–47, h1–19 Addition and subtraction without regrouping: f47–61, g15–21 Addition and subtraction with regrouping: h31–61
Multiplication and Division of Whole Numbers			Multiplication concepts: g49–61
Fractional Numbers		Introduction to fractions, $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$: d25–33	Halves, thirds, and fourths: h21–29
Problem Solving and Applications		Solving money picture stories: c47–51 Discussions: a2, 14, 26, 42; b2, 16, 26, 40; c16, 36, 46, 54; d2, 14, 26, 46, 56 Picture story for zero: a20	Money problems: e25–31, g14 Addition and subtraction problems: e60; f2 32, 34, 54, 60; g20, 46, 60; h16, 18, 45, 58, 6 Multiplication problems: g59–60 Addition and subtraction problems with regrouping: h45, h58, h60 Discussions: e2, 26, 42; f48
Estimation			Introductory concepts leading to regrouping: h31, h39
Number Theory		Skip counting sequences: d54 Skip counting even numbers: d62	Odd and even numbers: e12 Skip counting: e62, f36, h38 Nomograph addition: f24 Square numbers: f62
Measurement		Concept of length: b53–57 Centimetres: b57–61 Length of paths: b62, d34 Readiness for area concepts: c62 Readiness for volume concepts: d24	Time: e33–39 Centimetres: f37–40 Perimeter: f41–42 Area: f43–44 Litres: f46 Volume: g48
	Recognition of simple shapes: P17, P27, P28, P30, P48	Number line: a47; b11, 21, 29, 35, 43, 47; d5, 19–22, 48, 54 Recognition of basic shapes: a12, a50–54 Open and closed curves: a55 Segments and paths: a57 Same size figures: a59–61 Symmetry: d25	Number line: e6, 46, 49; f12, 16, 21, 29; g54, 58; h35 Triangles, squares, rectangles, circles: g23–26 Segments: g27–28 Congruence: g28–30 Similarity: g31–32 Graphing: h48
		Telling time: c27–33 Money: c45–51 Let's have fun: a12, 24, 40, 50, 62; b14, 24, 38, 52, 62; c14, 26, 34, 44, 52, 62; d12, 24, 34, 44, 54, 62	Time: e33–39 Roman numerals: e32–39 Money: e25–31, g14 Magic squares: h8 Let's have fun: e12, 24, 32, 40, 52, 62; f10, 24, 36, 46, 62; g14, 22, 34, 4 h8, 20, 30, 38, 48, 62

[Continued on inside ba

Teachers' Edition to accompany

Investigating School Mathematics

ROBERT E. EICHOLZ

PHARES G. O'DAFFER

CHARLES R. FLEENOR

Collaborator, Teachers' Edition
THERESA BURKE

Collaborators, Reference Material and Metrication
JOHN BATES J. NORMAN C. SHARP
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Foreword

The *Investigating School Mathematics* series co-ordinates the precise concepts of modern mathematics with an approach that stimulates the child to actively participate in his own learning experiences. The series provides for the necessary mastery of basic number skills, and presents the material in a way that emphasizes the exciting, creative nature of mathematics. As the child becomes involved in exciting explorations and investigations, the structure and beauty of mathematics unfolds. The children are encouraged to investigate and discover ideas for themselves, to look for interesting patterns and relationships, and to develop their own generalizations. New and fascinating topics are explored not solely for their mathematical value, but also because they stimulate interest and motivate children to put forth their best efforts.

In our view, the development of a sound mathematical structure need not be hindered by an exciting, activity-oriented approach. Rather, the activity approach can and should reinforce the child's experiences as he investigates mathematical topics in an orderly, structured fashion. The same, sound mathematical structure that was called "modern" in the 1960's is present in *Investigating School Mathematics*. The important difference in this new series lies in its approach. The child learns through continual active participation in activities and investigations that lead to the discovery of each new idea.

As each new concept unfolds, the child is given an opportunity to investigate the ideas by using a wide variety of manipulative materials and activities. Then, through guided discussion, he is led to a deeper understanding of the ideas and their relation to the overall structure of mathematics. Following the investigation and discussion, he is provided with sufficient problem-solving practice to develop speed and accuracy.

The *Investigating School Mathematics* series is unprecedented in its careful provision for individual differences. Throughout each text, the child is challenged to do what he *can* do, not what someone else *thinks* he can do. Each child has the opportunity to experience individual success in an environment that stresses co-operation and communication rather than competition. This careful provision for individual differences makes the *Investigating School Mathematics* series unusually adaptable to such diverse teaching situations as ungraded schools, individual or small-group instruction, or whole-class instruction.

The essence of the *Investigating School Mathematics* series is reflected in the beliefs to which we are committed: that there are fundamental mathematical concepts which can be isolated and set forth with sharpness and clarity; that these concepts, when truly understood, provide powerful tools for extending knowledge; that children of every level should be encouraged to actively participate, to think, to question, and to seek understanding; that, although a certain body of knowledge must be passed on to each generation from preceding generations, the individual creativity of each new generation must not be stifled by pedagogy which forces upon its pupils patterns of thought which have served us well in the past but which may be inadequate for the future.

Mathematics can be successfully taught in this spirit. At every stage in the learning of mathematics the discovery of new relationships can be a delight. It is in this spirit that *Investigating School Mathematics* has been written.

The authors wish to express their appreciation to Ball State University and to the Educational Research Council of Greater Cleveland, where many of the ideas were generated and tested for the *Elementary School Mathematics* series, which served as a forerunner of *Investigating School Mathematics;* to Edith Biggs and the Nuffield Project in England, for their leadership in bringing the activity-oriented laboratory approach into prominence; to Mrs. Nancy Hildebrand, whose contributions to the teachers' manuals for *Elementary School Mathematics* are still reflected in this manual; to Theresa Burke, who assisted in the preparation of this manual by bringing, from a wealth of classroom experience, many of the activities and teaching suggestions found in each lesson; and finally, to the many teachers and children who have proved that studying mathematics can be an exciting and stimulating experience in the elementary school.

Contents

The Book 3 Program

Mathematics of the Book 3 Program

Book 3 of the *Investigating School Mathematics* series is organized according to a careful sequential development of mathematical concepts. The children begin the general mathematical development by studying the numbers and numerations systems, and then move on to operations. You will notice that the process of developing operations is divided into two parts: the children first study the **concept** of an operation, for example, addition; and then, later on, study the **techniques** of the operation, for example, adding. In this introductory work, we distinguish between the study of the concept and the study of the techniques by using terms such as multiplication (concept) and then multiplying (technique).

In the preparation of Book 3 we have assumed that the children have mastered all addition and subtraction facts through 18. You will notice, however, that the concepts and basic facts of both addition and subtraction are reviewed. Even if the children mastered these facts in Books one and two, the review at this point is valuable. Though it may seem that the early chapters of Book 3 devote much time to simple, factual information, you will note that reviewing these basic concepts early in the children's experience will pay dividends as they proceed to explore more difficult concepts.

The development in Book 3 does not depend on previous exposure to or background in multiplication. The basic concepts and facts of multiplication are reintroduced in the Book 3 program, and children should work toward a thorough understanding and mastery of them. In the presentation of subtraction and division facts, emphasis is placed on the children's ability to think of differences as missing addends and quotients as missing factors. We intend for the children to develop facility in finding differences and quotients by thinking about the inverse operations.

Periodically, you may find it helpful to have additional exercises similar to those provided in the body of the text. Supplementary exercises can be found in the back of the pupil's text, and they are reproduced with answers in the back of this manual. Assign these exercises to children who will clearly benefit from the extra practice.

General Bibliography

To gain a broader comprehension of the overall mathematical development in the *Investigating School Mathematics* series, the list of books that follows should prove extremely helpful.

Biggs, Edith, and James MacLean, *Freedom to Learn: An Active Learning Approach to Mathematics* (Don Mills, Ont.: Addison-Wesley, 1969).

Boyer, Carl B., *A History of Mathematics* (New York: John Wiley & Sons, Inc., 1968).

Copeland, Richard, *How Children Learn Mathematics: Teaching Implications of Piaget's Research* (New York: The Macmillan Co., 1970).

Dienes, Z. P., *Building Up Mathematics* (London: Hutchinson Educational Ltd., 1960. Available from Humanities Press, New York).

Elliott, H. A., James MacLean, and Janet Jorden, *Geometry in the Classroom: New Concepts and Methods* (Toronto, Ontario: Holt, Rinehart and Winston of Canada, Ltd., 1968).

Forbes, Jack, and Robert Eicholz, *Mathematics for Elementary Teachers* (Menlo Park, Cal.: Addison-Wesley, 1971).

Members of the Association of Teachers of Mathematics, *Notes on Mathematics in Primary Schools* (New York: Cambridge University Press, 1967).

National Council of Teachers of Mathematics, *Insights into Modern Mathematics* (23rd Yearbook, 1957); *The Growth of Mathematical Ideas,* Grades K–12 (24th Yearbook, 1959); *Enrichment Mathematics for the Grades* (27th Yearbook, 1963); *Topics in Mathematics for Elementary School Teachers* (29th Yearbook, 1964); *More Topics in Mathematics for Elementary School Teachers* (30th Yearbook, 1969) (Washington, D.C.: National Council of Teachers of Mathematics).

Newman, J. R., *The World of Mathematics* (New York: Simon and Schuster, 1956).

Nuffield Mathematics Project, *I Do, and I understand* (New York: John Wiley & Sons, Inc., 1967).

School Mathematics Study Group, *Studies in Mathematics, Vol. IX, A Brief Course in Mathematics for Elementary School Teachers,* Revised Edition (Stanford University, 1963).

The Schools Council, *Mathematics in Primary Schools* (Curriculum Bulletin No. 1. Available from Selective Educational Equipment, Newton, Mass., 1964).

Williams, E. M., and Hilary Shuard, *A Resource for Teachers: Primary Mathematics Today,* GRADES 1–8 (Menlo Park, Cal.: Addison-Wesley, 1972).

Teaching the Book 3 Program

Design Features of Book 3

Each lesson is titled with either a question or a provocative phrase inviting the child to explore a given idea. The core lessons of Book 3 are designed in one of two ways: those lessons which include an investigation activity begin on the left-hand page with the investigation, followed by a set of discussion exercises and then by a set of exercises on the right-hand page for the children to work independently. The titles of these sections are *Investigating the Ideas, Discussing the Ideas,* and *Using the Ideas,* respectively; other lessons are designed around a set of discussion exercises, titled *Discussing the Ideas,* and a set of independent exercises, titled *Using the Ideas.*

Many lessons throughout the book contain starred exercises for enrichment and a *Think* problem for the more able or interested children. Each chapter contains a chapter review *(Reviewing the Ideas)* and every chapter, except the first, contains at least one cumulative review *(Keeping in Touch).*

Though each lesson in Book 3 is presented on facing pages, this format may be treated with considerable flexibility. You may find that for some lessons you will want to spend your entire allotment of time for mathematics on the Investigation and Discussion, saving the exercises for another day. In other lessons, however, you may find that you can cover as many as two facing-page lessons in one day. Keep in mind, though, that, in general, the core lessons are designed to be used for a single mathematics lesson in one day.

Color is used functionally throughout the text whenever it is felt that color-coding of numerals and symbols will facilitate learning and understanding of key concepts.

Teaching Strategies for Book 3

While specific teaching strategies will be made clear through your teachers' edition notes, there is a broad general plan for the teaching strategy throughout the book. The organization of the teachers' manual, as well as the material in the child's book, continually suggests this strategy. It is intended that each day's lesson in which the child is presented with a new concept be divided into four parts: Preparation, Investigation, Discussion, and Using the Exercises. The preparation usually should be kept fairly short, and care should be taken to see that this work does not preempt either the Investigation or the Discussion. Generally, the preparation should do nothing more than provide the children with that readiness which they need before they begin Investigation. The Investigation presents the rudiment of the concept treated in the lesson and should be the "main event" in terms of pupil activity and involvement in the unfolding of the concept.

In general, it is expected that the Investigation will be done by the children either individually or in small groups. Think of the Investigation as student-centred activity. It is fully anticipated that the students will grope, question, search, and explore. Most Investigations are designed to provide for individual differences; that is, the child is frequently asked to perform a certain task as many ways as *he* can, or to find how many ways he can do a certain thing. By presenting the child with this type of challenge, at least some degree of success is assured. That is, your slowest student will find that he can do something more than one way, while your more able children will find many ways to do a given task. Thus, as you guide the children through an investigation, it is important for you to recognize that they will achieve in widely differing ways, and that you should give recognition for all levels of achievement. Perhaps the most important thing to remember in working with the children during the Investigation is to encourage them to do the thinking and exploring. It is their section. Do not help them too much.

Following the Investigation, the children are given an opportunity in the Discussion section to talk about what they did and to summarize the mathematical ideas of the lesson in preparation for working independently in the Using the Ideas section. Generally, the beginning discussion exercises are designed to stimulate the children to talk about what they did in the Investigation. You should encourage them to discuss the various methods that they used to investigate and explore the concepts. Also, you should follow your teachers' guide carefully to be sure that whatever mathematical ideas are to be developed in the section are actually summarized and understood by the children.

The section titled *Using the Ideas* (called *Using the Exercises* in the teachers' manual) does exactly what the title implies. The children, having come through the first three parts of the day's lesson —the Preparation, the Investigation, and the Discussion—should be ready to work on their own to use the ideas of the lesson. Again, you should provide for individual differences in assigning work in this section; in other words, base your assignments on the needs and abilities of the children. The exercise sets in this section are generally graded, beginning with the easier exercises and ending with the more challenging (starred) problems. Often, there will be a challenging puzzle-type problem for a given lesson. From time to time, a fairly easy challenge problem is provided to encourage less able children to attempt it. Also, quite often, all of the children will benefit from a discussion of the challenge problem.

At the top of each left-hand page of the teachers' edition, under the heading *Objective(s),* the goal of

the lesson is stated in terms of what the child should be able to do as a result of the lesson. This objective summarizes the key idea of the lesson in terms of the child's performance. Throughout the teaching suggestions for the Investigation and Discussion, the most important ideas of each lesson are reemphasized and clarified. It is important that you carefully consider the objective of each lesson so that you know how to direct the children through the development of ideas to the desired goal.

Following the statement of the objective for each lesson, the teaching suggestions follow the format of the general teaching strategy mentioned above. The Preparation lists any materials the child will need for the Investigation, and, occasionally, materials recommended for use by the teacher during the Discussion. The preparation section is designed to review ideas, motivate children, or give them necessary background information regarding the Investigation. Depending on your classroom organization, you may choose to use only those Preparations which are essential to the Investigation. At times it is recommended that you have children begin immediately with the Investigation.

The next three sections of the teaching suggestions correspond to the sections of the child's text:

Investigating the Ideas – Investigation

Discussing the Ideas – Discussion

Using the Ideas – Using the Exercises

Each section contains teaching suggestions related specifically to the corresponding section in the child's text. When lessons in the child's text deviate from the standard format, the teaching suggestions fall under the Discussion, and/or the Using the Exercises heading.

A section titled *Mathematics* is included in certain lessons to provide background or to clarify for the teacher the principal mathematical concepts treated in the lesson. This section is strictly for the teacher; terms used here need not be used with the children other than as indicated in the teaching suggestions or in the child's text.

The *Follow-up* suggests various activities, games, or worksheets to help reinforce the concepts developed in the lesson. These are included only as suggestions to be used as your schedule allows. However, some of these activities, and many of the materials recommended in the *Resources for Active Learning* may be used by the children independently at various activity or free periods throughout the day.

Since effective use of the investigative approach requires knowledge of the materials available for activity-oriented classrooms, suggestions are provided in the section titled *Resources for Active Learning*. The lists of resources in the Chapter Introductions and in most of the lessons offer an ample start. Provided some of these resources are available, you should be able to use or adapt the ideas contained in them to various situations.

The "Manipulative Devices" can be used to support the lessons. The "Commercial Games" and "General Activities" in the Chapter Introduction should be useful as continuous activities throughout the chapter, for review or practice of basic skills and concepts. The Resources listed for specific lessons are directly related to those lessons' objectives. Choose one or two, and try them out in a variety of situations.

At the time of this writing, the authors cited those resources which are of high quality and which directly complement the active-learning approach. Familiarize yourself with those materials which have been marketed subsequently. Check with your principal or supervisor for more recent materials to support this type of learning approach. Such materials can often be obtained from your local school supplier. Additional sources are listed on page xi.

Provision for Individual Differences

Minimum, average, and maximum assignments are provided for each lesson other than review lessons. These assignments are given to assist you in providing for the individual needs of the children. It is *not* intended that you give the minimum assignment to the slower children, the average assignment to the average children, and the maximum assignment to the more able children. Rather, these designations are given to assist you in making individual assignments according to needs, abilities, and time available for each individual child. For example, if time is short and you need to move rapidly through a particular lesson, you may choose to use the minimum assignment for all children. The minimum assignment will, in general, provide the children with sufficient practice and mastery of skills to move ahead to the next lessons. On the other hand, you may sometimes choose to use the maximum assignment with slower children over a period of two or three days. Also, it is highly likely that you will not want to use the maximum assignment for the more able children, since quite often they need less practice than some average and below average children. For example, when your more able children demonstrate the ability to perform a particular skill with great efficiency, they should not be made to drill excessively in that skill. In some cases, an asterisk is placed beside an assignment to indicate that the lesson could be omitted without loss of continuity in the flow of ideas.

Long-range Planning Chart

The long-range planning chart is designed to provide the teacher with some guidelines for planning basic, average, and maximum course coverage of *Investigating School Mathematics*.

The basic course outline covers all the essential parts of the pro-

LONG-RANGE PLANNING CHART						
Chapter	**Basic Course**		**Average Course**		**Maximum Course**	
1	4–13, 28	3 weeks	4–13, 16–25, 28, 29, 14, 15	3 weeks	4–25, 28, 29, 26, 27	3 weeks
2	30–41, 48, 49	4 weeks	30–43, 48, 49, 44, 45	2 weeks	30–40, 46–49, 44–45	2 weeks
3	50–67, 72, 73, 68–71	5 weeks	50–73	3 weeks	50–73	$2\frac{1}{2}$ weeks
4	74–79, 82, 83, 88, 89, 86, 87	$2\frac{1}{2}$ weeks	74–83, 86–89, 84, 85	2 weeks	74–89	2 weeks
5	90–107, 112, 113, 116, 118, 121	6 weeks	90–109, 112, 113, 116–118, 120, 121	5 weeks	90–113, 116–121, 114, 115	$4\frac{1}{2}$ weeks
6	122–139, 142, 143, 146–155, 166–168, 134, 161	5 weeks	122–157, 160, 161, 166–168, 158, 159	$4\frac{1}{2}$ weeks	122–161, 166–169, 162–165	$4\frac{1}{2}$ weeks
7	170–175, 178–189, 186–188, 192–194	$4\frac{1}{2}$ weeks	170–189, 192–195	$3\frac{1}{2}$ weeks	170–189, 192–195, 190, 191	$3\frac{1}{2}$ weeks
8	196–203, 212 213, 204–205	2 weeks	196–207, 212 213, 210, 211	2 weeks	196–213	2 weeks
9	214–215, 218–221, 224, 225	1 week	214–219, 224, 225, 222, 223	$1\frac{1}{2}$ weeks	214–225	2 weeks
10	226–241, 252–255, 242, 247	3 weeks	226–249, 252–255, 250, 251	4 weeks	226–255	$4\frac{1}{2}$ weeks
11			256–261, 270, 271, 262–265	$2\frac{1}{2}$ weeks	256–299, 304–307, 272–273	$2\frac{1}{2}$ weeks
12			276–290, 304–307	3 weeks	276–299, 304–307, 300–303	3 weeks

gram, but provides little in-depth or extension material. The average course covers all the material of the basic course plus considerable extension material. The maximum course provides for nearly total coverage of all topics presented in the text. Optional material for each of the three courses is shown by page numbers shaded gray.

The suggested time schedule, covering 36 weeks, should be viewed only as an *aid* in helping you plan a time schedule that allows for the individual differences within your own class. You should not view it as a rigid schedule. For example, the basic course may be used with children who are achieving below the average grade level. This same course might be used in other ways, such as with a brighter child who may move rapidly through these same pages to make up for time lost through an absence.

General Suggestions

We offer two general suggestions regarding the use of the chapter and page lesson notes:

(1) Read and consider each point as it applies to the immediate objectives for the lesson and the overall objectives of the unit.

(2) Do not allow the teachers' manual notes to deter you from using your own effective teaching methods or to stifle creative efforts.

Your manual does not attempt to dictate all of the activities in the day-to-day handling of your class and the individuals in it. You should

use your manual as a guide to be co-ordinated with those methods which you have found to be most effective in teaching mathematics in the past. One of the key techniques in presenting a structured system of mathematics consists of developing a topic and pursuing it until you have reached a desired level of understanding with regard to that topic. This is one of the guiding philosophies behind the development of *Investigating School Mathematics*. You will notice this philosophy particularly in the development of some of the more intense sections of work. That is, when the topic is being introduced, it is explored in great detail; however, we have tried to provide interesting activities to relieve the intensity of these longer sections. Treat the relief materials with a light touch and in the spirit of having fun with mathematics.

Classroom Organization

The Book 3 program can be used in situations where the entire class works together on the same lesson, where small groups work together on the same lesson, or where individual students are allowed to proceed at their own rate of speed. You will want to employ the type of classroom organization that best suits the physical facilities of your particular situation. Teachers often find it stimulating to vary group sizes for different lessons and units of work. Smaller-size groups can often work more effectively together and allow greater opportunity to participate in the Investigation and the Discussion. Whatever class organization you choose, keep in mind that the key to the *Investigating School Mathematics* program is active student participation.

Evaluation of Progress

A child's attitude toward mathematics is often influenced by the methods used for evaluating his progress. All too often, evaluation procedures focus attention on what the child *did not* understand or master, rather than on what the child *did* accomplish. In evaluating a child's progress, try to mantain a positive view, one which capitalizes on successes and develops confidence.

Achievement and diagnostic tests for Book 3 may be obtained from the publisher. Chapter reviews are provided in the text, as well as cumulative reviews, to aid in evaluation of the child's progress. However, you may find that a day-to-day evaluation of the children, often involving personal interviews, will help you determine how well a child grasps the concepts and is able to apply them.

Children's Bibliography

General Interest Books

Books listed in the Books to Explore section of the pupil's text (pages A38–A40) were selected to appeal to the interests of children, as well as to provide historical background and reference material.

Additional books which may be of interest are listed below.

Adler, Irving and Ruth, *Shadows* (New York: John Day Co., 1968). Here is a way to bring science and math together by measuring shadows to find out the length of tall objects.

Baer, Howard, *Now This, Now That* (New York: Holiday House, 1957). The author helps the reader see how things look from distances and angles, and also provides background for probability.

Carroll, Lewis, *Through the Looking Glass* (New York: Grosset & Dunlap, 1963). The author, a distinguished mathematician as well as a fine writer, might have been influenced by the obvious association between mirrors and mathematics to create Alice's adventures.

Feravolo, Rocco, *Wonder of Mathematics* (New York: Dodd, Mead & Co., 1962). The author offers activities and problems for developing numeration systems other than base ten.

Hogben, Lancelot, *The Wonderful World of Mathematics* (New York: Doubleday & Co., 1968 edition). This book shows that the story of man becoming civilized is also the story of the growth of mathematics as a science.

Jonas, Arthur, *New Ways in Math* (Englewood Cliffs, N.J.: Prentice-Hall, 1962). This book tells the story of mathematics, including sets, probability, and algebra, in cartoon style. Mathematical greats from Pythagoras to VonNeumann are represented in a section called "Men in Math." Mr. Jonas also has a new book, *More New Ways in Math*.

Leeming, Joseph, *Fun with Puzzles* (Philadelphia: J. P. Lippincott, 1946) (Also available in paperback from Scholastic Book Service, 1966). This book contains more than 200 puzzles and their solutions, including cutout, paper-and-pencil, match, word, and coin puzzles.

Simon, Leonard, *The Day the Numbers Disappeared* (New York: McGraw-Hill Book Co., 1963). The author compares Egyptian, Greek, and Roman methods for writing numbers.

Storybooks

Auerbach, Marjorie, *Seven Uncles Come to Dinner* (New York: Alfred A. Knopf, 1963). Money.

Barr, Donald, *Arithmetic for Billy Goats* (New York: Harcourt Brace Jovanovich, 1966). Base 2.

Behn, Harry, *All Kinds of Time* (New York: Harcourt Brace Jovanovich, 1950). Time.

Branley, Franklyn M., *Big Tracks, Little Tracks* (New York: Thomas Y. Crowell, 1960). Comparison.

Brenner, Barbara, *The Five Pennies* (New York: Alfred A. Knopf, 1963). Coins.

Caudill, Rebecca, *A Pocketful of Cricket* (New York: Holt, Rinehart and Winston, 1964). Number-line Cricket.

Elkin, Benjamin, *Six Foolish Fishermen* (Chicago: Children's Press, 1965 edition). Cardinal and ordinal numbers.

Friskey, Margaret, *About Measurement* (Chicago: Children's Press, 1965). Length.

Hoberman, Mary Ann and Norman, *All My Shoes Come in Twos* (Boston: Little Brown and Co., 1957). Multiples.

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House, 1961). Large numbers.

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Suppliers of Resources for Active Learning

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Creative Publications, Palo Alto, California

Cuisenaire Company of America, Inc., New Rochelle, New York

Edmund Scientific Co., Barrington, New Jersey

Educational Supply Co., Toronto.

Educational Teaching Aids, Chicago

J. L. Hammett Co., Braintree, Mass.

Herder and Herder, Inc., New York (Available from Methuen Publications of Canada Ltd., Agincourt, Ontario).

Holt, Rinehart and Winston of Canada Ltd., Toronto.

LaPine Scientific Co., Chicago.

The Macmillan Co., New York (Available from Collier-Macmillan Canada Ltd., Don Mills, Ontario.

Math Media Inc., Danbury, Conn.

Metric Aids Ltd., Toronto.

Milton Bradley, Springfield, Mass.

Moyer-Vico Ltd., Weston, Ontario.

Responsive Environments Corp., Englewood Cliffs, New Jersey.

Sargent-Welch Scientific Co. of Canada Ltd., Weston, Ontario.

Scholar's Choice, Stratford, Ontario.

Selective Educational Equipment, Newton, Mass.

Sigma Division, Scott Scientific Inc., Fort Collins, Colorado.

TUF, Rowayton, Connecticut.

Walker Educational Book Corp., New York (Available from Fitzhenry & Whiteside Ltd., Don Mills, Ontario).

Webster Division, McGraw-Hill Ryerson Ltd., Scarborough, Ontario.

World Wide Games, Delaware, Ohio.

PAGES 2–3

Objective

Given a typical lesson in the text, the child will be able to distinguish its three principal parts: Investigating the Ideas, Discussing the Ideas, and Using the Ideas.

Preparation

This first step of the four-part teaching plan for each lesson in the text is the briefest. However, it will often be the key to inspiring children with an enthusiastic approach to the lesson. In some lessons, it is essential to the investigation; in other lessons, it simply serves as a review. In almost all cases the preparation should be limited to a maximum of five minutes. Brisk and lively, it should ensure that each lesson is begun with a positive attitude.

For this introductory lesson, you might simply encourage the children to flip through the pages of the book, stopping to look at whatever interests them. After one or two minutes, refer them to the title and author page and the table of contents, asking them what chapters they think they will enjoy most. Finally, refer them to the introductory lesson on pages 2 and 3.

Investigation

As stated in the introductory material, the investigation presents the child with an opportunity to explore, often with minimum teacher direction, the ideas presented. Sometimes this exploration consists of an investigation with concrete objects; at other times, the child simply uses paper and pencil to explore ideas with number. In any case, this section should be totally child-centred. It is the child's opportunity to explore new ideas, or known ideas in a new way; it launches the lesson. The attitude to be fostered is that the child cannot make a mistake; since he will often learn as much from his "incorrect" approaches as from finding the correct methods, he should be encouraged to explore things on his own.

In this introductory lesson, you will probably want to read with the children the paragraph at the top of the page. Observe with them the question in the box and explain that the main question of every investigation is shown in the same way. Read this question together; then direct them to try to answer it.

Move around the room encouraging children who find the required sections and suggesting that they look for more. Stress the importance of recording their answers even when the text does not explicitly request it.

Discussion

The discussion section provides an opportunity for children to discuss the ideas they investigated and for you to guide the discussion so that the emphasis is on the main point of the lesson. It is in this section that you will sometimes want to demonstrate various computational skills or use visual materials to highlight a particular concept. The questions in the text help the children focus on the most important concepts, but your skill in guiding discussion and presenting ideas will be of inestimable value in this section.

Ask several children to share

Let's explore your mathematics book.

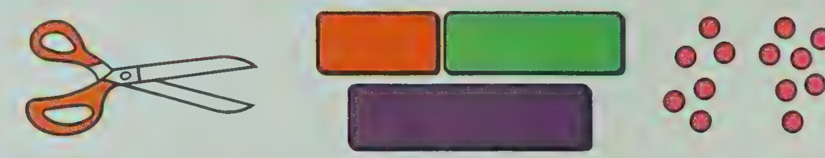
Investigating the Ideas

This is a sample lesson. It will help you understand how you will use your book.

In this part of a lesson there are things for you to **investigate** and discover.

? Can you find some Investigations where you would use these objects?

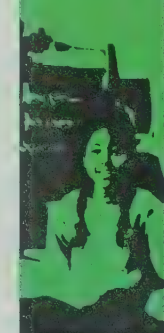
(See sample answers below.)



Discussing the Ideas

In this part of a lesson you will **discuss the ideas** you investigated. You will be sharing your ideas with others. You are getting ready to **use the ideas.**

1. Look through your book. Can you explain an easy way to find the "Investigating the Ideas" sections?
   They always appear in the upper part of left-hand page with a strip colored some shade of blue.
2. Can you explain an easy way to find the "Discussing the Ideas" sections in your book?
   They always appear on left-hand page, usually in lower portion, with a strip colored some shade of green.
3. Find a "Discussing the Ideas" section that begins at the top of a page. Sample answer: p.32
4. Find a page called "Keeping in Touch." Sample answer: p.49
   What do you think this means?
   A chance to study concepts and skills presented in previous portions of the text (a cumulative review)



2

Using the Ideas

In this part of the lesson you will **use the ideas**. You will work problems to improve your understanding of the ideas you have discussed. Try these.

1. How many "Investigating the Ideas" sections are in Chapter 4? 6
2. Find the number of "Discussing the Ideas" sections in Chapter 6. 16
3. How many "Using the Ideas" sections are in Chapter 8? 8
4. Look up *prime numbers* in your index. What page numbers are given? 222-223

Problems in these boxes are a **special challenge** for you. Be sure to try some of them. See if you can do this one.

think

The faces of a block are numbered from 1 to 6. What is the sum of the numbers you cannot see?

3

their findings from the investigation. Then in exercises 1 and 2 have them identify the color bands which correspond to the "Investigating" and the "Discussing" sections. Note that almost every investigation section is followed by a discussion section. However, in some lessons no investigation precedes the discussion section. Point out that each chapter ends with a lesson called "Reviewing the Ideas," which reviews the concepts studied in that chapter. Also, at the ends of chapters and occasionally within chapters, there are "Keeping in Touch" lessons, which review other important concepts.

Using the Exercises

The section in the child's text entitled "Using the Ideas" is referred to in this teachers' manual as "Using the Exercises." As indicated by its title, this section is intended to provide the children with an opportunity to use the ideas they have investigated and discussed. Ordinarily, children should be expected to work by themselves for this section. Occasionally, however, you might want to use a few of the exercises as a basis for introduction, directions, and discussion. Exercises on these pages may be assigned selectively; it is rarely essential that each child do all of the exercises. On many pages, however, the exercises progress developmentally, so children should do at least some parts of each exercise, excepting those which are starred. Starred exercises are those which extend the concept of the lesson or give more difficult applications of it. They are not intended for all children.

Explain to the children the purpose of this section and assign the exercises. Due to the amount of reading in some of these sections, you might find it desirable to read through the material with some children. As explained in the text, the *Think* problem is a special challenge. Encourage children to try these, even though they are intended primarily for enrichment.

CHAPTER 1 | Measurement

Pages 4–29

General Objectives

To introduce the concept of length
To provide experiences in approximation
To develop a feeling for selecting appropriate units
To introduce the concept of area
To introduce the concept of volume
To provide experiences in working with cubic centimetres and litres

In this chapter, three basic types of measurement are explored: length, area, and volume.

Measurement in these lessons is divided into two parts: (1) counting distinct objects within a set and (2) counting which does not involve distinct objects. When specific objects cannot be counted, approximation or estimation is used. In the exercises in this chapter, measurements for area or volume can usually be found by counting objects, and estimation is not required. Estimates or approximations *are* required, however, when the children are asked to determine the length of given objects.

After volume is introduced by means of arrays, the children work with volume given in litres. Although the concepts are essentially the same, the children may find this kind of measuring quite different from that of counting cubes. Note that both concepts involve finding volume expressed in terms of a given unit.

Mathematics

Measurement is essentially a counting process, but it is not always immediately apparent what it is we are to count or how we are to count. Regardless of the object or set to be measured, the measurement process consists of the following three steps:

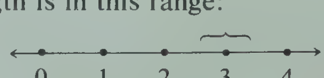
1. Select an appropriate *unit*. For length we choose a segment such as a centimetre. For volume we choose a cube, and for angle measure a unit angle such as the degree or radian.
2. Divide the object to be measured into parts congruent to the chosen unit.
3. Count the number of units contained in the object.

In this chapter, the children are not asked to measure more accurately than to the nearest half centimetre. When a child arrives at a measure of $4\frac{1}{2}$ cm for a given object, he has counted 4 one-cm segments and 1 half-cm segment. Even though the object may not be exactly $4\frac{1}{2}$ cm long, the counting process is still important.

Suppose we ask a child to measure to the nearest tenth of a centimetre an object that is $4\frac{7}{10}$ cm long. He could arrive at the measure of $4\frac{7}{10}$ by counting 4 one-cm segments and 7 tenth-centimetre segments. The process can be carried to any desired degree of accuracy by counting smaller and smaller unit segments to arrive at a closer approximation. But whether or not a child can arrive at the exact measure of a given object by continually breaking apart the unit segment into smaller segments is unimportant at this time. What *is* important is to have the children recognize that the process of measurement involves counting.

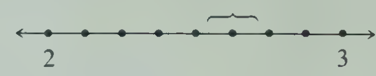
The following diagrams illustrate the concept of measurement to the nearest unit or nearest fraction of a unit.

If the length of an object is given as 3 to the nearest unit, the true length is in this range:



The length is closer, or as close, to 3 as it is to 4 or to 2.

If the length is given as $2\frac{5}{8}$ to the nearest $\frac{1}{8}$ unit, the true length is in this range:



The length is closer, or as close, to $2\frac{5}{8}$ as it is to $2\frac{4}{8}$ or $2\frac{6}{8}$.

Teaching the Chapter

Materials

Bucket of sand or substitute
Centimetre rulers (1 per child)
Colored strips
Containers for studying volume
Construction paper
Cubes of sugar, wood, or the like for studying volume
Graph paper, 1-cm grid
Metre stick
Scissors
Squares of cardboard, floor tiles, or similar objects for studying area
String
Tagboard
Tape measures

Vocabulary

approximation	measurement
area	metre
centimetre	metric
cube	region
cubic unit	ruler
decimetre	square
estimation	square centimetre
gram	surface
kilogram	unit
kilometre	volume
length	
litre	

Because children can explore best through physical activities, the investigations in this chapter are activity-oriented and require the child to trace, cut, fold, count, measure, etc. Allow the children considerable freedom in these activities, once you are sure they know what they are to do.

Have available as many supple-

mentary materials as possible; even common materials such as wire, rope, and string may be used to extend an activity involving length measurement. Encourage children to pursue any topic in which they become especially interested.

In the vocabulary section, all the words pertinent to the discussions in this chapter are listed. Although the children already are familiar with many of the words, you should be alert to stress those which are new to your class.

Lesson Schedule

Because so many different activities can enhance the investigation of measurement concepts, it would be easy to allot too much time for this chapter. It is but an *introduction* to ideas of measurement, however, so you should not expect children to master or memorize all the facts and concepts presented. These topics are explored in greater depth in Books 4, 5, and 6. Plan to spend, at most, three weeks on the chapter, and attempt to limit the time to two-and-a-half weeks without greatly restricting the activities.

Evaluation of Progress

Although the ideas of the chapter are important to the full development of the child's overall experience in mathematics, they do not constitute essential background material for the remainder of the year's work. In judging a child's achievement in this chapter, remember that only the exploration and recognition of the ideas and concepts are of primary importance. Take special care to see that the class has a successful and enjoyable experience.

The chapter review on pages 28 and 29 can be used for evaluating achievement or for constructing a test similar to the review. On the other hand, you may choose to use these pages simply as review, omitting any formal evaluation.

Resources for Active Learning

General Activities

(This list includes ideas and activities that can make your classroom come alive at the beginning of the year. If you have one of the books listed, choose a project or two to investigate during the lessons in Chapter 1.)

Franklin Series: *Learning About Measurement,* "Measuring Curves," pp. 15–19, Lyons and Carnahan (Available from McGraw-Hill Ryerson)

Freedom to Learn, "Length," pp. 121–127; "Perimeter and Area," pp. 127–131; "Volume and Capacity," pp. 131–134, Addison-Wesley

Madison Project: *Discovery in Mathematics,* "Area," pp. 83–87 of Student Discussion Guide, Addison-Wesley

Mathex: Measurement No. 10, "Area – Activities 1 and 2," p. 9, Encyclopaedia Britannica Publications Ltd.

Mathex: Measurement and Estimation No. 5, "Linear Measurement," pp. 15–27; "Perimeter and Area," pp. 28–34, Encyclopaedia Britannica Publications Ltd.

Nuffield Project: *Beginnings 1,* "Length and Area," pp. 68–81, Wiley

Nuffield Project: *Computation and Structure 2,* "Length." pp. 6–23, Wiley

Nuffield Project: *Shape and Size 2,* "Area," pp. 61–77, Wiley

Manipulative Devices

(The following devices will aid the child in his study of linear, area, or volume measurement.)

Caliper (Edmund Scientific; Selective Educational Equipment)

Cubical Counting Blocks (Milton Bradley; school supplier)

Cuisenaire Cubes, Squares, and Rods (Cuisenaire Co.)

ETA Discovery Blocks (Educational Teaching Aids)

Geo Blocks (Selective Educational Equipment; Webster, McGraw-Hill)

Geoboards (Addison-Wesley)

Height Measure (Selective Educational Equipment)

Lake and Island Board (Math Media)

Liquid Capacity Measures (Educational Teaching Aids; school supplier)

Pattern Blocks (Selective Educational Equipment; Webster, McGraw-Hill)

Trundle Wheels, metre (Math Media; Selective Educational Equipment)

Commercial Games

Which Is More? (Selective Educational Equipment)

PAGES 4–5

Objective

Given an appropriate object such as a desk top, the child will be able to measure its length or width by counting nonstandard linear units.

Preparation

Materials

sets of colored strips

To stimulate interest, ask the class what kinds of things they would expect to do in math class. Then refer to the colored strips, which children may recall from previous years. Tell the children that in this lesson they will have a chance to study counting and measuring and that they will be able to use the strips also.

Investigation

One of the main purposes of this investigation is to give children an opportunity to experiment with a nonstandard unit to measure length. Although the strips for the individual child should be the same length, the excitement of the investigation will heighten if different children use strips of different lengths. Remind the children to keep a written record of their measurements. Also encourage children to use the picture as a guide. Circulate among the children, giving help only where needed; for example, suggest that the children take care in laying the strips end to end. If time permits, you might suggest that some children measure classroom objects as well as parts of their desks.

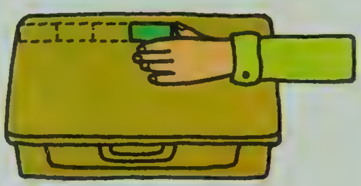
1 Counting and Measurement

Can you measure by counting?

Investigating the Ideas

Choose one of your colored strips.

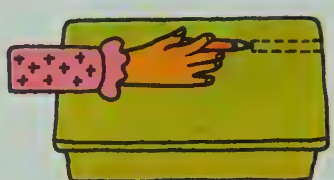
Count how many times you have to put your strip down to move it across your desk top.

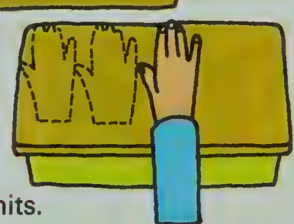


Can you use one of your strips to measure some other parts of your desk?

See Investigation.

Discussing the Ideas





1. How did you use your strip to measure your desk?
   See Discussion.
2. Jane said, "My desk is about 4 pencils wide." What did Jane mean?
   Sample answer: Her desk is as wide as 4 pencils laid end to end.
3. How many hands wide is your desk? Answers will vary.
4. Objects used for measuring are called units. What are the three units used so far in this lesson?
   strips, pencils, hands

4

Discussion

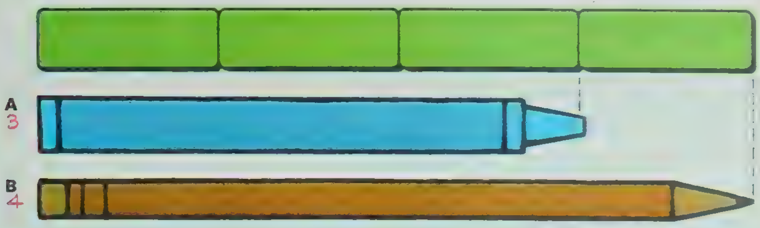
After the children have measured several objects, encourage discussion of how they did the measuring and of the results they recorded. The idea to be emphasized here is that they were counting units; the concept of length will be stressed in a subsequent lesson. If a child gives a measurement in halves or fourths, accept it without too much concern for fractional accuracy.

For exercise 2, encourage the class to measure their desks with pencils and compare their measure with Jane's. Then let the children talk about how the measures differed for each unit they used. It would be helpful to make a chalkboard record of some of the measurements for each unit.

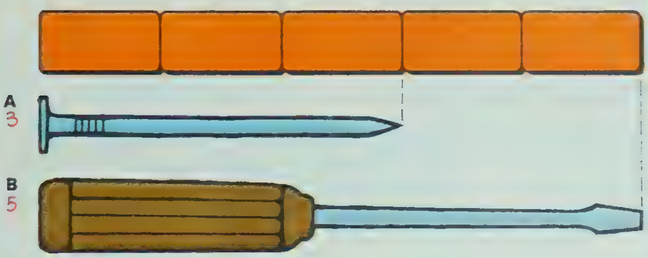
One of the important objects of this lesson is to give children an understanding of the term *unit*. Read exercise 4 carefully with the children, helping them see that the strips, the pencils, and their hand widths are the units by which they measured their desks and other objects. Also discuss other items that could have been used as units.

Using the Ideas

1. Count the number of strip units for the measure of each object.



2. How many strip units long is each object?



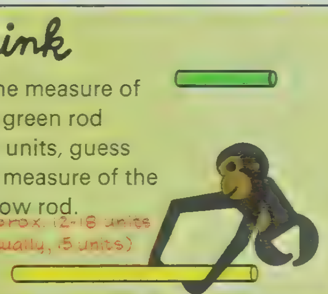
3. How many paper clip units long is the straw? 4

4. Count the units to measure each rod.

think

If the measure of the green rod is 6 units, guess the measure of the yellow rod.

Approx. 12-18 units (Actually, 15 units)



5

Using the Exercises

Direct the children to do the exercises on page 5 independently. When they are finished, discuss the answers with the children and again stress the idea that finding the measure of an object involves counting units.

The *Think* problem is for more capable children. Any answer from 12 to 18 is acceptable: but the "correct" answer is 15, since the length of the yellow rod is about $2\frac{1}{2}$ times that of the green one.

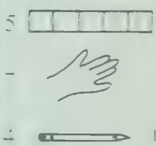
Assignments (page 5)
Minimum: 1–4. Average: 1–4.
Maximum: 1–4.

Mathematics
This lesson and the next explore two ideas basic to the study of measurement: the choice of a unit and the counting of units. The selection of the unit is usually based on its convenience relative to the size of the object to be measured. Aside from this consideration, the selection of a unit is an arbitrary matter. We can select any unit we please to measure a given object, but if we wish to communicate with others concerning this measurement, everyone concerned must know the unit used.

The children are introduced to the selection of different-sized units to arrive at the measurement of given objects. They explore intuitively the idea that the smaller the unit of measure, the larger the number for length.

Follow-up
You might like to devote a bulletin board to this chapter on measurement, since many of the activities readily lend themselves to illustration. For example, the children might like to show the results of their investigation on a wall chart, illustrating each object measured and the units used.

Object	How long
Desk top	5 (strips)
	[illegible] (hands)
	4 (pencils)



Also, you might have the children make a list of interesting but familiar objects found at school or home. Then the children might choose a unit, measure each listed object, and record the results for the unit they chose.

Resources for Active Learning
Developmental Math Cards. E²3, E²8, F²11. Addison-Wesley. [Some exciting experience activities in measurement]
Franklin Series: *Learning About Measurement*. "Measuring Line Segments." pp. 4, 5. Lyons and Carnahan. (Available from McGraw-Hill Ryerson)
Math Activity Cards. "Lengths . . . ." A 25, 26, 27, 28, 29, 30. Macmillan.

PAGES 6–7

Objective

Given a nonstandard unit, the child will be able to make his own ruler and use it to measure length.

Preparation

Materials

plain strips of paper or tagboard, at least 20 cm long and 3 to 5 cm wide; colored strips

Review with the children the process, presented in the previous lesson, of laying down and counting objects used as units. Discuss the awkwardness of picking up and putting down the unit object. Ask the children if they can think of an easier way to count units. If someone mentions a ruler, explain that in this lesson they will find out how to make and use their own ruler.

Investigation

This investigation continues the study of the choice of unit and relates it more specifically to measurement of length.

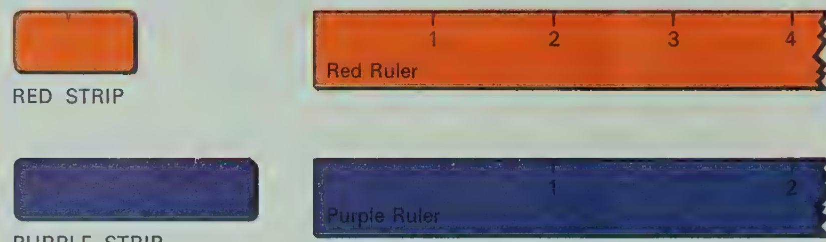
As the children make their rulers, guide them in placing their strips and marking the rulers carefully. They will need to move their strip after marking each unit, and you may find that close supervision and some help from you are required. Encourage them to mark off at least 4 units on their rulers.

Most children will accept and understand the word *length* from its context without any formal explanation. Give the children ample time to compare the results of their measuring. The discovery that their answers differ should lead to a quickened interest and curiosity that should be encouraged as you introduce the questions in the discussion section.

- *How can you make and use your own ruler?*

Investigating the Ideas

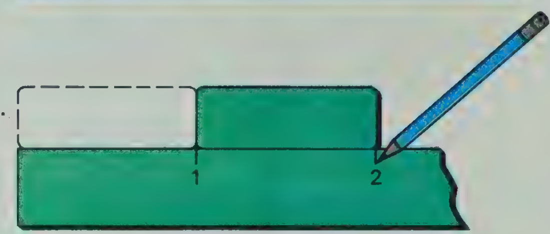
Each of these rulers uses a different strip as unit.



RED STRIP

PURPLE STRIP

Cut a long strip of paper. Choose one of these strips and make a ruler as shown.



Making a Light Green Ruler

Can you use your ruler to find the length of your pencil? See Investigation.

Discussing the Ideas

1. Which unit would give the largest number for the length of your pencil? Red strip unit
2. Which unit would give the smallest number for the length of your pencil? Purple strip unit
3. Joe measured his crayon using a red ruler. He found it was 4 units long. What is the length of his crayon using a purple ruler? 2 units
4. Bill and Jane each made a ruler. Jane used a unit as wide as her hand. Bill used a unit as wide as his finger. If they both measure the same thing, who will get the larger number? Bill

6

Discussion

One of the basic ideas of measurement is brought out in this lesson: The length of an object depends upon the choice of unit. Some children will discover through the investigation and ensuing discussion that, given two units, one of which is smaller than the other, the smaller unit will give a larger number for the measure of the object; do not expect all children to understand this concept, however.

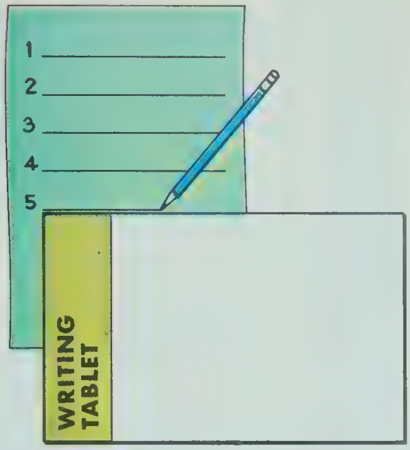
In order to discuss questions 1 and 2, you will need to have some children work in pairs or in small groups measuring one pencil with the different units they have chosen. Have each child keep a record of the measures his group finds. Then discuss their results.

You might like to have the children measure an object with their hand and then with their finger to help them see the answer to exercise 4.

Using the Ideas

For exercises 1-3, see Using the Exercises.

1. Draw five lines on your paper as shown. Number your lines 1 to 5.



2. On the top line use your ruler to mark 2 **points** that are 4 units apart. Name your points *A* and *B*.

You have a **segment** *AB* that is 4 units long.

3. **A** On line 2 mark a segment *CD* that is 3 units long.

   B On line 3 mark a segment *EF* that is 5 units long.

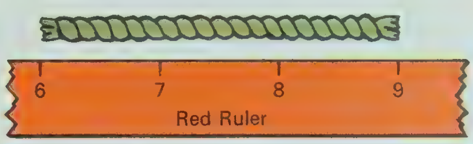
   C On line 4 mark a segment *GH* that is more than 3 units and less than 4 units long.

   D On line 5 mark a segment that is more than 2 units and less than 3 units long.

4. Try this one without using any of the rulers you made.

   A How long is the rope if the red strip is the unit? 3 units

   B How long is the rope if the light green strip is the unit?
   Light green strip : 2 units



Follow-up

The class would probably enjoy measuring distances by counting the number of steps different children take to walk from one place to another. Organize the class into pairs or small groups of children, with one as the official recorder of data. To make the data interesting, choose the tallest child and the shortest child for one pair; two or three children of the same size for another group; etc. Tabulate the results for all the children to see, and then discuss the differences in the number of steps for the same distance. If practical, you may wish to conduct this activity in a large area such as the multipurpose room, on a paved portion of the playground, or on the sidewalk.

Also, if you began making the wall chart suggested in the previous lesson, you might want to have the children add to it by recording the length of the desk top or other objects in terms of the new units.

Resources for Active Learning

Franklin Series: *Learning About Measurement*, pp. 6, 7, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Math Activity Cards, "Lengths," A24, Macmillan.

Workbook, page 1

Using the Exercises

Direct the children to use the picture as a suggestion for drawing the lines, but caution them that the lines must be drawn across the full width of their paper. Then assign the exercises as independent work. Stress the importance of numbering the 5 lines and of labelling the segments with the letters suggested. Help any children with the labelling as needed. When the class is finished, it would be helpful for them to exchange papers to see the differences in the lengths of the line segments. Encourage a discussion of the problems that can result from using different units of measure.

Assignments (page 7)

Minimum: 1–3. Average: 1–3
Maximum: 1–4.

PAGES 8–9

Objective

Given the terms centimetre, metre, *and* kilometre, *the child will be able to interpret them in measuring and recognize them as special, or standard, units.*

Preparation

Materials

plain strips of paper or tagboard, about 15-by-5 cm; colored strips (yellow); centimetre rulers (1 per child if possible); metre stick.

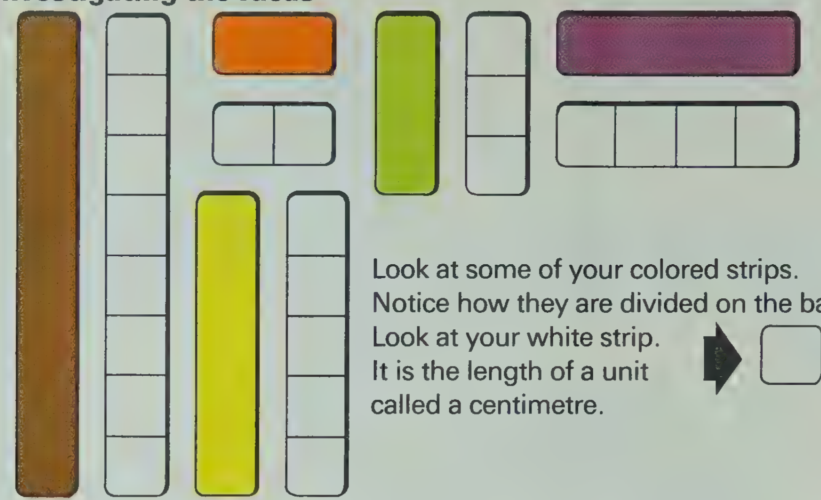
To introduce this lesson, you may wish to review briefly with the class the problems encountered in the last lesson when different units of measure were used. Then ask the children to suggest how we might avoid these problems. When it has been established that it would be a good idea for everyone to use the same unit of measure, explain to the class that in this lesson they will make a ruler that has standard unit markings. (The rulers they make should be retained for use in subsequent lessons.)

Investigation

Read the directions with the class, calling special attention to the fact that the unit they will use to mark their new rulers is called a centimetre. Though the procedure the children should use to make their centimetre rulers is the same as that used to make their rulers in the previous lesson, you will need to supervise their work carefully and give any help necessary.

Let's think about some special units.

Investigating the Ideas

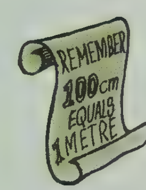


Look at some of your colored strips.
Notice how they are divided on the back.
Look at your white strip.
It is the length of a unit called a centimetre.

? Can you use the centimetre unit and a strip of paper to make a ruler?

See Investigation.

Discussing the Ideas



1. How does your ruler compare to a centimetre ruler?
   See Discussion.
2. Can you name some objects that are usually measured in
   A centimetres? B metres?
   your height football field
3. A kilometre is 1000 metres. A train of 72 boxcars is about 1 kilometre long. What are some distances that are usually measured in kilometres? Sample answer: distances between cities
4. Which unit would you choose to measure
   A a hockey arena? B a pencil? C the width of your room?
   metre centimetre metre

8

Discussion

In discussing exercise 1 with the children, take care that they are not overly concerned about the precision of the rulers they made as compared to the commercial rulers. The important point is that now they all have a measuring device that has more or less the same unit markings. With their new rulers they can all measure the same object and expect to get similar results.

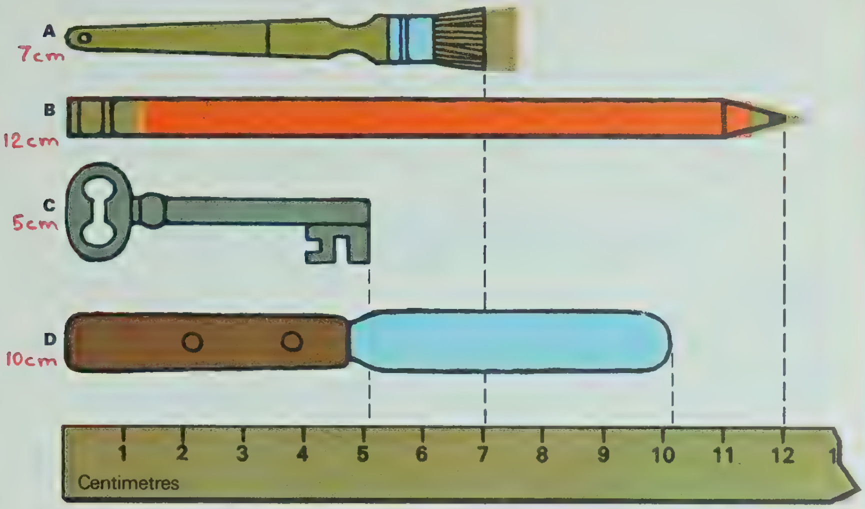
In exercises 2 and 3, have the children measure a variety of distances, using their rulers and a metre stick as they choose. These activities should help give the children a feeling for the sizes of some standard units and the way they compare to each other.

In exercise 4, the children might enjoy imagining how absurd it would be to try to measure the length of an arena with a ruler, or the distance from the earth to the moon with a metre stick. Measuring the length of the classroom first with a ruler and then with a metre stick would be instructive.

Before concluding the discussion, review the idea that the units in this lesson are standard units, units whose size people have agreed upon in order to be able to discuss such measurements as length and

Using the Ideas

1. Find the length in centimetres of each object.

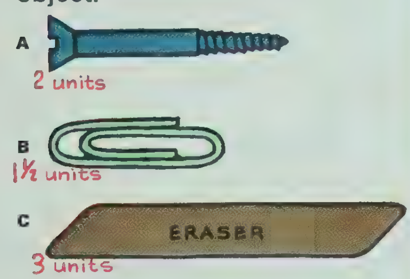


2. Use your centimetre ruler to measure each segment.

A ——————————— 8cm

B ——————————————— 10cm

★**3.** If this ├——┤ is your unit, find the measure of each object.



think

The bug went around one time. He traveled 12 units. About how long is this segment?

4 units

width in terms that mean the same thing to everyone.

Using the Exercises

Remind the children that they will need the centimetre ruler to do exercises 1 and 2. (Point out that the unit markings on rulers often do not extend all the way to each end of the ruler.) You may wish to have everyone try the starred problem.

The *Think* problem is for more capable children. If necessary, help them to see that the long sides are about twice the width, so if the total distance around is 12 units, the length of each long side must be 4.

Assignments (page 9)

Minimum: 1–2. Average: 1–2. Maximum: 1–3.

Follow-up

With the introduction of some basic units of measurement, some children might like to do some measurement reports or projects. For example, good readers might report to the class on the history of measurement, particularly regarding the units introduced here. Others might choose to report on interesting uses of measurement in various kinds of work, such as laying bricks, building bridges, or carpentry. Someone interested in sports might illustrate the lengths of various playing fields (football field, soccer field, basketball court, etc.). A child interested in aviation might illustrate lengths of airfield landing strips, heights at which planes fly, or the like.

Several books mentioned in the Books to Explore section (pages A38–A40) provide interesting information about measurement. Among them is Philip Carona's *Things That Measure.*

Resources for Active Learning

Franklin Series: *Learning About Measurement,* pp. 7–8, 12–14, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

PAGES 10–11

Objective

Given a centimetre segment and an arbitrary unit segment, the child will be able to use both units to measure length and will recognize which is the longer unit.

Preparation

Materials

colored strips (red); strips of tagboard for rulers (1 per child, 5 cm wide and at least 15 cm long); centimetre ruler

As a review of the standard units discussed in the previous lesson, you might name some familiar objects and have the children respond with the unit they would use to measure each.

Investigation

The children may find it easier to use the red-strip segment in their book as a guide in making their rulers; the strip itself might be difficult for some children to handle efficiently. Encourage the children to measure other objects in the classroom, using their red-strip ruler for each object.

Investigating the Ideas

This is a centimetre unit.

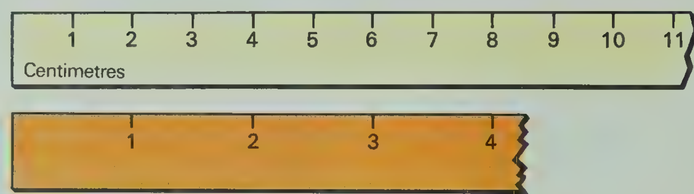
This is a unit, too.

Find one of your strips that is just as long as the second unit.

?

Can you use that strip to make a ruler ?
Now use your ruler to find the length of this brush ? 10 cm

Discussing the Ideas



1. Which is longer, the red strip or the centimetre ? red strip
2. Which ruler will give the greater number for the length of the brush ? Centimetre ruler
3. The crayon is about 4 red strips long. Measure it with a centimetre ruler.

Did you get more or less than 4 ? More (8 cm)

10

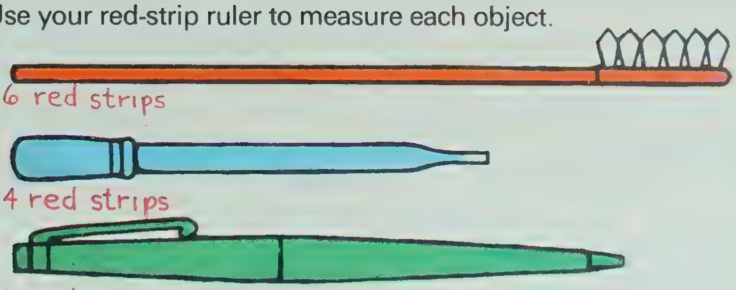
Discussion

In the investigation, the children tangibly explored the new unit introduced in this lesson – the red-strip. After you have discussed the rulers pictured above the exercises on page 10, and the exercises themselves, you might want to have the children use their centimetre rulers and red-strip rulers to compare the measurements of some other objects. Guide the children's discussion so that they clearly understand that centimetres are shorter than red strips. Encourage them to tell you that, in measuring a given length, a greater number of shorter units is needed than of longer units.

Discussion exercise 3 stresses the idea that more centimetres than red strips are needed to describe the length of an object. You might also remind the children how important it is to name the unit when they give a measurement.

Using the Ideas

1. Use your red-strip ruler to measure each object.



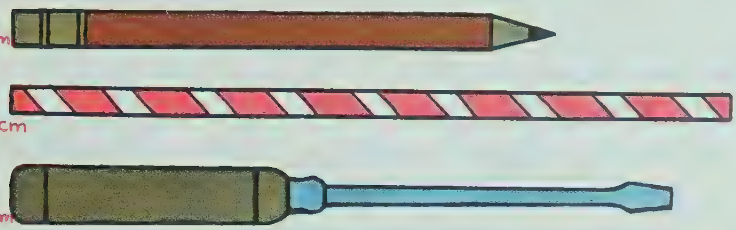
6 red strips

4 red strips

C

5 red strips

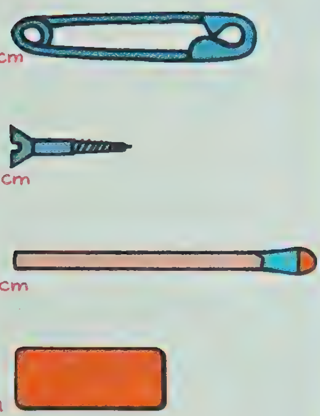
2. Use your centimetre ruler to measure each object.



A 9 cm

B 12 cm

C 11 cm



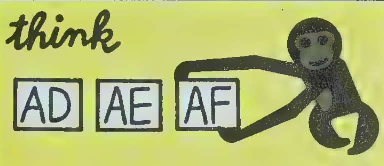
D 4 cm

E 2 cm

F 5 cm

★ G $2\frac{1}{2}$ cm

think



What comes next?

1. AA, AB, AC, AD, AE, ? AF
2. AR, BS, CT, DU, EV, ? FW
3. AZ, BY, CX, DW, EV, ? FU
4. AA, AB, BB, BC, CC, ? CD
5. AB, DE, GH, JK, MN, ? PQ
6. AB, DC, EF, HG, IJ, ? LK

11

Using the Exercises

Exercises 1 and 2 on page 11 offer good opportunities for the children to use their rulers independently. If you prefer, the children might use commercial rulers for exercise 2. When the children have finished, read the correct answers and allow time to discuss the questions.

The starred exercise (2*G*) gives the children a chance to estimate a length which is not an exact number of units. Give them credit if they say the figure measures about three centimetres. Actually, it is slightly more than two and a half centimetres.

The *Think* problem gives those who wish to try it a chance to work with patterns. You might wish to have some children explain to the class the patterns that they found.

Assignments (page 11)

Minimum: 1–2F. Average: 1–2F. Maximum: 1–2.

Follow-up

This lesson might stimulate more ideas for projects. For example, some children might invent a unit of their own, name it, and compare it to the centimetre.

Again, much of this material lends itself to illustration on a chart or a bulletin board.

Resources for Active Learning

Franklin Series: *Learning About Measurement*, p. 11, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Workbook, page 2

PAGES 12–13

Objective

Given an appropriate object to measure, the child will be able to measure it to the nearest centimetre using the half-centimetre mark as a guide. (This implies that he will also begin to recognize that measurements are not exact.)

Preparation

Materials

the centimetre rulers made in the previous lessons

To prepare for this lesson, review the terms *unit* and *half centimetre.* Have the children take out the centimetre rulers they made in the previous lessons. Review the term *unit* by mentioning a measurement made with the centimetre ruler and explaining that the unit used is the centimetre; likewise when the red-strip ruler is used, the unit is the red strip. You might review halves by writing ½ on the chalkboard and asking for examples of halves, such as a half dollar, half of a candy bar, one half of the class, etc. Then lead into the use of the half centimetre in the investigation.

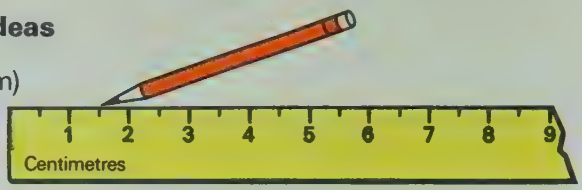
Investigation

In this investigation, the children add half-centimetre marks to the centimetre rulers they made previously. Read the directions with the class, stressing the importance of making the half-centimetre marks carefully. Then let the children mark their rulers and measure the clothespin by themselves. Give help to any children who definitely need it, but encourage the children to follow the directions and tackle the investigations independently. As the children measure other objects in the room, remind them to be sure to keep a record of the measurements, in whole numbers to the nearest centimetre.

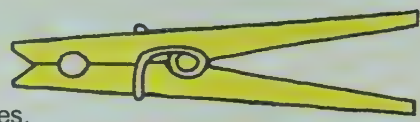
● *How can you measure to the nearest unit?*

Investigating the Ideas

Half-centimetre ($\frac{1}{2}$ cm) marks on your ruler help you find measures to the nearest centimetre.



Put half-centimetre marks on your ruler. Use it to tell whether the clothespin is closer to 6 or to 7 centimetres.



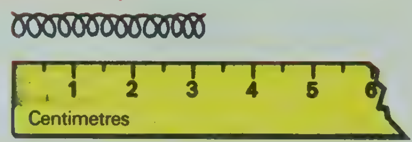
See Investigation.

? Can you measure other objects to the nearest centimetre ?	List them and record their measures.

Discussing the Ideas

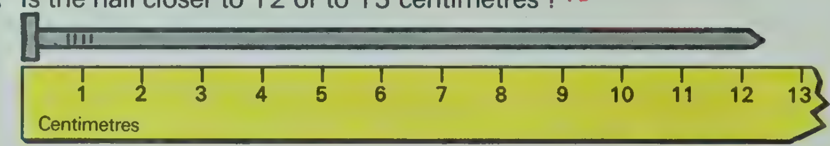
1. Explain how you could tell that the clothespin is closer to 7 cm. We say: The length to the nearest centimetre is 7.
Sample answer: It is longer than 6½ cm, so it must be closer to 7 than to 6 cm.

2. The length of the spring is **more than** 3 cm but **less than** 4 cm. Is it closer to 3 or to 4 ? 3



3. The length of the spring (to the nearest centimetre) is ▮ cm. 3

4. Is the nail closer to 12 or to 13 centimetres ? 12



5. The length of the nail (to the nearest centimetre) is ▮ cm. 12

12

Discussion

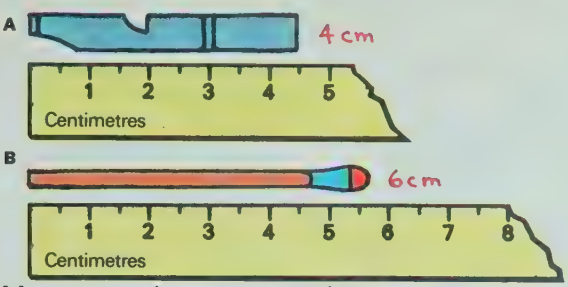
In the investigation the children were given the opportunity to use the half centimetre to measure objects to the nearest centimetre. Seeing objects in relation to half-centimetre marks makes it easier to see which centimetre the object is nearer. There may be some confusion, however, when an object appears to fall exactly halfway between two units. Explain that an agreement must be reached as to whether such a measure should be expressed as the next greater whole number of units or as the smaller whole number of units. Allow children some freedom in deciding which they want to call it, but have the children be consistent in their usage.

When you discuss the exercises, encourage the children to use the phrase "to the nearest centimetre." For example, when they give the answer to exercise 2, you might remark: "Since the spring is closer to 3, we say, 'its length *to the nearest centimetre* is 3 centimetres.'"

Continue to stress the idea that all measurements, as the children know them, are approximations.

Using the Ideas

1. Give the length of each object to the nearest centimetre.



2. Measure each segment to the nearest centimetre.

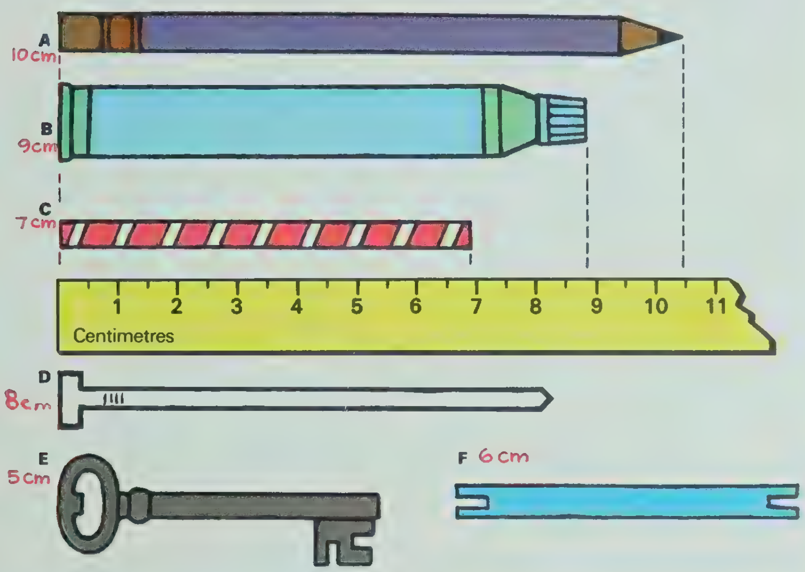
A 4cm ______ B 6cm ______

C 12cm ______

D 11cm ______

E 3cm ______ F 2cm ______ G 5cm ______

3. Give the measure of each object to the nearest centimetre.



More practice, page A-1, Set 1

Using the Exercises

Give the children a chance to do the exercises on page 13 on their own. When the children have finished, check the answers together and allow time for any necessary discussion.

Mathematics

Since the lengths of many objects are not exact multiples of the chosen unit and since all measurement is subject to error, the need for giving measures to the nearest unit or fraction of a unit is understandable. For example, saying that the length of an object is 5 to the nearest unit means that the true length is between $4\frac{1}{2}$ and $5\frac{1}{2}$. (It is closer to 5 than it is to either 4 or 6.) If the length to the nearest one-sixteenth unit is given as $3\frac{7}{16}$, the true length is at least as close to $3\frac{7}{16}$ as it is to $3\frac{6}{16}$ or $3\frac{8}{16}$.

Follow-up

Since the children have measured various objects in the room to the nearest centimetre, you might suggest that the more capable children now measure these same objects to the nearest red strip. It would be helpful for them to record both measurements side by side for each object. This list might be posted for the whole class to see.

For the children who had difficulty with this lesson, you might pass out duplicated worksheets containing pictures or line segments of appropriate lengths and have the children measure these to the nearest centimetre.

Resources for Active Learning

Franklin Series: *Learning About Measurement*, pp. 8, 9, Lyons and Carnahan. (Available from McGraw-Hill Ryerson.)

Mathex: Measurement and Estimation No. 5, pupil page 12, Encyclopaedia Britannica Publications Ltd.

Duplicator Masters, page 1

Assignments (page 13)
Minimum: 1-3. Average: 1-3.
Maximum: 1-3.

PAGES 14–15

Objective

Given an appropriate object to measure it to the nearest half centimetre.

Preparation

Materials

centimetre rulers (commercial or those made by the children in previous lessons)

A brief review of how children can use their rulers to find measures to the nearest centimetre should provide sufficient preparation for this lesson.

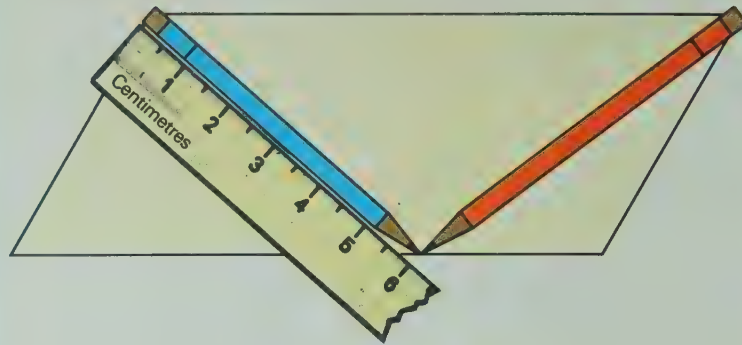
Investigation

In an investigation of this type some children will need more guidance than others. Read the directions together. Make sure the children realize that after they have made and recorded their guesses they should check their guesses by measuring each of the pencils *to the nearest centimetre.*

Most of the children will probably have guessed that both the pencils are about the same length, but the measures given to the nearest centimetre are 6 centimetres for the blue pencil and 7 centimetres for the red pencil. In discussing these results with the children, bring to their attention the fact that the red pencil is not exactly 7 centimetres in length. Ask the children if there is a mark on their rulers that is closer to the measure of the red pencil than the 7-cm mark is. Help them to see that the measure of the red pencil is closer to 6½ centimetres than it is to 7 centimetres. That is, that the red pencil measures 6½ centimetres to the nearest half centimetre.

How can you measure to the nearest half unit?

Investigating the Ideas

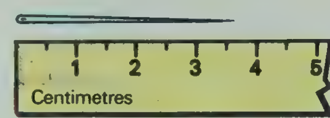


? Look at the **blue** pencil and the ruler. Which pencil do you think is longer? Use your ruler to check your guess.
Red (See Investigation.)

Discussing the Ideas

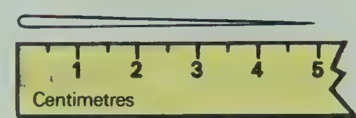
1. A Is the end of the needle closer to $3\frac{1}{2}$ or to 4? (Read $3\frac{1}{2}$ as "three and one half.") $3\frac{1}{2}$

   B What is the length of the needle **to the nearest half centimetre**? $3\frac{1}{2}$ cm

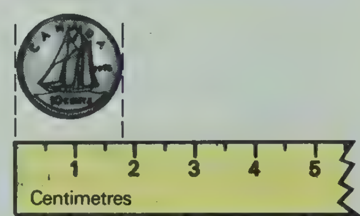


2. A Is the end of the toothpick closer to $4\frac{1}{2}$ or to 5? 5

   B What is the length of the toothpick **to the nearest half centimetre**? 5 cm



3. Does the figure show that the width of the dime is nearer to $1\frac{1}{2}$ centimetres or to 2 centimetres? 2 cm



14

Discussion

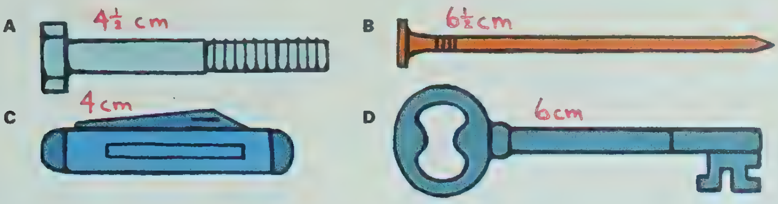
Use the investigation results to discuss measurement to the nearest half centimetre. Point out how the measure of the blue pencil is 6 centimetres and the measure of the red pencil is 6½ centimetres if we measure each to the nearest *half* unit. Commercial rulers provide marks between the half-unit marks to assist in measuring to the half unit. Direct the children to add these marks to their own rulers.

As you continue with the discussion exercises, encourage children to use the phrase *to the nearest half centimetre* in their answers. In exercise 2, help the children see that although the measure is given as the whole number 5, the measurement was made to the nearest half unit, the half centimetre. This simply points out that we know the true measure within a smaller neighborhood.

It would be helpful to present for discussion additional examples of measuring to the half unit on the chalkboard or with the aid of the overhead projector.

Using the Ideas

1. Use your centimetre ruler to measure each object to the nearest half centimetre.



2. Use your ruler to help you answer these questions. (Answer to the nearest half centimetre.)
   - A How long is a dollar bill ? 15 cm
   - B How wide is a quarter ? $2\frac{1}{2}$ cm
   - C How thick is your pencil ? $\frac{1}{2}$ cm

think

Can other things be used to measure in centimetres ?

A nickel is 2 cm wide.
How long is this page in nickels ? In centimetres ?

How wide is a dollar bill ?
Can it be used as a measuring unit ?
Check the length of this page in dollar bills.

15

Using the Exercises

Direct the children to do the exercises on page 15 on their own.

When the children have finished, allow ample time for discussion.

The *Think* problem is for more capable children, but the whole class might enjoy an explanation by the children who work out the answers.

Follow-up

Some children might enjoy using other non-standard units. They might measure objects around the room and compare the results with previous measurements. These comparisons should reaffirm that the smaller units lead to a narrowing of the measurement limits and thus to a more accurate measure.

Children who had difficulty in measuring to the nearest half unit would benefit from further practice of measuring with the standard units.

Resources for Active Learning

Franklin Series: *Learning About Measurement*, p. 10, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Mathematics in Modules, M1, Addison-Wesley.

Duplicator Masters, page 2
Workbook, page 3

Assignments (page 15)*
Minimum: 1–2. Average: 1–2.
Maximum: 1–1.

PAGES 16–17

Objective

Given a unit for area measure, the child will be able to count units and find the area of certain bounded regions.

Preparation

Materials

scissors; tracing paper (transparent); graph paper

Since the next few lessons deal with area, introduce the terms *region* and *surface*. Point out a desk or table top as an example of a flat surface or region. Also point out that this region is bounded; it does not continue indefinitely. The children may mention other examples of bounded flat regions such as a basketball or tennis court, the floor of the classroom, and so on. Then direct them to the region they will trace and cut in the investigation.

Investigation

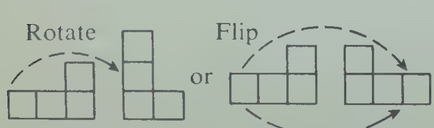
As you read the directions with the children, point out that the region they will trace is a square and that the four regions they will cut will be squares.

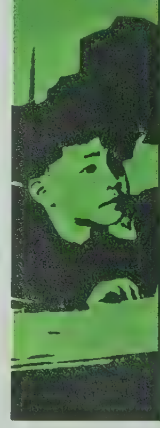
Before the children begin, it would be helpful to make a few ground rules, such as not to use shapes like the following:

Agree that to be an acceptable shape each square has to have one side in common with at least one other square, and squares that touch must touch along the entire side. Allow plenty of time for children to discover the shapes and draw them.

Following are the shapes the children may discover:

If a child suggest that flips or rotations of the same shape are different, have him cut out the two shapes and see if they are the same. For example:

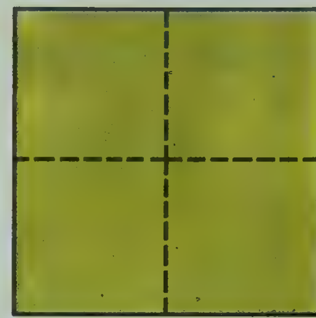




What does it mean to find area?

Investigating the Ideas

Trace this region and cut it into four squares.



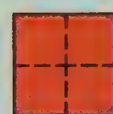
?

How many different-shaped regions can you make using the 4 squares? Draw a picture of each one you find. See Investigation.

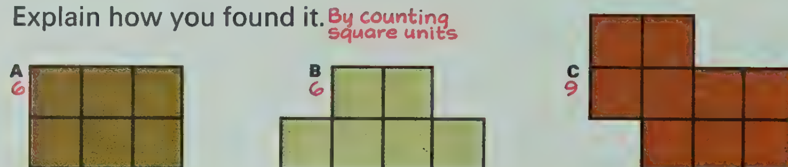
Discussing the Ideas

1. We find the **area** of a region by counting **square units**.

IF	—— is the unit	THEN	this length ———— is ▥. 4
IF	■ is the unit	THEN	this region has an area of ▥. 4



2. What is the area of each region you found in the Investigation? 4 sq units
3. If each small square is a unit, find each area. Explain how you found it. By counting square units



16

Discussion

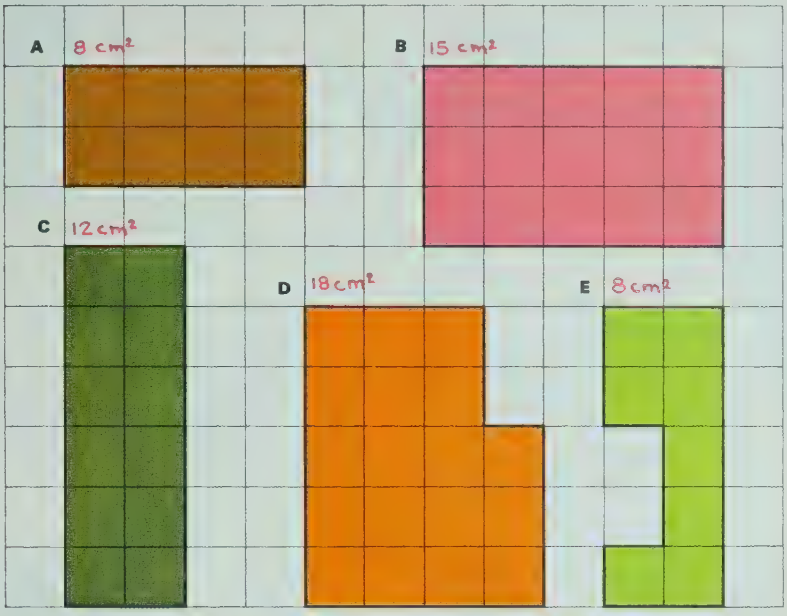
The discussion of exercise 1 should emphasize this important point: just as length is found by counting line segments of unit length, area is found by counting squares of unit area. A common square unit is the *square centimetre*, which is introduced in the exercises on page 17.

To enhance discussion of exercise 2, it would be helpful for a few children to sketch on the chalkboard some regions they discovered with the squares, or you might have them place 4 squares on the flannelboard in the various arrangements. Although the regions have different shapes, counting reveals that the number of squares in each region, that is, the area, remains the same.

Exercise 3 reinforces the main idea of counting square units in order to find area.

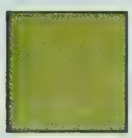
Using the Ideas

1. The unit in these exercises is the square centimetre.
Find the area of each shaded region.

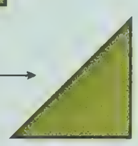


2. Draw a region that has an area of 6. Any region of 6 cm²
Use the same unit as in exercise 1.

★ **3.** If the area of this region is 1, →



what is the area of this region? 1/2 →



17

Follow-up/"Find the Area"
Give each child a piece of graph paper (commercial or duplicated). Tell everyone to design at least six figures by marking the grid lines and lightly shading the inner regions with a crayon. Then instruct everyone to trade papers with a classmate and try to find the area of each figure on his classmate's paper by counting the number of units. It is a good idea to require two ground rules for this session to help eliminate figures too difficult for the children at this time:
1. All lines must be straight lines.
2. Each child must be able to find and verify the area of all the regions he designs.

Resources for Active Learning
Franklin Series: *Learning About Measurement,* "Measuring Area," pp. 20–25, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)
Franklin Series: *Pencil and Paper Geometry,* "Area," pp. 75–78, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)
Math Activity Cards, "Area: Tiles, Grid," A 40, 41, Macmillan.
Mathematics in Modules, M6, Addison-Wesley.

Using the Exercises
Have the children do the exercises on page 17 on their own. Point out that the unit for area measure in these exercises is the square centimetre. Have graph paper available for exercise 2, even though some children may choose to trace the region from the grid in the book. If time permits, you might like to develop exercise 3 in discussion; however, the concepts involved here will be treated in the next lesson.

Assignments (page 17)
Minimum: 1. Average: 1–2.
Maximum: 1–3.

PAGES 18–19

Objective

Given nonrectangular regions bounded by straight lines, the child will be able to use halves of a square unit to find the area.

Preparation

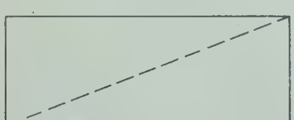
Materials

scissors; graph paper

Remind the children that in the previous lesson they found area by counting square units. Then display a square and a rectangle and ask the children to describe the difference. Be sure they see that in a square all four sides are equal in length and in a rectangle the opposite pairs of sides are equal in length but the sides which meet at a corner are not. Explain that in this investigation they will use both of these shapes.

Investigation

In this investigation the children have the opportunity to experiment with rectangular regions. There are innumerable ways to fold both the square and the rectangle into 2 parts of equal size. However, it is the rare child who will realize this. Just encourage each child to find as many ways as he can, and praise him for his efforts. Note that in the following picture the rectangle folds into two equal-size sections even though one section does not fall exactly upon the other.

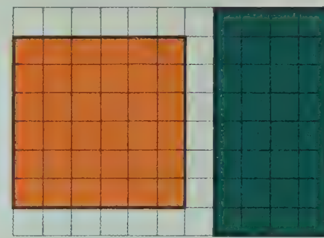


If possible, you might have some children do this investigation by finding possible ways of dividing a square and a rectangle on the geoboard.

● *Let's explore using fractions in finding area.*

Investigating the Ideas

Use graph paper to cut out a square and a rectangle.

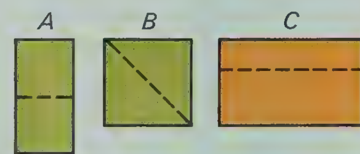


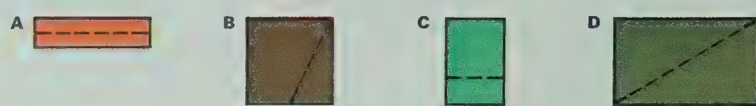
? How many ways can you fold each figure into two parts of the same size?

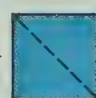
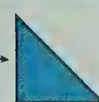
See Investigation.

Discussing the Ideas

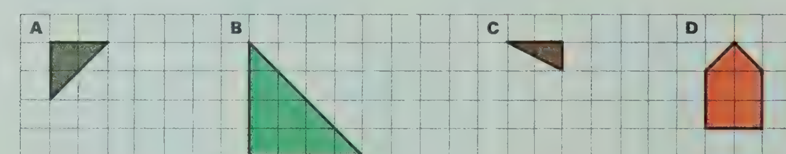
1. Regions *A* and *B* are divided into **halves**. Region *C* is not divided into **halves**. Which regions below are divided into halves? A and D





2. If this region has an area of 4,  what is the area of this region?  2

3. Explain how to find the area of each shaded region. Each small square is 1 unit.



2 sq units | 8 sq units | 1 sq unit | 5 sq units

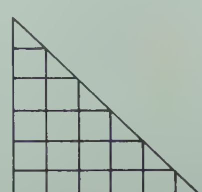
18

Discussion

One of the main purposes of this investigation is to help children realize that halves of rectangular or square regions can be used to help determine the area of triangular regions.

Ask children to share the ways of folding that they discovered in the investigation. Then guide the discussion of exercise 1, stressing the fact that 2 parts of a region must be the same size in order for them to be halves.

As an adjunct to the discussion of exercises 2 and 3, you might wish to direct attention to the squares and rectangles the children folded for the investigation. Instruct the children to cut carefully along a diagonal of the square they folded. Their result will be a triangle similar to this.

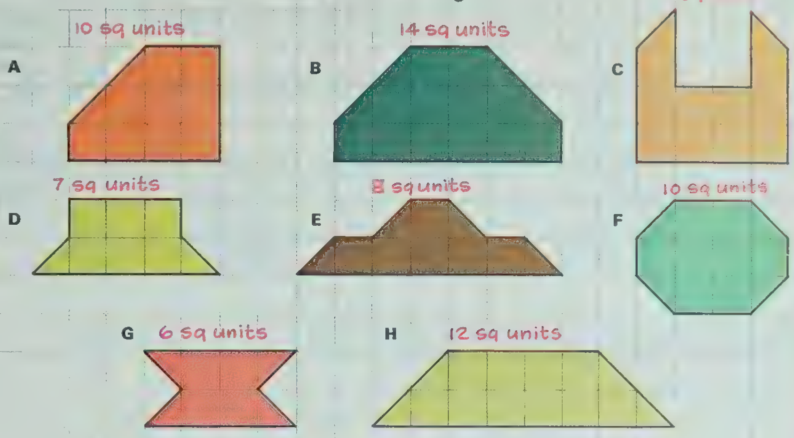


Ask the children to explain how to find the area of this region. If necessary, suggest that 2 of the triangles would make 1 square.

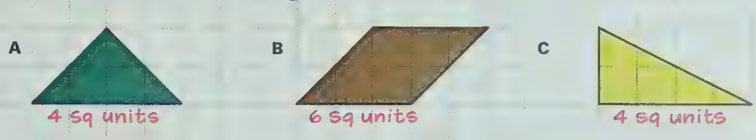
It would be helpful to use the chalkboard or the overhead pro-

Using the Ideas

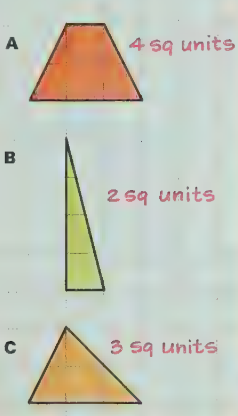
1. The units for these exercises are marked with gray lines. Find the area of each shaded region.

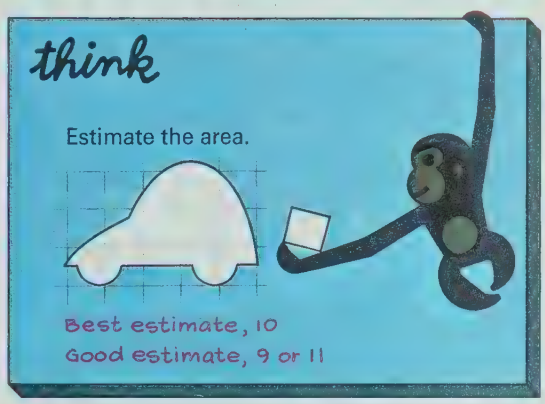


2. Find the area of each region.



★ **3.** Find the area of each region.





More practice, page A-2, Set 3

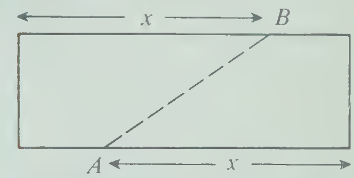
19

jector to demonstrate how to find the area of regions such as those pictured in exercise 3. Throughout the discussions, it is important to stress counting units to find area and matching 2 halves together to count them as 1 square.

Using the Exercises
If necessary, work through the first few exercises on page 19 together. When the children have finished the exercises, give them an opportunity to state or demonstrate the way they found the given area. Some children may need to cut apart graph paper on which they have reproduced the designs and actually manipulate the parts of the regions to verify some of the areas.

Assignments (page 19)
Minimum: 1–2. Average: 1–2.
Maximum: 1–3.

Mathematics
It was stated earlier that there are innumerable ways to fold the square and rectangle into 2 parts of equal size. To see this, consider the following:



Any rectangle cut along a segment such as segment AB will be divided into two parts of equal size provided the two distances marked x are the same length. There are infinitely many choices for the length x; hence, there are infinitely many ways of folding the rectangle into two parts that have the same size.

Follow-up/Parquet Designs
Provide each pupil with sets of poster board squares and triangles (half squares made by cutting squares on the diagonal to form two congruent triangles), rubber cement, and a large sheet of colored construction paper. The squares, about 5 by 5 cm, should be cut from brightly colored poster board. Give a child squares and triangles in varied patterns. When they make one or several patterns that are pleasing, instruct them to glue the triangles and squares to the poster board, and to write the area beside each figure. Let the children choose the designs they like best for a bulletin board display on the area theme. (Also, it would be appropriate to display on the same board some of the different ways of folding that the children discovered in the investigation.)

Resources for Active Learning
Developmental Math Cards, D18, Addison-Wesley.
Franklin Series: *Learning About Measurement*, "Measuring Area," pp. 20–25, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)
Inquiry in Mathematics via the Geoboard, "Area," Geo-Cards 11/1-5, Walker. (Available from Fitzhenry & Whiteside)

Duplicator Masters, page 3
Workbook, page 4

PAGES 20–21

Objective

Given simple objects and basic geometric shapes, the child will be able to use halves and fourths in finding area and measuring length.

Preparation

Materials

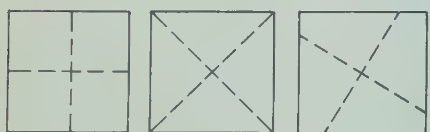
paper for tracing and cutting; scissors

Since this lesson immediately follows a similar lesson involving folding, you may wish to begin at once with the investigation in the text.

Investigation

As in the preceding lesson, the children are given the opportunity to manipulate regions. Here the activity is directed toward developing an understanding of what we mean by *fourths.*

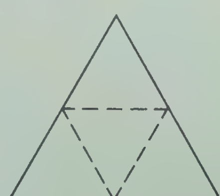
Again, only the unusual child will see that there are innumerable ways to fold the square into four equal parts (as suggested by the figure at the right, below). Most of the children, however, will discover on their own the folds which yield the four smaller squares or the triangles.



Since the chief purpose of this investigation is simply to give the children a feeling for the idea of fourths, you may prefer to avoid mentioning the more complex folds.

For the circle, on the other hand, you will probably find that most of the children will discover that the number of possible folds is limitless. Do not expect verbal generalizations about this fact, but be sure the children see that the folds all must include the centre point.

Folding the equilateral triangle into four regions of the same size and shape will be a challenge to the children. Again keeping in mind the central purpose of the activity, you should not attach importance to the success or failure of the children's efforts to find the correct folds (shown below).



Children who do find the proper folds should be commended, but those who do not should not be allowed to feel discouraged.

Allow the children plenty of time and opportunity to experiment with a variety of ways of folding and to share their results.

Discussion

Exercise 1 stresses the idea of halves in measurement of length. It would be helpful for the children to use their centimetre ruler and refer to the half-centimetre marks. Recall with them that the length of the paper clip and pin are given "to the nearest half centimetre." Since the clip and pin align with a half-centimetre mark, tenths are not shown as guidelines, and need not be mentioned. Exercise 2 uses fractions to find the area of parts of a region. In this connection, emphasize again that in order for four parts to be fourths they must be equal in size.

How can you use halves and fourths in measurement?

Investigating the Ideas

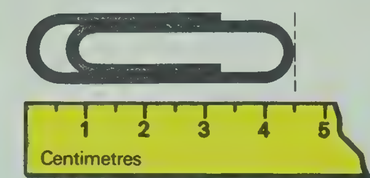
Trace each region on paper and then cut it out.

? Can you fold each region into 4 parts that are the same size and shape?

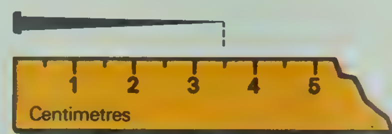
See Investigation.

Discussing the Ideas

1. The centimetre ruler has been divided into **halves**.

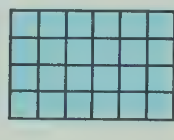


The length of the paper clip is $4\frac{1}{2}$ centimetres.



What is the length of the pin? $3\frac{1}{2}$ cm

2. A Find the area. 24 sq units



B If you fold the rectangle into fourths, what is the area of each fourth? 6 sq units

C What is the area of $\frac{3}{4}$ of the region? 18 sq units

20

Using the Ideas

1. Find each length to the nearest half centimetre.

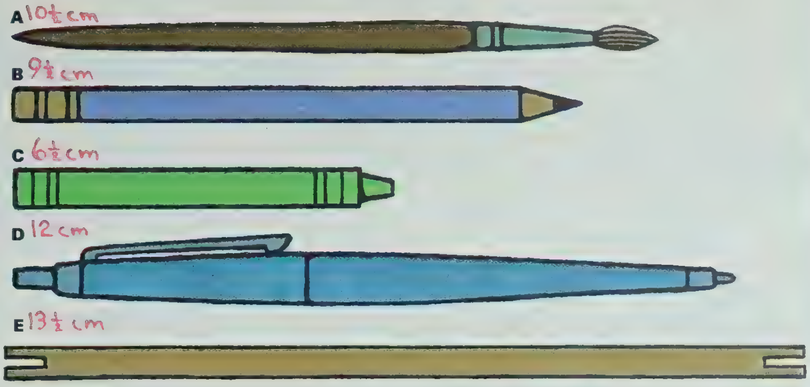
A $10\frac{1}{2}$ cm

B $9\frac{1}{2}$ cm

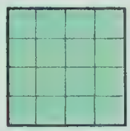
C $6\frac{1}{2}$ cm

D 12 cm

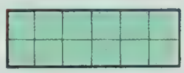
E $13\frac{1}{2}$ cm



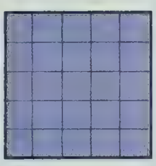
2. A Find the area of this square. 16
   B What is the area of $\frac{1}{2}$ of it? 8
   C What is the area of $\frac{1}{4}$ of it? 4



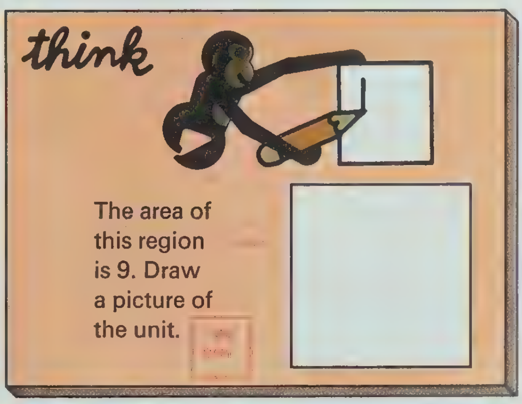
3. A Find the area of this rectangle. 12
   B What is the area of $\frac{1}{2}$ of it? 6
   C What is the area of $\frac{1}{4}$ of it? 3
   ★ D What is the area of $\frac{3}{4}$ of it? 9



4. A What is the area of this square? 25



   ★ B What is the area of $\frac{1}{2}$ of it? $12\frac{1}{2}$
   ★ C What is the area of $\frac{1}{4}$ of it? $6\frac{1}{4}$



21

Follow-up
An interesting and worthwhile activity is to have everyone trace a hand or foot on graph paper and then try to figure out the area it covers. Since the traced figures will not divide the unit squares evenly, the children will have to approximate the area as best they can. Children will enjoy seeing the drawings of their feet and hands displayed around the room.

Resources for Active Learning
Developmental Math Cards, F¹16, F²18, Addison-Wesley.
Math Activity Cards, "Halving," B17, Macmillan.

Workbook, page 5

Using the Exercises
Assign the exercises on page 21 according to the needs and capabilities of the class. You might like to work through part of exercise 1 together. The children may use the centimetre ruler they made in the lesson on page 8. For exercises 2 and 3 some children may need to cut apart graph paper on which they have reproduced the regions and manipulate the parts of the regions to verify some of the areas. This method might be particularly helpful for the starred exercises although these are intended primarily for more capable children.

If anyone has difficulty with the *Think* problem, you might want to give him a hint to measure the sides with a centimetre ruler.

Assignments (page 21)
Minimum: 1–3. Average: 1–4A.
Maximum: 1–4.

PAGES 22–23

Objective

Given a simple region, such as a rectangle, triangle, or circle, the child will be able to identify a fractional part of the region by giving a fraction to describe the part.

Preparation

Materials

scissors; paper to be cut into strips and used for folding

To prepare the class for this lesson review simple geometric figures (square, rectangle, circle, triangle). It would also be appropriate to review the term *fraction*. You might do both by displaying a square, a rectangle, and a circle. Then fold one of the figures in half and write "$\frac{1}{2}$" on the chalkboard. Continue similarly for $\frac{1}{4}$ and explain that $\frac{1}{2}$ and $\frac{1}{4}$ are called fractions. In this preparatory activity, use only halves and fourths; other fractional parts will be introduced in this lesson.

Investigation

You might want to read the investigation exercises with the class. However, let each child do his own thinking and let him record his answers for 1 and 2 independently. Allow the children to struggle (if need be) with the investigation question rather than show them what to do. You might ask those who finish quickly to try folding the strip so that each part is $\frac{1}{16}$ or $\frac{1}{32}$. Or have them investigate folding the paper into thirds or fifths.

Let's find out more about fractions.

Investigating the Ideas

Give the missing fraction.

1. If you fold like this, each part is ▒ of the paper. ½

2. If you fold like this, each part is ▒ of the paper. ¼

? Cut out a strip of paper. Can you fold it so that each part is $\frac{1}{8}$ (one eighth) of the paper?

3 folds needed (See Investigation.)

Discussing the Ideas

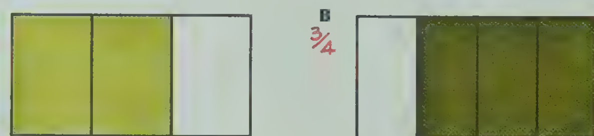
1. Explain what you would do if you wanted to color $\frac{3}{8}$ (three eighths) of the paper you folded.
   Sample answer: Color 3 of the 8 parts shown by the fold lines.

2. A Give the fraction that tells what part of this strip is colored. 5/8

   B What part of the strip is not colored? 3/8

3. What part of each region is shaded?

A 2/3 B 3/4



22

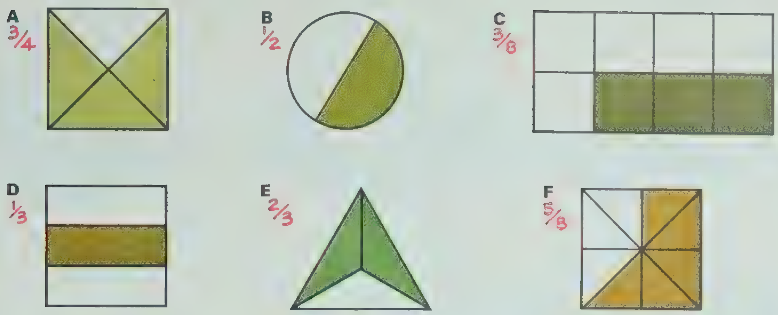
Discussion

When the children have folded the strip from the investigation into eighths ask someone to sketch on the chalkboard the way his strip looks. Stress the fact that since each of the eight parts has the same size, each is called one eighth.

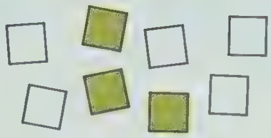
As you discuss the exercises, help the children to read the fractions carefully, saying "five eighths," "two thirds," etc. It would be helpful to develop the idea that the terms eighths and thirds tell us into how many parts the region has been divided. For example, in exercise 2A you might say something like this: "The whole strip has been divided into 8 parts or eighths. Five of these parts have been colored, so we say '$\frac{5}{8}$ (five eighths) of the strip is colored.'"

Using the Ideas

1. Give the fraction that tells what part of each region is colored.

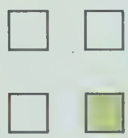


2. Jim folded a rectangle into eighths. He colored part of it and then cut it into squares like these.



   A What fraction tells the part of the rectangle he colored? 3/8
   B What part of the set of squares is not colored? 5/8

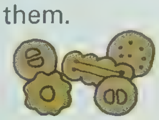
3. Give the fraction that tells what part of this set of squares is colored. 1/4



Short Stories

1 Had 6 baseball cards. Gave away $\frac{1}{2}$ of them. How many left? 3

2 Store had 6 lollipops. Bought $\frac{1}{3}$ of them. Bought how many? 2



3 8 children. $\frac{1}{4}$ of them wear glasses. How many wear glasses? 2

4 12 cookies. $\frac{1}{4}$ of them are chocolate. How many are chocolate? 3

More practice, page A-3, Set 4

Using the Exercises

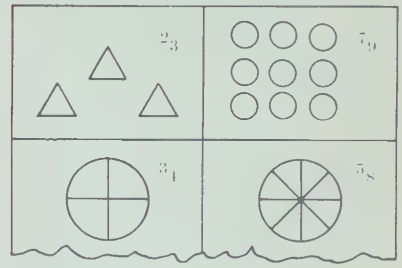
You may assign exercise 1 on page 23 as independent work. Exercise 2 introduces the concept of fractional parts of a set. You might want the children to do what the problem describes that Jim did. After coloring and then cutting a rectangle into eighths, the problem can be viewed as one of identifying how many parts of a set are colored rather than what fractional part of a region is colored. A discussion of this concept would be helpful before the children continue with exercise 3 and the Short Stories.

Suggest that the children draw the sets specified in the stories and then outline groups of the same size or partition sets of objects into subsets of the same size. By grouping the objects in this way, the children should be able to answer the questions in the Short Stories independently.

Assignments (page 23)
Minimum: 1-3. Average: 1–3, short stories. Maximum: 1–3, short stories.

Follow-up

You might want to provide the children with duplicated worksheets and ask them to color enough objects so that the illustration represents the indicated fraction.



You might also create some short stories like those on page 23 and ask the children to draw pictures on the chalkboard to illustrate the information given in each story. Then have them partition and shade the sets to show what happens when the conditions of the problem are met.

Sample short stories to use for such an activity follow.

1. 8 sandwich cookies.
   $\frac{1}{2}$ are chocolate.
   How many are chocolate?

2. 8 jacks.
   $\frac{1}{4}$ of them are red.
   How many red jacks?

3. 6 children.
   $\frac{1}{3}$ of them are girls.
   How many girls?

Resources for Active Learning
Developmental Math Cards, E-10, Addison-Wesley. [A way to investigate the meaning of fractions]

Duplicator Masters, page 4
Workbook, page 6

PAGES 24–25

Objective

Given pictorial representations of special figures made up of cube units, the child will be able to count the cube units to find the volume of such special figures.

Preparation

Materials

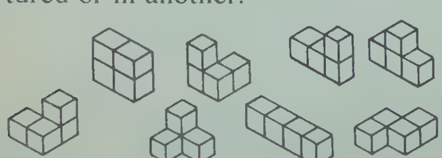
2-cm cubes, blocks, sugar cubes, or any substitute set of cubes that are of uniform size (minimum: 4 per child; preferred: 12 per child)

To prepare the children for this lesson, remind them of the kinds of measuring they have been studying, namely, length and area. Then spend not more than two or three minutes posing questions such as: "Have you ever wanted to know how much your desk (or closet, drawers, etc.) would hold? How would you find out?" Explain that in this lesson they will study what it means to find *volume*.

Investigation

One of the important points in this investigation is to help children visualize pictorial representations of three-dimensional objects. If you do not have at least 12 blocks per child, let the children figure out that the best way to answer the first question is to work together and build with their blocks the shapes pictured in A, B, and C, and then count the number of blocks they used. Even if the children work the first question in groups, have them work on the second question individually. You will probably want to make ground rules to preclude shapes like these:

Such rules might be: (1) each cube must have one face in common with at least one other cube; (2) faces that touch must touch fully. Below are shapes which the children may find, either in the position pictured or in another.

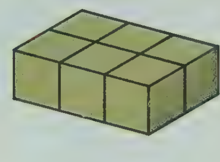


What does it mean to find volume?

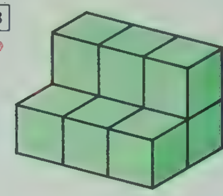
Investigating the Ideas

How many blocks does it take to make each figure?

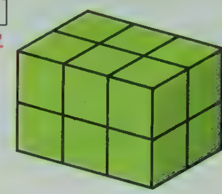
A 6



B 9



C 12



? How many different-shaped figures can you make using four blocks?

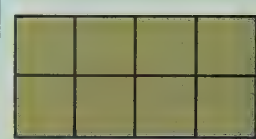
See Investigation.

Discussing the Ideas

1. Give the missing numbers. Explain your answers.

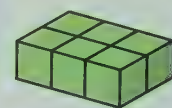
A IF this is the unit THEN this length is ▥. 6

B IF this is the unit THEN



this area is ▥. 8

C IF this  is the unit THEN



this volume is ▥. 6

2. What is the volume of each figure in the Investigation? 4

24

Discussion

The first discussion exercise stresses the idea that, like length and area, the concept of volume involves choosing an appropriate unit and counting. You might have the children try this exercise on their own before discussing it.

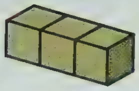
In discussing exercise 2, stress the idea that, although the figures that were constructed with the four blocks had different shapes, each had the same volume.

It would also be helpful to discuss the volume of figures A, B, and C illustrated at the top of page 24. Help children realize that when they consider the volume of such figures they must imagine the cubes that are hidden.

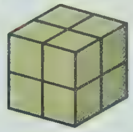
Explain the term *cube* in reference to the blocks or other materials the children used, telling them that a cube is a block whose edges all have the same length. Though usage is the best way for third-graders to grasp the meaning of the terms length, area, and volume, you might clarify their understanding by explaining each in terms of the questions they answer. (Units of *length* answer "How *long* is it?" Units of *area* answer "How much *surface* or *region* is there?" Units of *volume* answer "How much *space* does it occupy?")

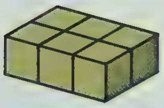
Using the Ideas

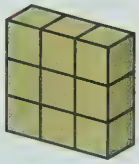
The unit used in these exercises is  . Find the number of cubic units (volume) in each figure.

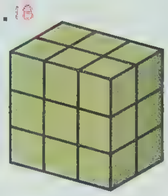
1. 3


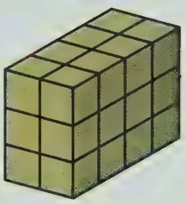
2. 4

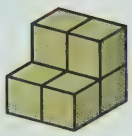

3. 8


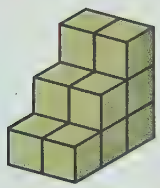
4. 6


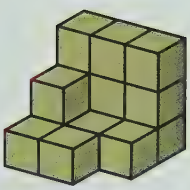
5. 9


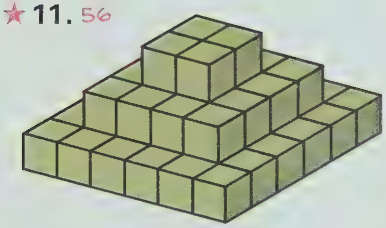
6. 18


7. 24


8. 6


9. 12


★ 10. 15


★ 11. 56


25

Using the Exercises

Give the children sufficient time to work the exercises on page 25 on their own. When they finish, let them give the volume of the figure in each exercise and provide cube demonstrations to verify their results.

Though exercises 10 and 11 are starred, make a game of trying to figure out the volume. Let small groups of children try to reproduce the stacks to verify their guesses. Some of the children may need to use a set of small blocks or sugar cubes in order to solve these problems.

Assignments (page 25)
Minimum: 1–6. Average : 1–9.
Maximum: 1–11.

Follow-up

To help children improve their perception of spatial relationships, it would be worthwhile for them to continue the type of activity suggested in the investigation. For example, you might arrange blocks in various arrays and have the children guess the number of blocks in each stack.

If you prefer a different type of follow-up, the children might make a chart listing objects which they could measure in terms of length, area, or volume.

Length	Area	Volume
rope	playground	room
necklace	floor	box
pencil	front yard	desk drawer
⋮	⋮	⋮

You may want to challenge some children by giving them a variety of different-sized boxes and asking them to figure the approximate volume with the help of the cubes.

You might also ask: "What is the volume of your desk?" "How much bookshelf space is there?" "How many cubes (2-cm) would be needed to fill a box that is 20 cm long, 20 cm wide, and 20 cm high?"

Resources for Active Learning

Developmental Math Cards, $F^3 7$, Addison-Wesley.

Franklin Series: *Learning About Measurement,* "Volume," pp. 54–59, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Math Activity Cards, "Volume: Cubes," A 38, Macmillan.

Mathex: Measurement and Estimation No. 5, "Introducing Standard Units," pp. 13–14, Encyclopaedia Britannica Publications Ltd.

Workbook, page 7

PAGES 26–27

Objective

Given word problems involving liquid measurement, the child will be able to work with litres to solve them.

Preparation

Materials

bucket of dry substance suitable for pouring, or several litres of water; containers of liquid measure: various-sized containers, from 50 ml to 2 *l*; soda pop bottles; funnel (if available)

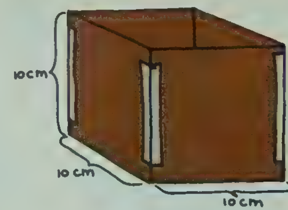
To introduce this lesson, talk about the different kinds of measurement the class has already studied. Then pour a small amount of water from one glass to another, and ask if anyone can tell you how much water you poured. Let the class decide whether centimetres, square units, or cubes would help. Then explain that this lesson will help them answer a question like "How much water was poured?"

Investigation

Give the children an opportunity to work in groups and investigate the volume unit shown here by using water or sand and actual containers. Though the measuring is carried out as a group activity, you should suggest that each child make his own liquid measure table. Afterward, allow time for the children to compare their findings to be sure the data in their tables are in agreement.

Investigating the Idea

A litre is the unit used to measure volume.
A litre would just fill a container this size —



?

Make a collection of containers that hold about 1 litre. Can you find any that hold 2 litres ? more than 2 litres ? half a litre ?
Answers will vary.

Discussing the Idea

Suppose you collected and labelled these 5 containers. Knowing their volumes try answering these questions.

1. How many jugs would be needed to fill the can ? 2

2. If you empty the can into jugs and cartons, and have only 1 jug, how many cartons will be needed ? 2

3. About how many bottles would fill the bag ? 6

★4. What containers would you use to hold the contents of the bag
   - A if you could use 2 ? a carton and a jug
   - B if you could use 3 ? 3 cartons or a jug and 2 bottles
   - C if you could use only 1 container ? the can

26

Discussion

As you work through the discussion exercises, stress the idea that a greater number of smaller units are needed to hold the same amount that can be held by one larger unit.

In exercise 4, help the children to see that 1 carton and 1 jug will hold the contents of the plastic bag or that 3 cartons will also hold the contents of the bag. However, if only 1 container can be used, it must be the can. It will not be full but it will hold the 3 litres from the bag.

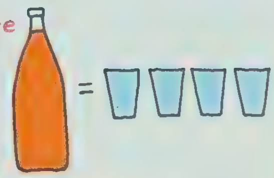
Using the Ideas

Shopping Problems

Sally's mother asked her
to go to the store.
Here is her shopping list: ⟶

Shopping List
Ice Cream
Bread (2 loaves)
Milk (2 litres)
Eggs (1 carton)
Orange Soda
Grape Drink (3 litres)

1. Sally picked up 3 half-litre cartons of milk. How much more should she get ? 1 half litre



2. Sally knew that the orange soda was for her little brother's birthday party. If each of the 6 boys at the party drinks 2 glasses of orange soda, how many bottles should Sally buy ? 3

3. Sally bought a carton of 6 half-litre bottles. How many litres did she buy ? 3

4. Which should Sally buy to save money — the 3-litre jug or the 3 one-litre bottles ? the 3-litre jug

5. One container of ice cream holds enough for 4 sundaes. There will be 6 boys at the party. How many containers should Sally buy ? 2

27

Follow-up
It would be interesting for the children to compare the amounts of liquid contained in familiar pop bottles with the standard containers. If you have a variety of bottles or cans, the children might work in groups–some in each group measuring and pouring, others recording the comparisons in chart form.

Resources for Active Learning
Developmental Math Cards, D²9, Addison-Wesley.
Franklin Series: *Learning About Measurement*, "Liquid Measure," pp. 60–65, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)
Math Activity Cards, "Volume: Bottles, Order," A36, 37, Macmillan.
Mathematics in Modules, M10, M12, Addison-Wesley.
Mathex: Measurement No. 10, "Volume," pp. 18–21, Encyclopaedia Britannica Publications Ltd.
Math Workshop: Games and Enrichment Activities, "Three Boxes," pp. 10–11, Encyclopaedia Britannica Educational Corp. [A game of logical reasoning through "If-then" situations]

Duplicator Masters, page 5
Workbook, page 8

Using the Exercises
For page 27, call on several children to take turns reading aloud the shopping list and then the exercises. Give everyone an opportunity to ask questions and talk about the problems, but avoid providing specific answers to the exercise questions. After they have finished, ask volunteers for the correct answers so that everyone can check his paper.

Assignments (page 27)*
Minimum: 1–4. Average: 1–4.
Maximum: 1–5.

PAGES 28–29

Objective

The child will demonstrate his ability to work with the concepts developed in this chapter.

Preparation

Materials

centimetre rulers (one per child); graph paper; scissors; blocks (for use as needed by some children)

Review the key topics of the chapter, giving the children an opportunity to discuss length, area, and volume. According to class need, provide demonstrations to review all three types of measurement. Include discussion of ideas such as the concepts that the smaller the unit used, the larger the number to express the measure; and that measurement is an approximation.

Provide graph paper and scissors and blocks or cubes for less capable children to manipulate.

Reviewing the Ideas

1. Bob takes steps about this long.
   Sue's steps are just half as long.
   A Who will take the most steps to cross the room? Sue
   B Who takes the longer step? Bob
   C If Bob takes 10 steps to cross the room, how many steps will Sue take? 20
   D If Sue takes 12 steps from the door to the teacher's desk, how many steps will Bob take? 6

2. A Give the length of each segment using the centimetre as your unit.
   15 cm (15 cm), 3 pentacents
   11 cm ($11\frac{1}{2}$ cm), 2 pentacents
   B Measure each segment to the nearest half centimetre. See above
   ☆ C Ted called his unit the pentacent ——— Pentacent
   Give the measure of each segment above, using the pentacent as your unit. Use your ruler if you like. See above

3. Does the pentacent or the centimetre give the larger number when you measure the same object? the centimetre

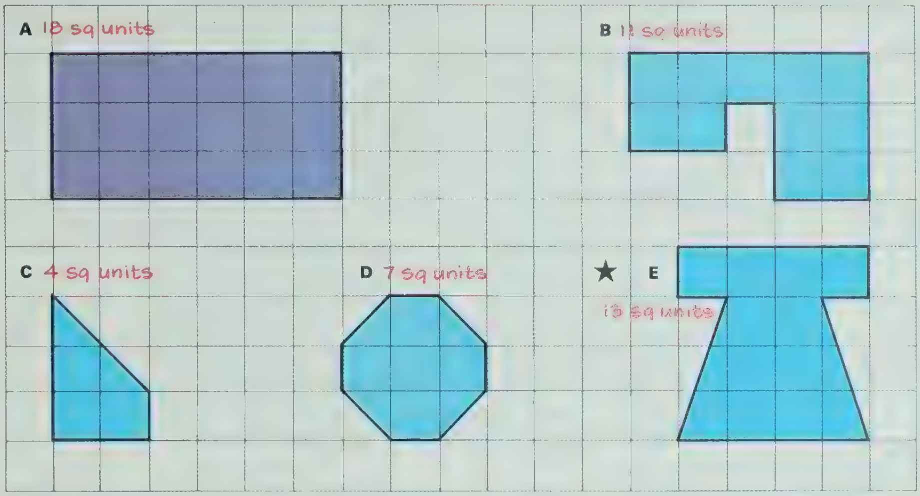
4. A Is 4 centimetres longer than 1 pentacent? No
   B Is 11 centimetres longer than 2 pentacents? Yes

28

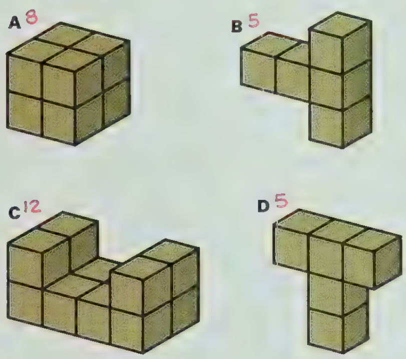
Discussion

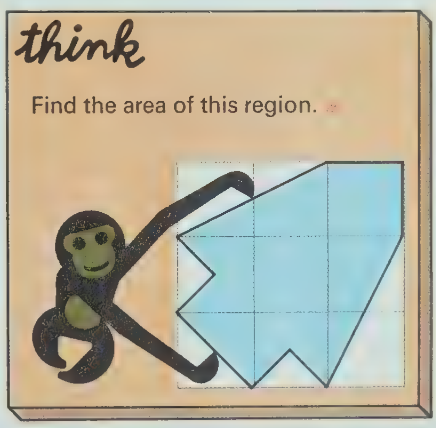
You may choose to use these pages as a review of the chapter or as an evaluation page. Follow up the exercises with a carefully planned discussion to strengthen any of the concepts or topics of the chapter which may have caused difficulty for the class. Stress those concepts mentioned in the preparation comments. Encourage the children to ask questions about those problems that caused difficulty.

5. Using the unit shown, give the area for each region.



6. Give the volume of each figure below.





29

Follow-up

As a culmination of the chapter you might let the children work on one of the activities mentioned previously which they enjoyed, or you might choose among these:

1. If the children are interested in designing figures and finding their areas, pass out graph paper and encourage experimentation with units and parts of units. More capable children may enjoy copying outlines of favorite cartoon characters. Show them how to put graph paper over a page from a coloring or comic book and tape this to a window to trace. When they finish tracing, they can color each figure and estimate its area, according to the units of the grid.
2. Pose several measuring situations and ask the children to decide whether length, area, or volume is to be measured, and what kind of units are appropriate for the task.
3. Have some children measure a surface or length, using for units a specified group of everyday items, such as scissors, a length of string, a twig. The measurement tools may even be limited to such odd units as a candy bar wrapper or a piece of newspaper.
4. For more capable children, pose the problem of measuring a curve, such as the circle in the centre of the gymnasium floor or the edge of a round or oval table. Tell them to choose a unit and any tools they can find at hand; but stress that they must devise a way to measure as accurately as possible and be able to demonstrate the unit, the technique of measuring, and the approximate length. No group should use a unit that another group has chosen. If the groups have difficulty getting started, give them a box containing such things as yarn, nonstretchable string, a cloth tape measure, a steel tape measure, masking tape, a wheel, and adding-machine tape; but give no hint about which device to use or how to use it.

Workbook, pages 9, 10

General Objectives

To review sets, numbers, and numerals
To provide a thorough understanding of place-value concepts
To introduce hundreds, thousands, *and* millions
To review and extend work with inequalities

A clear understanding of place value is one of the most important objectives in this mathematics program. Without such understanding, there is no hope of teaching the algorithms of arithmetic and decimal notation with any comprehension.

We begin Chapter 2 by having children investigate grouping sets of objects by tens, and then using these sets and two-digit numerals to review basic place-value ideas. Pennies and dimes are used to reinforce place-value concepts. Following this review, *hundreds* and *thousands* are introduced and carefully explored. The children are given many opportunities to work with place-value concepts in identifying the number represented by a given digit in a symbol and in working with inequalities. When a child is asked to write the symbol for the larger of two numbers, and when the symbols for these numbers differ only in the hundreds' place, he is led to see that if one number has more hundreds than the other, it is greater than the other.

At this time, children are not taught the meaning of individual places in the place-value system beyond the thousands' place. To introduce 5-digit and 6-digit numbers, we simply point out that the three digits in the fourth, fifth, and sixth places name the number of thousands. By this procedure, we arrive at the meaning of large numbers with minimum effort, and simplify the reading of large numbers.

Mathematics

The *digits* are the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. A *numeral* is any symbol that stands for a number. Thus the symbol 4 is a digit, and it is also a numeral. The symbol 57 consists of two digits and is a numeral for the number fifty-seven.

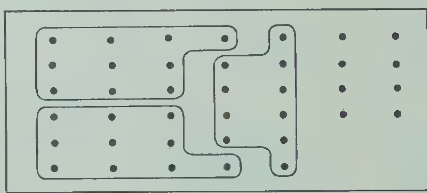
Correct usage of the words *number, numeral,* and *digit* is often awkward. For example, you might ask a child to write the "number" 1486, or you might refer to 1486 as a "4-digit number." This terminology is an abuse of the language because you cannot write numbers, nor do numbers have digits. You write the *symbol,* or *numeral;* and it is the symbol, or numeral, that has digits. However, the expressions noted above are acceptable since children will no doubt understand that you mean for them to write the symbol for this number, and that you are referring to the 4-digit symbol which represents this number.

There are, on the other hand, certain abuses of the number-numeral terminology which are objectionable because they may confuse the children. Whenever you are in doubt about which word to use, use *number.* Avoid making an issue of these words with the children. If a child points at his paper and says, "That is the number fifty-seven," do not criticize or correct his remark. Children have an intuitive grasp of the difference between *number* and *numeral,* and a lengthy discussion of these ideas may serve only to confuse something which was previously clear.

An important property of our numeration system is its utilization of place value. Place value simply means that the number a digit represents depends upon the place it occupies in the symbol.

Another important fact is that we can represent any number by using only ten symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Each of these digits used by itself represents a single number. It is only when we write symbols for numbers greater than nine that a given digit may stand for two or more numbers. Thus, in 636, one 6 stands for 600 and the other for 6.

The place-value scheme that we utilize has a base of ten. Base ten means that we group by tens. For example, given a collection of objects, we might ask how many disjoint groups of ten can be formed. Consider the set of dots shown below. There are 3 disjoint groups of ten, and 8 left over.



The importance of place value is evident when we attempt to write the numeral for this number of dots. Instead of writing "3 groups of ten and 8 more," we simply write 38 and agree that the digit in the second place (in this case, 3) is to represent groups of ten.

When working with larger numbers, we must group the groups of ten by tens. We then have groups of 10 tens, each of which is called one *hundred.* For example, we might have a set of objects grouped as follows: 5 groups of one hundred, 3 groups of ten, and 7 more. We would write 537.

Teaching the Chapter

Materials

Advertisements or price lists of toys, sports items, automobiles
Counters, or a substitute such as bottle caps, toothpicks, etc. (between 25 and 45 per child)
Beans (at least 1000 for five children)
Flannelboard

Hundred board, commercial; or 100 paper fasteners, 100 key tags, 1-by-1-metre piece of cardboard
Hundred-squares, felt, 25-by-25 cm marked into 100 units (10 each)
Magazines for children, such as back issues of *My Weekly Reader*
Paper bags, 4 sizes
Paper cups (approximately 1 for every 12 children)
Poster board or suitable material for wall display
Reference books, such as primary science and geography books
Sets to demonstrate place value (pencils, pipe cleaners, and so on)
Sheets of paper, A4
Toothpicks (at least 1000 for every 5 children)
Ten-strips, felt, 2-by-25 cm marked into 10 units (10 each)
Unit-squares, felt, 2-by-2 cm (10 each)

Vocabulary

digit	number
dime	numeral
greater than	penny
hundred	place value
less than	set
million	thousand

names of numbers through 99

Symbols

$>$, greater than, inequality symbol
$<$, less than, inequality symbol
$=$, equals, equality symbol
. . ., signifies an omission or continuation in the same manner

Although many children at this level can grasp abstract ideas from pictures shown on the printed page, there is virtually no substitute for actual manipulation of concrete materials to facilitate understanding of place-value concepts. Likewise, demonstration materials play an important role in the presentation of these concepts. During the investigations, encourage the children to manipulate materials beyond what is suggested. During discussions, use demonstration sets to illustrate concepts.

When *hundreds* and *thousands* are introduced, you may find it helpful to have ready different-sized boxes or paper bags labelled *100* and *1000*. For example, you can show two large and three small boxes, four bundles of ten pencils, and five loose pencils. Tell the children to imagine that each of the large boxes contains 1000 pencils and the smaller boxes 100 pencils so they can visualize the number illustrated by your demonstration.

In this chapter, it is important that the children become accustomed to distinguishing between the words *digit* and *numeral*. They should recognize that when you talk about a digit you mean *one* of the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and when you talk about a numeral you mean *any* symbol which stands for a number.

In the lessons covering numerals of more than four digits, the vocabulary problem increases. Make it clear to the children that the three digits in the fourth, fifth, and sixth places name the thousands, and that the three digits in the seventh, eighth, and ninth places name the millions. Thus they should quickly recognize that reading large numerals is no more difficult than reading hundreds and determining whether the number is in the millions or the thousands.

A set demonstration illustrating inequalities is not practical when comparing numbers in the hundreds or thousands. However, when comparing large numbers, such as 4357 with 4157, we can observe that both contain the same number of thousands, tens, and ones. But 4357 contains more hundreds than 4157, so $4357 > 4157$. Likewise, we can conclude that $4157 < 4357$.

Lesson Schedule

Your time schedule for this chapter will depend primarily on the children's background. If they are new to this program, you may need to take more time earlier in the chapter to develop the concepts. Plan to spend a maximum of two and a half weeks on the chapter, but attempt to cover it in two weeks.

Evaluation of Progress

The chapter review on page 49 should be useful for evaluating the class's progress in this chapter. However, keep in mind that the most important items to evaluate are found in the early lessons of the chapter. The reading and comparison of 5- and 6-digit numbers should be considered enrichment, rather than more important items of the chapter. Recognition of the place-value concepts in 2- and 3-digit numbers must be the first item in your evaluation of the children's progress.

Resources for Active Learning

General Activities

Chip Trading Activities—Set 1, "Banker's Game," Cards 6–10, Sigma, Scott Scientific
Dienes Multibase Arithmetic Blocks, Tasks and Manual, Cards 1-5, Herder and Herder. (Available from Methuen Publications)
Freedom to Learn, "Place Value," pp. 116–117, Addison-Wesley
Nuffield Project: *Computation and Structure 2,* "Counting Toward Addition," pp. 42–57; "Place Value," pp. 70–74, Wiley

Manipulative Devices

Abacus or Abacus Board (school supplier)
Cuisenaire Rods (Cuisenaire Co.)
Dienes Multibase Arithmetic Blocks (Herder and Herder)
Grid Kit (Sigma Scientific)
Hundred Board and Cylinders (Educational Teaching Aids; Responsive Environments Corp.)
Peas and Particles (Selective Educational Equipment; Webster, McGraw-Hill)
SEE Calculator (Selective Educational Equipment)
SEE Chips (Selective Educational Equipment)
Unifix Math Lab Kit (Educational Teaching Aids; Math Media; Responsive Environments Corp.)

Commercial Games

Kalah (Creative Publications; World Wide Games) A game of counting and strategy.

PAGES 30–31

Objective

Given a set of objects less than 100, the child will be able to group them by tens and use a 2-digit numeral to express how many.

Preparation

Materials (optional)

counters or a substitute such as bottle caps, toothpicks, etc. (between 25 and 45 per child)

To introduce this lesson, ask the children how many times a day they make a guess about something. For example, they might guess how much their lunch or some candy bars might cost, or what the time of day is, or how many blocks they walked taking a new route to school. Discuss the different clues that they use to help them in guessing. Then direct them to the text.

Investigation

This investigation gives children an opportunity to appreciate the usefulness and importance of grouping by tens. Make sure they read the directions first, so that no one starts counting any of the stars. You may want to read the first directions with them to be certain that they know what to do. Remind them to record their answers in writing.

When children have drawn the set of dots allow them to work with a partner and give in writing the number of dots in each other's sets. Encourage them to continue with other sets similarly. If some do not grasp that grouping the dots by ten will help the person guessing, you might have them work with a child who does, but let the children discover this from their own work or from one another.

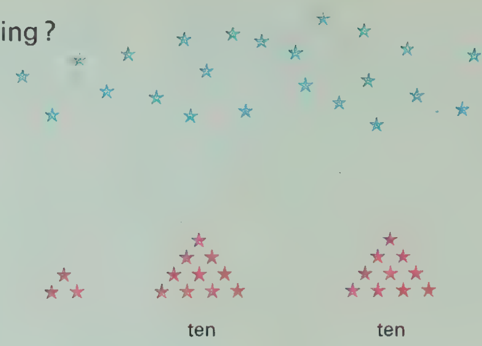
2 Place Value

Can grouping by tens help you find the number in a set?

Investigating the Ideas

How good are you at guessing?

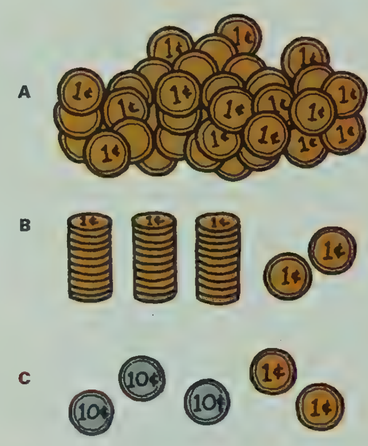
1. Guess the number of blue stars quickly. Guesses will vary. (Actually, 23)
2. Give the number of red stars quickly. 23
3. Check your answers. Which was easier?



? Can you draw a set of dots (between 30 and 50) so that a classmate can give the number quickly? See Investigation.

Discussing the Ideas

1. Each set of coins in A, B, and C is worth the same amount.
   - A What is the value of each set? 32¢
   - B Which two sets are easiest to count? Sets B and C Explain your answer.
2. Suppose you had 40 checkers to count. Could you count these by only counting to ten once? Explain. Sample answer: Count and stack one set of ten and make three matching stacks.



30

Discussion

When children have finished the investigation, ask them to explain why it was easier to find the number of red stars.

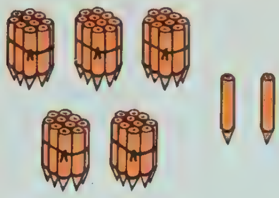
Emphasize the usefulness of grouping by tens as you continue through a discussion of exercise 1. Also, ask the children to write the symbol for this number of objects. Review the place-value meaning of each digit; remind the children that by using place value we can write the numeral for any number with only the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

For the discussion of exercise 2, let the children actually count these according to the directions. If necessary, guide them by appropriate questions so that they will count one stack of ten and then measure three other stacks with this one. Again, have the children write the numeral. Point out that without counting beyond ten they found that there were 40 checkers—that is, 4 tens and 0. Provide other demonstrations using sets of varying numbers of objects (all less than 100) and writing the numeral with an explanation of each digit.

Using the Ideas

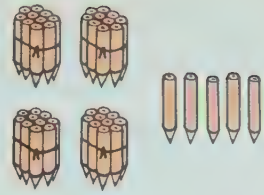
1. Write the numeral for each set of pencils.

A



5 tens and 2
We write ▒. 52

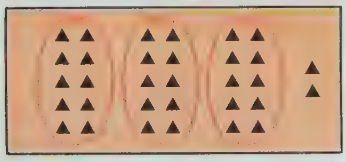
B



4 tens and 5
We write ▒. 45

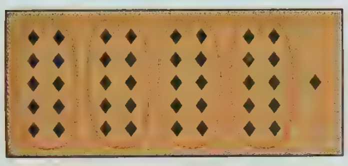
2. In the pictures below there are 10 objects in each ring.
Give the number of objects in each box.

A 32

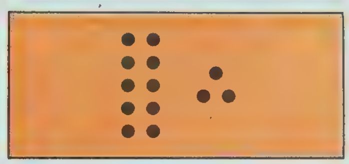


B 24

C 41



D 13



3. Write the 2-digit numeral for each of these.

A 2 tens and 3 23
B 4 tens and 2 42
C 3 tens and 4 34
D 6 tens and 7 67
E 9 tens and 3 93
F 5 tens and 7 57
G 1 ten and 1 11
H 1 ten and 0 10
I 2 tens and 0 20

4. Give the correct digit for each ▒.

A 37 means 3 tens and ▒. 7
B 48 means ▒ tens and 8. 4
C 82 means 8 tens and ▒. 2
D 50 means ▒ tens and 0. 5
E 93 means ▒ tens and 3. 9
F 15 means ▒ tens and 5. 1
G 67 means 6 tens and ▒. 7
H 76 means ▒ tens and 6. 7

More practice, page A-4, Set 5

31

Using the Exercises

You might use exercise 1 on page 31 to emphasize the importance of the position of the digit when writing a numeral. Help the children to see that although the digit 5 is used in both numerals, its value in each numeral is different and that the position of the digit is what expresses this difference.

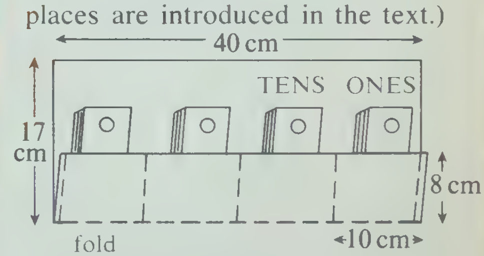
Instruct the children to finish the exercises independently. When the children have finished, check the answers with them and allow time for questions and discussion as needed.

Assignments (page 31)
Minimum: 1–3. Average: 1–4.
Maximum: 1–4.

Mathematics
In this lesson, the study of numerals is directed in a manner which points out that the very concept of place value centres on the marks that we put on paper (e.g., "37 means 3 tens and 7"). When we discuss the meaning of a given numeral with regard to place value, we are actually concerned with the digits and their position with respect to other digits. The concept of the number does not enter into place-value notation. Thus, the entire study of place value merely involves arbitrary agreements about the writing of symbols.

Follow-up/Place-value Charts
Four-place individual pocket charts may help children in their work with place value this year. Give each child a 25-by-40 cm piece of tagboard and forty small index cards.

Show the children how to fold and staple the tagboard to make four pockets. Next, have them make four packs of cards, ten cards in each pack. Ask them to write the digits 0 through 9 in order, one digit for each card, using a different colored crayon for each pack. Then have the children label the tens' and ones' pockets on their chart. (They can label the other pockets as these places are introduced in the text.)



Such a chart has many uses. For example, name several numbers and ask the children to show the numeral for each on their charts.

Resources for Active Learning

Developmental Math Cards, C¹17, C¹13, Addison-Wesley.

Discovery, Section I, Activity 2, pp. 2–3, Encyclopaedia Britannica Educational Corp.

Math Workshop: Games and Enrichment Activities, "The Game of Tens and Ones," pp. 19–20, Encyclopaedia Britannica Educational Corp.

Mathematics in Modules, WN6, Addison-Wesley.

PAGES 32–33

Objective

Given a 2-digit numeral, the child will be able to explain the place value for each digit. Also, the child will be able to count to 100 by tens, fives, and twos.

Preparation

Since much of this lesson is to be treated orally, you might begin immediately with the text. If you want to use an oral game to review place value, you may find opportunity to do so during the discussion.

Do you know the number names up to one hundred?

Discussing the Ideas

1. There are ten sticks in each bundle. Tell the number of tens and then the number of sticks in all.

2. Read the number. Then tell how many tens and how many ones.

A 63 6,3 D 57 5,7 G 96 9,6 J 60 6,0 M 17 1,7 P 11 1,1 S 48 4,8 V 35 3,5
B 47 4,7 E 21 2,1 H 85 8,5 K 51 5,1 N 42 4,2 Q 10 1,0 T 80 8,0 W 89 8,9
C 39 3,9 F 16 1,6 I 43 4,3 L 30 3,0 O 78 7,8 R 99 9,9 U 27 2,7 X 70 7,0

3. Give the word name for each of these.

A 6 tens and 5 sixty-five E 8 tens and 3 eighty-three I 7 tens and 0
B 5 tens and 9 fifty-nine F 3 tens and 0 thirty J 4 tens and 9
C 4 tens and 2 forty-two G 4 tens and 3 forty-three K 8 tens and 5
D 5 tens and 4 fifty-four H 1 ten and 0 ten L 7 tens and 3

I seventy
J forty-nine
K eighty-five
L seventy-three

4. Count by tens to one hundred.
ten, twenty, thirty, . . . forty, fifty, . . . one hundred

5. Count by fives to one hundred.
five, ten, fifteen, twenty, . . . twenty-five, thirty, . . . one hundred

6. Count by twos to one hundred.
two, four, six, eight, ten, twelve, . . . fourteen, sixteen, . . . one hundred

32

Discussion

Exercise 1 reviews the number names twenty through ninety. As the children give the number of tens and the number of sticks in all, use the chalkboard to write the numeral and its word name to make sure the children can read it. For example, with exercise 1E, you might write the following:

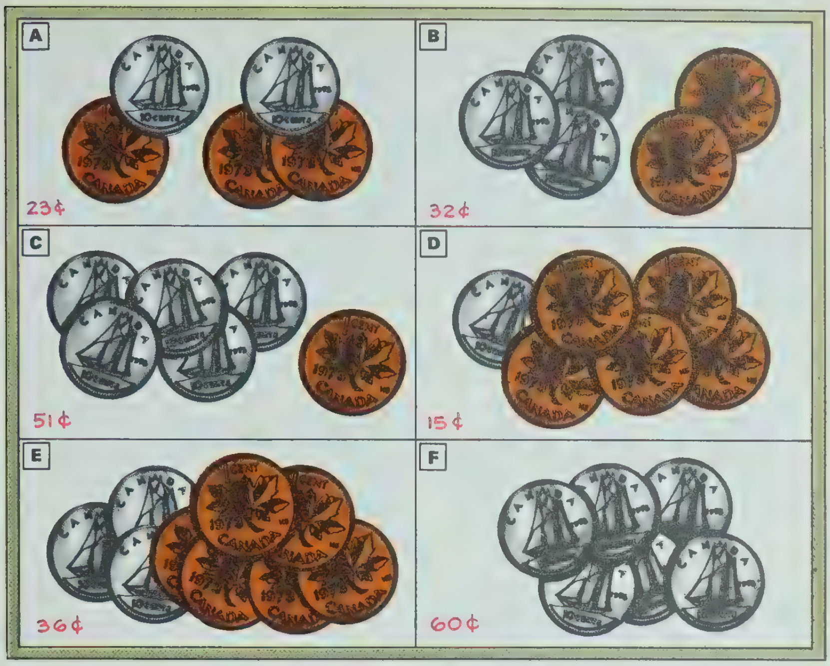
6 tens = 60 = sixty

The rest of the exercises should be treated orally. If possible, present additional counting opportunities beyond those suggested. Have children look for things in the classroom which they might count. It might be interesting for the children to count themselves in groups of tens and then in groups of fives and twos.

Since exercises 5 and 6 are fairly long, divide the counting sequence among several children. You might make this into a game where the child who is counting holds an object and then hands it to someone else as a sign for him to proceed with counting.

Using the Ideas

1. Find the value of each collection.

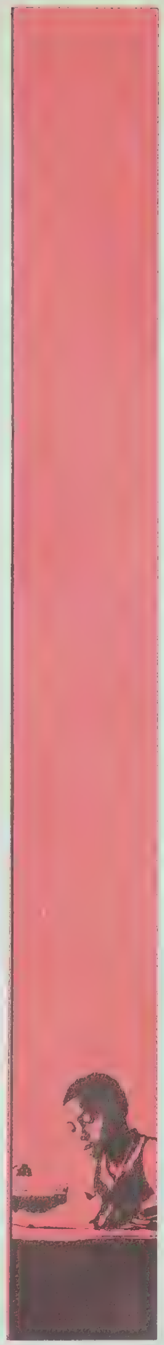


2. Copy each row, giving the missing numbers. See Answers, T.E. p. 33.

A 4, 5, 6, 7, ▮, ▮, ▮, ▮, 12, 13
B 24, 25, 26, 27, ▮, ▮, ▮, ▮, 32, 33
C 52, 53, 54, 55, 56, 57, ▮, ▮, ▮, ▮
D 90, 91, ▮, ▮, ▮, ▮, 96, 97, 98, 99
E 72, 73, 74, ▮, ▮, ▮, ▮, 79, 80, 81
F ▮, ▮, ▮, ▮, 48, 49, 50, 51, 52, 53
G 38, ▮, ▮, ▮, ▮, 43, 44, 45, 46, 47
H ▮, ▮, ▮, ▮, 62, 63, 64, 65, 66, 67
I ▮, ▮, ▮, ▮, 4, 5, 6, 7, 8, 9
J ▮, ▮, ▮, ▮, 22, 23, 24, 25, 26, 27

think
See Using the Exercises.

1. Using the digits 1, 2, 7, 9, how many 2-digit numerals can you write? 16
2. Which of these numerals names the largest number? 99
3. Which names the smallest? 11



More practice, page A-4, Set 6

33

Using the Exercises

On page 33, call the children's attention to the sets of coins in exercise 1. Help them see that 2 dimes and 3 pennies is much the same as 2 tens and 3 ones, or 23, in our place-value system.

Ask the children to copy each row in exercise 2, and to include the numerals for the missing numbers. Point out that they may need to count backwards for parts 2F, H, I, and J.

Although the *Think* problem is intended as independent work for the more capable, the rest of the class may benefit from having several children who solve it explain their findings. Be sure to tell those who attempt the problem that they may repeat any of the four given digits. The following sixteen 2-digit numerals can be made in answer to part 1.

11, 12, 17, 19, 21, 22, 27, 29, 71, 72, 77, 79, 91, 92, 97, 99

Answers, Exercise 2, page 33

2. **A** 8, 9, 10, 11 **F** 44, 45, 46, 47
B 28, 29, 30, 31 **G** 39, 40, 41, 42
C 58, 59, 60, 61 **H** 58, 59, 60, 61
D 92, 93, 94, 95 **I** 0, 1, 2, 3
E 75, 76, 77, 78 **J** 18, 19, 20, 21

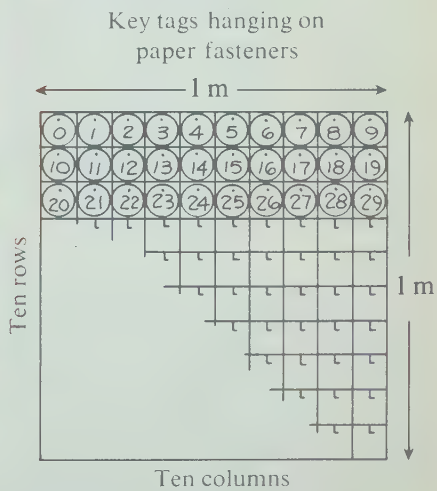
Assignments (page 33)

Minimum: 1–2E. Average: 1–2. Maximum: 1–2.

Follow-up/A Hundred Board

For more review in counting and number sequences to 100, display a hundred board on which all the tags are turned to the blank side. You may use a commerical board or chart, or make one by arranging 100 brass paper fasteners in 10 rows of ten on a 1-metre square of heavy cardboard. Label key tags or perforated cardboard discs on one side with the numerals 0 through 99. In class, turn over a tag, the tag bearing 69, for example, and ask what number comes next. When a child answers "Seventy," allow him to come up and turn over the tag for 70.

After each child has had a turn, return the tags to the blank side and say, "I'm thinking of the number that is 1 more than 39," or "I'm thinking of the number that is 1 more than 3 tens and 9. Who can find and turn over its tag?" Or, adapt the game to practice other weaknesses your pupils have in counting and order for numbers between 0 and 100.



Resources for Active Learning

Developmental Math Cards, C[1]14, E[1]9, Addison-Wesley.

Mathex: Numeration No. 2, pupil page 19, Encyclopaedia Britannica Publications Ltd. [A "Hundred Square" activity]

Mathex: Operations No. 3, "Hundreds and Thousands – Activity 3," pp. 13–14, Encyclopaedia Britannica Publications Ltd.

Duplicator Masters, page 6
Workbook, page 11
Skill Masters, page 6

PAGES 34–35

Objective

Given the numeral 100, the child will recognize it to be 10 tens and have an intuitive grasp of how large 100 is.

Preparation

Materials

flannelboard; felt hundred-squares, ten-strips

In order to help the children understand the word "estimate," you might do something like this. Ask the children to imagine 1 of their favorite things; then to think of 10 of those things, then 20, 30, 50, then 100 of them. Talk about ways of checking their imaginations. Tell them that they *estimated* how large 100 is. Teach the word "estimate" as meaning "to make a careful guess." Explain that in today's lesson they will estimate 100 in different ways and that they will be able to check their estimates to see how close they came.

Investigation

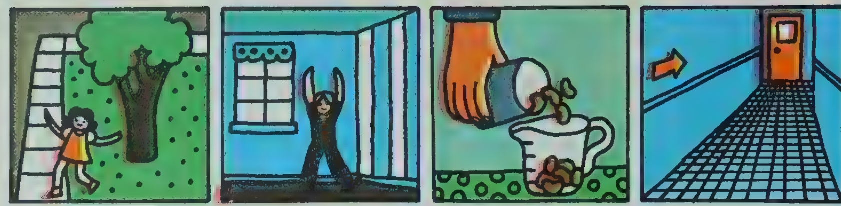
This investigation would best be handled by dividing the class into small groups. Then each group should do at least one of the investigations. Remind the children to record each child's estimate so they can check to see how close their estimate was to the actual result.

If your teaching situation makes it difficult to do the investigations outside the classroom, encourage the children to invent others. In fact, inventing other investigations would be helpful for them even if they can do all those suggested in the text. Suggestions that may be handled in the classroom include "How long a distance would 100 pieces of chalk cover if they were lined up end to end?" or "How thick a bundle would 100 pieces of hair make?"

As you move from group to group, notice how the children are counting as they check their estimates. If the children are counting by ones, and counting by groups would be more appropriate, as with the beans, ask them if there is another, easier way to count.

How large is a hundred?

Investigating the Ideas



How far down the sidewalk will 100 steps take me?

How high up the wall is 100 cm?

How full will the measuring cup be with 100 beans?

How far down the hall is the 100th tile from here?

? How well can you estimate 100? Try one of the questions above. See Investigation.

Discussing the Ideas

1. Give the missing numerals in the table.

We see	We think	We write
	8 tens	80
	9 tens	A 90
	10 tens	B 100

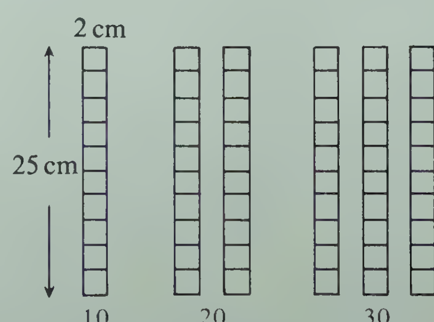
2. Explain how to count one hundred pencils without counting higher than ten.
Count out 10 groups of 10 pencils per group.
3. How would you draw one hundred dots without counting higher than ten?
Draw 10 sets of 10 dots per set.
4. Can you draw one hundred twenty-three dots without counting higher than ten? Explain.
Draw 10 sets plus 2 sets of 10 dots per set plus 3 more dots.

34

Discussion

Let each group share with the class the results of their investigation. Encourage the children to explain how they checked their estimates when they did their counting to 100. Guide the discussion so that the children understand 100 as 10 groups of ten.

Have children write the answers to exercise 1 on the chalkboard. Or you may prefer to demonstrate these groups of ten with 2-by-25-cm strips of felt marked into ten units. For example, you could place 8 of these strips on the flannelboard and have a child use felt numerals to put 80 under the strip. When you come to tens, you can replace the ten single strips with one felt square (25-by-25 cm) marked into single units.



Felt Ten-Strips

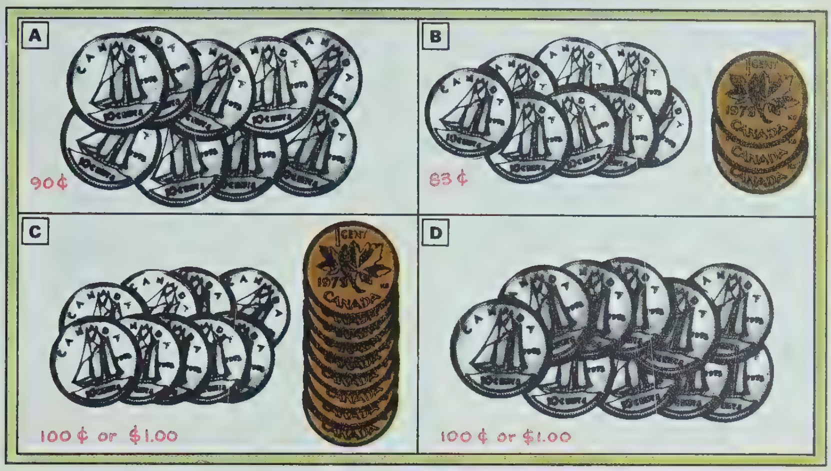
For exercise 2, let some children count out 100 pencils in groups of

Using the Ideas

1. Give the missing numerals.

- **A** For 9 tens and 6, we write ||||. 96
- **B** For 9 tens and 7, we write ||||. 97
- **C** For 9 tens and 8, we write ||||. 98
- **D** For 9 tens and 9, we write ||||. 99
- **E** For 9 tens and 10, we write ||||. 100
- **F** For 10 tens and 0, we write ||||. 100

2. Find the total number of cents in each box.



3. Write the missing numerals.

For 10 tens, we write 100.
For 20 tens, we write 200.

- **A** For 30 tens, we write ||||. 300
- **B** For 40 tens, we write ||||. 400
- **C** For 80 tens, we write ||||. 800

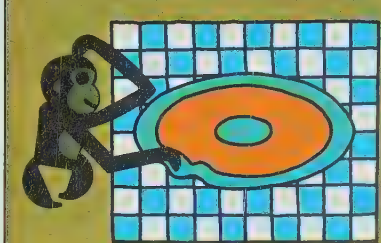
4. Write the missing numerals.

For 10 tens, we write 100.
For 11 tens, we write 110.
For 12 tens, we write 120.

- **A** For 13 tens, we write ||||. 130
- **B** For 14 tens, we write ||||. 140
- **C** For 15 tens, we write ||||. 150
- **D** For 16 tens, we write ||||. 160

think

The rug covers some of the tiles. How many tiles on this floor?



35

10 and have others count them by ones to show clearly that 10 tens is 100. You might want to have the children draw the dots discussed in exercises 3 and 4.

Using the Exercises

Assign the exercises on page 35 as independent work, but allow ample time for discussion afterward to make sure the children see that 100 can be thought of as 1 more than 99 (as in exercises 1E and 2C) or as 10 groups of ten (as in exercises 1F and 2D). To check these exercises, have some children write the answers on the chalkboard.

The *Think* problem involves the idea that we can think of the floor as 8 groups of 10 squares.

Assignments (page 35)

Minimum: 1-2. Average: 1-2.
Maximum: 1–4.

Mathematics

Once children master grouping by tens, it is easier for them to understand 2-digit numerals. Extending their comprehension to include three-digit numerals is more difficult, but it is based on the same principle. That is, children must learn to group by hundreds. Because the whole place-value system follows a pattern of grouping by powers of ten, the progression from 2-digit to 3-digit numerals is important.

Follow-up

As an activity to reinforce the concept of 100 as 10 tens, you might have the children use the ten-strips to see how many they need to equal 100 single units. For example, have them work in groups of five or so and combine their strips. Then they may not only count 100 by using ten-strips, but also, they might show other numbers with the strips. Write numbers such as 84, 75, 112, 234, on the chalkboard and have each group of children arrange their strips in tens and ones, or hundreds (bundles of ten strips), tens, and ones. It would be appropriate to have them show the answers for some of the exercises in "Using the Ideas" in this manner.

PAGES 36–37

Objective

Given a 3-digit numeral, the child will be able to read it, write it, and explain the place value of each digit.

Preparation

Materials

felt squares, strips, and single units (10 each); small index cards (9 per child)

Before beginning the investigation activity, conduct a short oral review of place-value concepts involving 2-digit numerals. Then supply each of the children with 9 index cards and guide them in labelling the cards with the numerals 1, 2, 3, 4, 5, 6, 7, 8, 9—one numeral per card. Ask the children to underscore each numeral for ease in distinguishing which is the bottom of the card. Tell them that the cards will be used in the game they will learn in the investigation for this lesson.

Investigation

Divide the class into groups of 3 and select one group to play a demonstration hand of High, Middle, Low as you read the rules with the class. You may wish to have several more groups play demonstration hands, to make sure the rules are understood, before having the groups play independently. To avoid possible misunderstandings, it would be a good idea to have the children write down their numerals for each hand before declaring "High," "Middle," or "Low."

Let's investigate 3-digit numerals.

Investigating the Ideas

High, Middle, Low Game

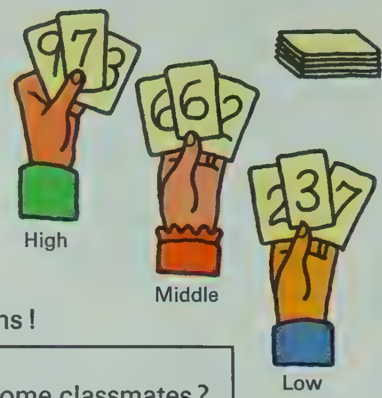
Directions:

Use 3 sets of 9 cards each with the digits 1 through 9. Shuffle the 27 cards and deal 3 cards to each of three players.

Each player forms a 3-digit numeral and guesses whether his number is **High**, **Middle**, or **Low**.

Then compare numbers and score 1 point for a correct guess.

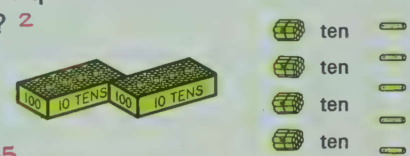
Shuffle and deal again. 10 points wins!



? Can you play this game with some classmates?
See Investigation.

Discussing the Ideas

1. John and Anne both said their numbers were **High**. Which player won? Anne
2. Would Nancy win if she had said **Middle**? Who was middle? No; John
3. Use the figure to answer the questions.
   - A How many hundreds? 2
   - B How many tens? 4
   - C How many ones? 5
   - D How many sticks, 254, 542, or 245? 245



4. Read the number. Then tell how many hundreds, tens, and ones.

A 278	C 512	E 765	G 318	I 380	K 900	M 707	O 437
B 346	D 923	F 492	H 640	J 704	L 506	N 770	P 864

Sample answers: A Read "two hundred seventy-eight"; 2 hundreds, 7 tens, 8 ones
J Read "Seven hundred four"; 7 hundreds, 0 tens, 4 ones

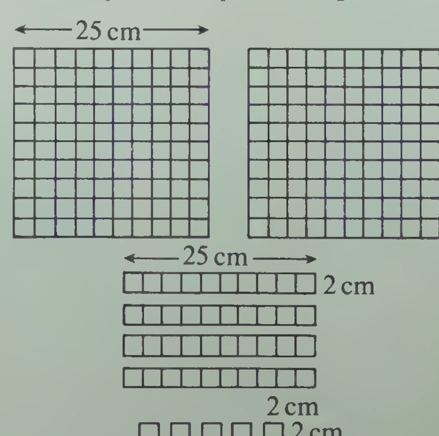
36

Discussion

The first two discussion exercises provide a transition from the small-group activity to the class discussion and also serve to indicate whether the children have grasped the place-value concepts involved in comparing 3-digit numerals.

As part of discussion exercise 3, designate some children to count out sets of 100, sets of 10, and the sets of 1 that total 245. Also, ask one of the children to explain how the numeral 245 represents the number of sticks pictured in the child's book. You might write out "245 means 2 hundreds, 4 tens, and 5 ones." Then illustrate this number by using the felt squares, strips, and single units as shown below.

Felt Squares, Strips, and Single Units



Using the Ideas

1. Write the numeral. (h stands for hundreds and t for tens.)

A 3 h, 2 t, and 6 326 D 6 h, 0 t, and 1 601 G 6 h, 2 t, and 7 627
B 4 h, 7 t, and 2 472 E 3 h, 2 t, and 0 320 H 1 h, 0 t, and 0 100
C 6 h, 5 t, and 0 650 F 3 h, 0 t, and 0 300 I 7 h, 6 t, and 5 765

2. Give the missing digit.

A 384 means 3 hundreds, ▒ tens, and 4 ones. 8
B 659 means ▒ hundreds, 5 tens, and 9 ones. 6
C 518 means 5 hundreds, 1 ten, and ▒ ones. 8
D 304 means ▒ hundreds, 0 tens, and 4 ones. 3
E 927 means 9 hundreds, ▒ tens, and 7 ones. 2

3. Write the numeral for each part.

A two hundred eighty-three 283 D six hundred fifty-four 654
B five hundred sixty-seven 567 E three hundred thirty-nine 339
C nine hundred forty-one 941 F seven hundred twenty-eight 728

4. Find the number that is 1 more than

A 9 10 D 49 50 G 89 90 J 119 120 M 199 200
B 19 20 E 69 70 H 99 100 K 139 140 N 699 700
C 29 30 F 79 80 I 109 110 L 439 440 O 899 900

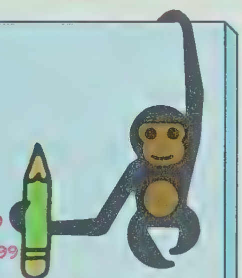
5. Copy each column and complete the counting.

A		B	
197		797	
198		798	
▒	199	799	
▒	200	▒	800
▒	201	▒	801
▒	202	▒	802
203		▒	803

think

Find the number that is 1 less than

1. one hundred. 99
2. one thousand. 999
3. ten tens. 99
4. ten hundreds. 999
5. one hundred hundreds. 9999



More practice, page A-5, Set 7

Follow-up

It would be helpful to prepare a worksheet for the children to reinforce their understanding of 3-digit numerals. Use exercises similar to exercise 4 at the bottom of page 36. You might add some which use expanded form of numerals as in this sample.

Match the following.	
400 + 60 + 9	237
900 + 40 + 6	469
200 + 30 + 7	946
300 + 70 + 2	851
800 + 50 + 1	796
700 + 90 + 6	372

Resources for Active Learning

Mathex: Operations No. 3, "Hundreds and Thousands—Activities 1, 2, 4, 5, 6," pp. 11–15 (pupil pages 34–35); "Other Activities and Games," p. 15, Encyclopaedia Britannica Publications Ltd.

Duplicator Masters, page 7
Workbook, page 12
Skill Masters, page 7

As the children orally work through exercise 4, illustrate some of the examples as shown above. This would be particularly helpful for numerals in which zero appears, such as 704 and 770.

Using the Exercises

According to class capability and need, assign the exercises on page 37 as independent work. If necessary, work through a few of the exercises orally, but have the children write the answers even if you work through them orally.

The *Think* problem on page 37 might stimulate more capable children to make up place-value riddles to share with one another.

Assignments (page 37)

Minimum: 1–3. Average: 1–4.
Maximum: 1–5.

PAGES 38–39

Objective

Given the numeral 1000, the child will be able to identify it as 10 groups of 100 and will have some intuitive grasp of how large a thousand is.

Preparation

Materials

sets of 1000 – beans, slips of paper, toothpicks, paper clips, or other counters; 10 felt hundred-squares

Talk with the children about the investigation in which they made estimates about 100, such as how far 100 steps would take them and how far up 100 centimetres would be. Ask them to name some 3-digit numerals greater than 100. Then ask what number comes after 999. When they say 1000, ask them what they think about counting to 1000 – would they be able to do it? Tell them that in this lesson they will have a chance to find out.

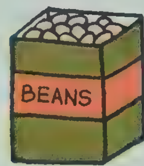
Investigation

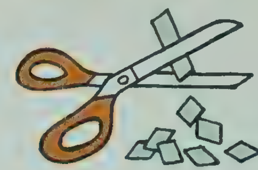
Although this investigation might appear as an exercise in counting, one of its main purposes is to lead the children to an intuitive grasp of the number 1000. Another purpose is to help the child see that 1000 can be thought of as 10 hundreds or 100 tens.

Divide the children into groups of five or six, perhaps appointing a "leader" or "captain" for each group. Plan for each group to conduct one of the investigations. Thus, if you have 30 children, you might have six groups of five children per group. You need two sets of materials for each way to count 1000 (10 hundreds or 100 tens). You may prefer to have the slips of paper cut beforehand so the children would be able to begin counting immediately, as they can with the beans and toothpicks. Encourage the children to find easy, quick ways of counting, such as having five children each count out 200 items. However, instead of giving explicit directions, use leading questions to let them discover their own ways of counting by groups.

How large is a thousand?

Investigating the Ideas

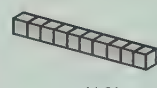




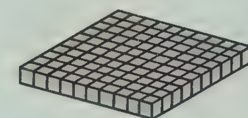
? How long do you think it would take you to count ten hundred objects? Try this with a set of objects such as one of those above.

See Investigation.

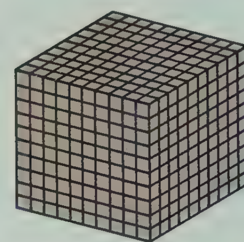
Discussing the Ideas



ten (10)



one hundred (100)



one thousand (1000)

1. A Which picture shows ten tens? Middle picture
   B Which one shows ten hundreds? Picture at right

2. A How many toothpicks should each of 10 children count to count a total of 1000 toothpicks? 100
   B How many groups of 10 toothpicks must each child count? 10

3. Can you count a thousand beans without ever counting higher than 10? Yes; count 10 groups of 10 ten times.

4. How many tens make a thousand? 100

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Discussion

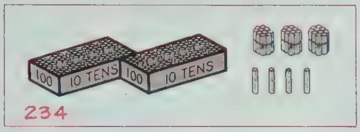
Let a child from each group explain the method of counting they used in the investigation. Even if the children did not use relevant grouping in the investigation, guide the discussion so that they will see that 1000 is the same as 10 hundreds. If, during the investigation, anyone tried counting by ones, stress the largeness of 1000 and the comparative ease of group counting.

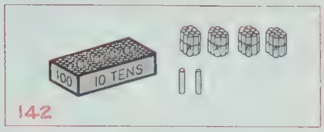
Exercises 2 and 4 develop the idea that 1000 may be thought of as 100 tens. If you think it necessary, have ten children each count 100 toothpicks, showing 10 groups of 100; or have 10 children each count out 100 beans by tens, each showing 10 groups of 10 – 100 groups of 10 in all.

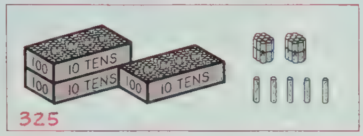
Depending on your class's need, you may wish to demonstrate these concepts further by using the felt hundred-squares. For example, place five of the hundred-squares on the flannelboard and ask the class how many hundreds there are. Have one child write the numeral 500 on the chalkboard. Continue to add hundred-squares through nine hundreds, having someone write it on the board. Then place the tenth hundred-square with the others. Stress that just as 100 is the same as 10 tens, now we have

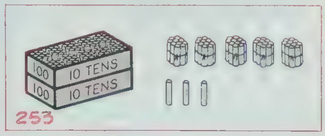
Using the Ideas

1. Give the number for each set.
There are 10 in each bundle and 100 in each box.

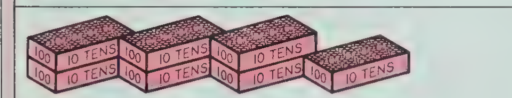
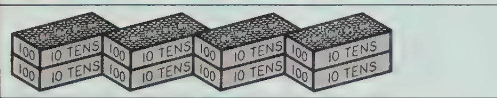
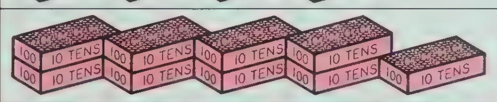
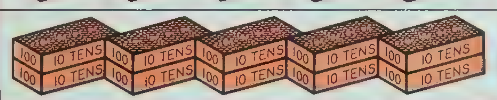
A


B


C


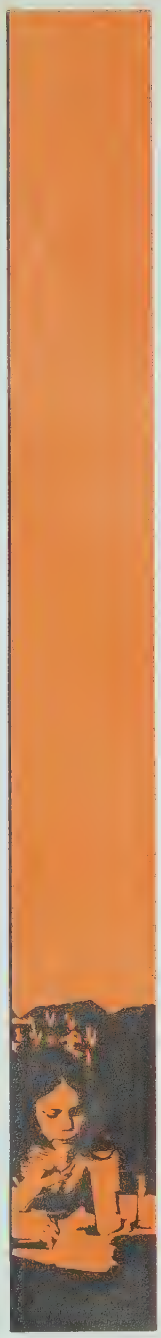
D


2. Give the missing numerals in the table.

We see	We think	We write
	7 hundreds	700
	8 hundreds	A 800
	9 hundreds	B 900
	10 hundreds	C 1000

3. Write the correct numerals.

A For 4 hundreds, 8 tens, and 3, we write ____. 483
B For 4 hundreds, 9 tens, and 7, we write ____. 497
C For 9 hundreds, 9 tens, and 7, we write ____. 997
D For 9 hundreds, 9 tens, and 8, we write ____. 998
E For 9 hundreds, 9 tens, and 9, we write ____. 999
F For 10 hundreds, 0 tens, and 0, we write ____. 1000



1000, which is 10 hundreds or 100 tens. Write 1000 on the chalkboard under 900. Count the 10 hundred squares and pin them together to signify that they are grouped to form 1000.

Using the Exercises
Assign the exercises on page 39 as independent work. You may wish to use this page to determine how well the children have understood place value to this point. When they have finished, check the answers with the children and allow ample time for discussion.

Assignments (page 39)
Minimum: 1–2. Average: 1–3.
Maximum: 1–3.

Mathematics
Once a transition has been made from tens to hundreds, enlarging the idea of place value to include thousands is little more than a formality. If children understand that 100 is 10 groups of ten, they should be able easily to extend the pattern in order to think of 1000 as 10 groups of one hundred.

Follow-up/"Bags for Place Value"
When the children understand that 1000 is 10 hundreds, you can illustrate 4-digit numerals by stuffing four sizes of grocery bags. Tie and label them: large-sized bags as 1000, medium-sized as 100, and the two smaller sizes as 10 and 1, respectively. The children can imagine that the bags contain the indicated number of objects. One or two capable children might use bags to show 4-digit numerals to a small group of children who need help in understanding place value. Or you might have a child write a 4-digit numeral on the chalkboard and ask other children to use the bags to demonstrate the place value of the given numeral or of a numeral 10 or 100 more or less than that given.

2386

PAGES 40–41

Objective

Given a 4-digit numeral, the child will be able to read it, write it, and explain the place-value meaning for each digit.

Preparation

Materials

newspapers; magazines; children's reference books

To prepare the children to look for 4-digit numerals, write 10, 100, and 1000 on the chalkboard. Ask the children to name the number and tell the number of digits in each. Repeat the question using numbers such as 25, 978, 3462. Then focus on four-digit numerals by asking them to guess how many 4-digit numerals there are. They might be surprised to learn that there are 9000 such numerals. In this lesson they will be able to look for as many as they can find in the materials available.

Investigation

Have a plentiful supply of advertising pages from newspapers and magazines, and reference books the children can read, such as a world atlas and geography and science books. If possible, distribute the materials so that each child may individually look for the numerals, but allow free sharing of ideas. If children need other examples or if materials are scarce, refer them to the tables of measures in the back of this book. Also, below are a few facts which involve 4-digit numerals the children may find interesting:

surface temperature of the sun, 6000°C

number of seconds in one hour, 3600

height of Mt. Logan, 5955 metres

length of Mackenzie River, 4210 kilometres

Let's explore 4-digit numerals.

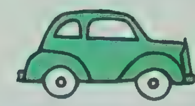
Investigating the Ideas

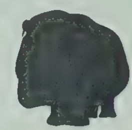
Look up one of the following.

A Price of your favorite car

B Weight of a hippopotamus

C Kilometres from Vancouver to Quebec

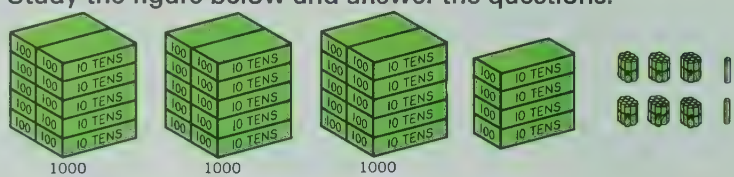




? Can you find some other examples of 4-digit numerals in newspapers, magazines, and books? See Investigation.

Discussing the Ideas

1. Study the figure below and answer the questions.



A How many thousands? 3

B How many hundreds? 4

C How many tens? 6

D How many ones? 2

E Explain the meaning of the numeral 3462.

2. A Can you read the numeral on the sign?

B How many thousands in the numeral? 6

C How many hundreds in the numeral? 0

D How many tens? 4 ones? 8

3. Read the number. Then tell how many thousands, hundreds, tens, and ones. See Discussion.

A 7264	E 1635	I 9025	M 8340	Q 9216
B 8315	F 7986	J 8840	N 2600	R 7007
C 9126	G 8204	K 7602	O 5000	S 6000
D 8427	H 3716	L 9100	P 8083	T 5080

Sample answers: A Read "seven thousand two hundred sixty-four"; 7 thousands, 2 hundreds, 6 tens, 4 ones
R Read "seven thousand seven"; 7 thousands, 0 hundreds, 0 tens, 7 ones

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Discussion

As children finish the investigation, have some of them go to the chalkboard and write the numerals they found for A, B, and C. It would be helpful to discuss a few of these numerals. For example, explain the place-value meaning of each digit in the numeral 4160, so that the children understand that 4160 means 4 thousands, 1 hundreds, 6 tens and 0 ones.

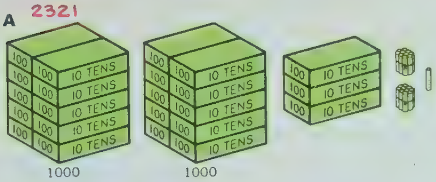
When discussing exercise 1, again stress the value given to each digit because of its position in the numeral.

Two of the important objectives of this lesson are for children to be able to read 4-digit numerals correctly and to understand the place-value meaning of each digit. In exercise 3, encourage the children by your example to read the numerals properly and give the groupings; in part A, for example: "Seven thousand two hundred sixty-four is 7 thousands, 2 hundreds, 6 tens, and 4 ones."

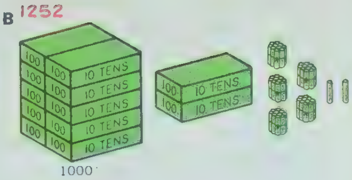
Using the Ideas

1. Give the 4-digit numeral for each set.

A 2321



B 1252



2. Write the 4-digit numeral for each of these. (*th* stands for thousands, *h* stands for hundreds, and *t* stands for tens.)
   - A 6 *th*, 5 *h*, 2 *t*, and 4 6524
   - B 9 *th*, 4 *h*, 2 *t*, and 1 9421
   - C 9 *th*, 4 *h*, 2 *t*, and 0 9420
   - D 9 *th*, 4 *h*, 0 *t*, and 0 9400
   - E 9 *th*, 0 *h*, 0 *t*, and 0 9000
   - F 6 *th*, 0 *h*, 8 *t*, and 0 6080

3. In each numeral below, one of the digits is red. Give the number for which that digit stands. For example, in exercise A the 7 stands for 700.
   - A 6728 700
   - B 4325 4000
   - C 4286 80
   - D 9515 5
   - E 7106 100
   - F 8732 700
   - G 9457 9000
   - H 1260 0
   - I 4037 0
   - J 5208 0

4. Find the missing digit for each of these.
   - A 6721 means ▒ hundreds, 6 thousands, 1 one, 2 tens. 7
   - B 4362 means 4 thousands, ▒ tens, 3 hundreds, 2 ones. 6
   - C 7820 means 2 tens, 8 hundreds, 0 ones, ▒ thousands. 7
   - D 5207 means ▒ hundreds, 0 tens, 5 thousands, 7 ones. 2

5. Copy each column and complete the counting.

A	
	996
	997
998	▒
999	▒
1000	▒
1001	▒
	1002

B	
	2396
	2397
2398	▒
2399	▒
2400	▒
2401	▒
2402	▒

think

1. What is the smallest 4-digit number that uses just 3 different digits? 1002
2. What is the largest such number? 9987

More practice, page A-6, Set 8

Follow-up/Place-Value Card Game
Form two teams with ten children on each team. Have printed on fairly large-sized cards two sets of the digits 0, 1, 2, . . . 9. Give each team a set of digits, so each member has a card to hold. As you call out a 3- or 4-digit numeral, each child who holds one of the digits called stands with the other 2 or 3 members of his team to form the numeral for all to see. The team whose members are the first to position themselves correctly gets a point. Those children not on a team may act as judges to see which team was quicker in forming the numerals.

Duplicator Masters, page 8
Workbook, page 13
Skill Masters, page 8

Using the Exercises
Before you assign the exercises on page 41 as independent work, you might want to work through exercise 1 and parts of exercise 3 orally to make sure the children know what to do.

After they finish the exercises, allow time for checking papers and discussing any difficulties the children had. It would be helpful for them to write the answers to exercise 2 on the chalkboard.

Many children may want to try the *Think* problem, and the whole class would benefit from a discussion of it. For example, you might discuss the role of zero in the numeral 1002, as the digit which tells us both how many tens and how many hundreds, and point out that if we wrote 0012, the 0 would not be necessary.

Assignments (page 41)
Minimum: 1–3. Average: 1–4.
Maximum: 1–5.

Objective

Given two numerals that have the same number of digits (fewer than 5), the child will be able to identify which numeral represents the greater number and use an inequality symbol to compare the numbers.

Preparation

Materials

advertisements of toys, sports items; children's reference books; stop-watch

To prepare for this lesson, make sure the children understand the meaning of "compare." For example, have the class observe the heights of two children. Let them talk about the differences they observe—one is taller, the other is shorter. The height of one is *greater than* the height of the other. Explain that many things can be compared and that in this lesson they will compare the "size" of numbers.

Investigation

If possible, let the children themselves find the numbers for comparison which are suggested in the text. However, if suitable references are unavailable, you might exhibit data such as those below.

Important dates: Founding of Quebec, 1608; Confederation, 1867; first man on the Moon, 1969.

Lengths of rivers: Mackenzie, 4210 km; St. Lawrence, 3300 km; Yukon, 2890 km.

Diameters of planets: Mars, 6760 km; Mercury, 4866 km.

For part C, let some children use your stopwatch or a wristwatch with a seconds hand to find the number of their heartbeats per minute. (Make sure they use the fingers to feel the pulse, as pictured.) The data below may be used if the children have difficulty.

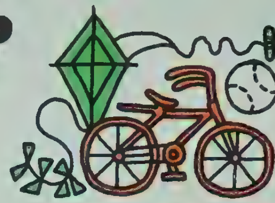
Average heartbeats per minute: baby, 120–140; child, 90; woman, 78; man, 72.

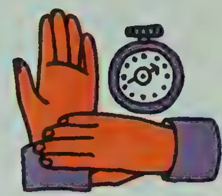
Average breaths per minute: baby, 40; child, 20; adult, 16.

Let's compare the "sizes" of numbers.

Investigating the Ideas

Choose one of these investigations.





A Find two important dates in history.

B Find the price at two stores of something you want to buy.

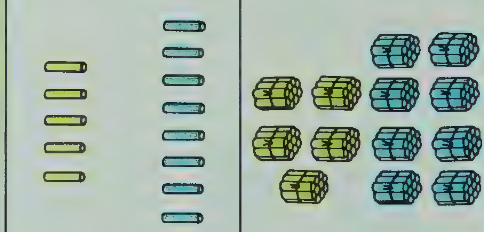
C Find the numbers of your heartbeats and breaths per minute.

? Can you tell which of the two numbers you found is greater? See Investigation.

Discussing the Ideas

1. Study the figure. Then answer the questions.
   - A Which is greater, 500 or 800? 800
   - B Which is greater, 105 or 108? 108
   - C Which is greater, 150 or 180? 180
   - D Which is greater, 524 or 824? 824

2. Explain an easy way to remember how to use the inequality marks (< and >). See Discussion.



5 is less than 8. $5 < 8$ 8 is greater than 5. $8 > 5$	50 is less than 80. $50 < 80$ 80 is greater than 50. $80 > 50$

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Discussion

Ask a few volunteers to put a pair of the numbers they investigated on the chalkboard and explain what they found. With each of these, introduce the inequality symbols, < (less than) or > (greater than). For example, the number of heartbeats a child has per minute is greater than the number of breaths a child takes per minute, and we show this by writing $90 > 20$ ("Ninety is greater than twenty").

In discussing exercise 1, focus attention on the place value of the digits 5 and 8. In each case, the 5 and 8 occupy the same place. It would be helpful to present an example where this is not the case, such as in 150 and 108. Point out to the children that they should first compare the hundreds; since these are the same, they should then compare the tens and note that 5 tens is more than 0 tens; hence, $150 > 108$.

For exercise 2, let the children devise their own ways of remembering how to use the inequality marks. If any have difficulty, you might suggest that they think of an arrow with its point always near the lesser number, or ask those children who discover their own method of remembering to share it with the others.

Using the Ideas

1. Which of the two numbers is greater?

A 9 or 7 9	D 80 or 20 80	G 900 or 300 900	J 2000 or 5000 5000
B 5 or 8 8	E 10 or 50 50	H 400 or 700 700	K 6000 or 4000 6000
C 6 or 2 6	F 60 or 40 60	I 600 or 500 600	L 8000 or 9000 9000

2. Answer true or false for each exercise.

A 9 is greater than 7. T	G 200 is less than 500. T
B 90 is greater than 70. T	H 300 is less than 200. F
C 50 is greater than 80. F	I 604 is less than 607. T
D 51 is greater than 50. T	J 820 is less than 850. T
E 68 is greater than 64. T	K 750 is less than 720. F
F 72 is greater than 73. F	L 930 is less than 940. T

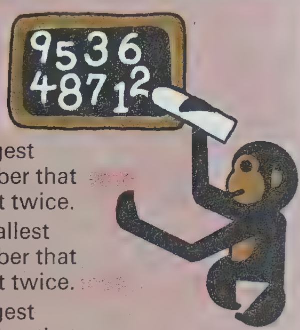
3. Which of the two numbers is greater?

A 8 or 3 8	D 82 or 32 82	G 68 or 63 68	J 628 or 623 628
B 6 or 9 9	E 67 or 97 97	H 625 or 925 925	K 6219 or 9219 9219
C 5 or 8 8	F 35 or 38 38	I 655 or 685 685	L 4519 or 4819 4819

4. Write each number pair on your paper. Then put the correct mark in place of the ▒. Write the two numbers in the order given.

A 2 ▒ 8 Answer: 2 < 8
B 80 ▒ 50 >
C 72 ▒ 92 <
D 72 ▒ 71 >
E 72 ▒ 78 <
F 200 ▒ 500 <
G 230 ▒ 530 <
H 837 ▒ 537 >
I 520 ▒ 580 <
J 684 ▒ 654 >
K 8237 ▒ 8537 <

think



1. Find the largest 4-digit number that uses no digit twice.
2. Find the smallest 4-digit number that uses no digit twice.
3. Find the largest 4-digit number that uses 2 digits twice each.

More practice, page A-6, Set 9

Follow-up

For more advanced children, you may wish to provide a duplicated worksheet with more difficult comparisons to make. Include an assortment of numerals up to four digits, as indicated by the sample.

Write <, >, or = in each blank to make a true statement.

27 ____ 73	999 ____ 1000
38 ____ 98	123 ____ 1023
75 ____ 717	4060 ____ 460
99 ____ 201	993 ____ 3012
325 ____ 56	6840 ____ 684
465 ____ 564	3070 ____ 3700
830 ____ 498	5505 ____ 5550

Also, the children might enjoy using the pocket chart described on page 31, with the labelling of hundreds and thousands completed. You may have the children use this chart in a variety of reviews and games. To review inequalities with 3- and 4-digit numerals, for example, you might play the following or a similar competitive game:

Divide the class into four or five equal teams, based on the class's seating plan by rows or tables. Provide a space for keeping score according to teams. Begin the game by writing a 3- or 4-digit numeral on the chalkboard. Then say to the first team "greater than," and stipulate that each member should show a number greater than the one on the chalkboard. The members of the other teams act as judges to see if each response is correct and merits a score point. Then continue similarly with the other teams, using another number, and varying "greater than" with "less than." After a few rounds, add the scores of each team to determine the winner.

Using the Exercises

On page 42, you may wish to use exercises 1 through 3 as the basis for continued discussion. They are carefully structured to lead the children to first compare the digits in the place having the greatest value. If these digits are not the same, the comparison can be made at once. If they are the same, the digits in the next place should be compared, and so on. Since the problems in exercise 4 are representative of the whole lesson, they should help you evaluate the children's skill if you assign them as written exercises.

Remind the children who do the *Think* problem that a 4-digit number is one between 999 and 10 000. Again, let those who work the problem explain their answers to the whole class.

Assignments (page 43)*

Minimum: 1–3. Average: 1–4.
Maximum: 1–4.

Resources for Active Learning

Mathex: Numeration No. 2, "Order Relations—Activity 6," pp. 20–21 (pupil pages 23–24), Encyclopaedia Britannica Publications Ltd.

Workbook, page 14

PAGES 44–45

Objective

Given numerals with as many as 9 digits, the child will be able to read them and tell how many millions, thousands, hundreds, tens, and ones they represent.

Preparation

Materials

reference materials containing a variety of statistical data; poster board or paper for wall display

In order for the investigation to be meaningful, the children must understand the meaning of the term *digit*. If you are confident they have sufficient understanding, begin immediately with the investigation. Otherwise, review the term with the children by writing on the chalkboard a few 4-digit numerals such as those that represent the current year or the number of seconds in an hour. Then ask someone to circle a digit in the tens' place or put a line under the digit in the thousands' place, to help the children recall that each symbol in the numeral is a digit.

Investigation

The children would benefit from using a variety of reference materials to search for numerals with 5 or more digits. Encourage them to note the contexts in which the numerals appear so that they will be able to tell their classmates how the numerals were used. You might suggest that volunteers record on the chalkboard a variety of their numerals, ranging from thousands through millions, for use in the ensuing discussion.

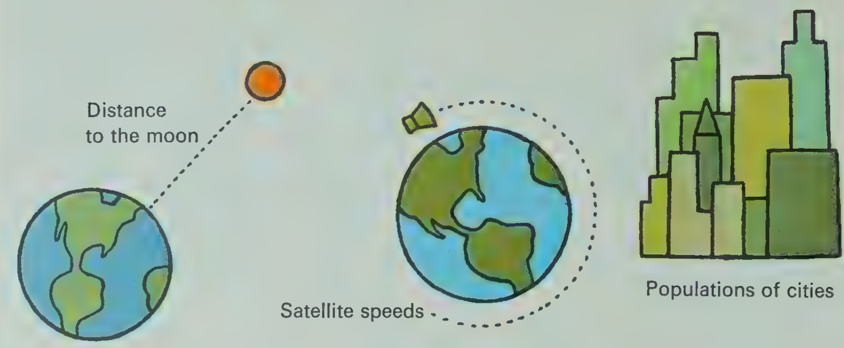
If the specific data suggested by the investigation are not appropriate to the type of reference material available, suggest alternatives. If the available reference material is wholly inadequate, you might write and discuss some of these:

Average Depth of Pacific Ocean – 4 260 m
Distance Around the Earth – 40 035 km
Speed of Light – 300 000 km/s
Population of Montreal – 2 720 413

Do you really use large numbers?

Investigating the Ideas

The pictures below suggest some large numbers.



? Can you find and read some numerals with 5 or more digits? See Investigation.

Discussing the Ideas

1. Give the number of thousands. See Discussion.
   - A 6287 Answer: 6
   - B 27 287 27
   - C 394 287 394
   - D 9564 9
   - E 37 564 37
   - F 291 564 291
   - G 53 486 53
   - H 7486 7
   - I 875 486 875
   - J 326 439 326
   - K 52 475 52
   - L 18 627 18

2. Write a numeral with 5 or 6 digits on the chalkboard. Have a classmate read it and tell how many thousands, hundreds, tens, and ones. See Discussion.

3. For 1 million, we write 1 000 000. Explain how to write
   - A 2 million. 2 000 000
   - B 3 million. 3 000 000
   - C 23 million. 23 000 000
   - D 672 million. 672 000 000

   Write 6 zeros after the digits that tell how many millions.

44

Discussion

As mentioned in the mathematics section for this lesson, the point to be stressed here is that the fourth, fifth, and sixth places name the number of thousands, while the seventh, eighth, and ninth places name the number of millions. Use several of the children's numerals from the investigation and a chalkboard demonstration to show this. For example, in the numeral 418 293 there are *418 thousands* (and we read, "*four hundred eighteen* thousand, two hundred ninety-three") and in the numeral 3 322 855 there are *3 millions* (and we read, "*three million,* three hundred twenty-two thousand, eight hundred fifty-five").

In the first discussion exercise, after children have given the number of thousands, encourage them to read the complete number.

The activity in exercise 2 should be extended until most of the children have had an opportunity to write and read a number. Then include some examples of your own, such as 400 004, 101 010, etc.

During this discussion, emphasize that the first three digits to the left of the space name the number of thousands and that the next three digits to the left name the number of millions.

Using the Ideas

1. Give the number of thousands. For part A, write 5.
For part B, write 38

A 5392 5	C 7682 7	E 53 007 53	G 100 005 100
B 38 467 38	D 23 487 23	F 467 265 467	H 999 999 999

2. Write the numeral for each exercise.

A seven thousand, two hundred twenty-six 7226
B fourteen thousand, five hundred eighty-three 14 583
C ninety-six thousand, four hundred thirty-eight 96 438
D one hundred twenty-six thousand, two hundred seventy-six 126 276
E three hundred eighty-six thousand, four hundred thirty 386 430
F nine hundred ninety-nine thousand, nine hundred ninety-nine 999 999

3. Give the next three numbers for each of these.

A	B	C	D
50 000	95 000	500 000	950 000
60 000	96 000	600 000	960 000
70 000	97 000	700 000	970 000
80 000	98 000	800 000	980 000
90 000	99 000	900 000	990 000
100 000	100 000	1 000 000	1 000 000

think

I think I'm big until
I spy
So many numbers
larger than I.
My name has a one
And zeros galore.
Seven digits in all,
and not one more.
1 000 000
WHO AM I?

See Using the Exercises.

4. Guess the correct answer to each of these questions.

A How long is 1 million seconds? (about a day; about 4 days; about 2 weeks)
B How many trips around the world would take you 1 million kilometres? (about 5; almost 10; about 25)
C How many full-sized cars weigh 1 million kilograms? (almost 5; about 40; almost 150)

Using the Exercises

Encourage the children to try to do the exercises on page 45 independently after you have ascertained that they know what is to be done. For exercise 3, be sure that they are able to recognize the pattern for each sequence.

Emphasize that for exercise 4 the children should simply try to *guess* the correct answers; they should not do any computation, and it is not expected that many children will make correct guesses. The exercise is designed primarily to develop children's consciousness of the relative size of one million. Allow ample time for discussing this and the other exercises after the children have worked through them on their own.

All the children may enjoy trying the *Think* problem, though you will probably need to discuss with them the meaning of "galore" in the phrase "zeros galore."

Assignments (page 45)*
Minimum: 1–2, oral. Average: 1–3. Maximum: 1–4.

Mathematics

In the place-value system, each place through the fourth place has a distinct name. (The first place is the ones' place, the second is the tens', the third is the hundreds', and the fourth is the thousands'.) Numerals of 5 or more digits, however, present a special problem in that there is not a unique name for each place in the thousands' period, the millions' period, and so on. We might claim that the fifth place is called the ten thousands' place and that the sixth place is called the hundred thousands' place, but actually the term *thousands* still applies to those two places. In these beginning lessons, do not call attention to such special names for the fifth and sixth places or for the eighth and ninth places; rather, stress that all three places in the thousands' period and in the millions' period name the number of thousands and the number of millions, respectively.

(In this book, the space is generally used only in numerals of 5 or more digits. Occasionally, however, a space is used in a 4-digit numeral to illustrate a certain point or to achieve uniformity in specific contexts.)

Follow-up

You might like to have the children make up a display from the results of their investigations for this lesson. Some children might make individual posters or charts illustrating a numeral of 5 or more digits and its use.

Resources for Active Learning

Developmental Math Cards, F18, Addison-Wesley. [An exploration of large numbers that can be found in the student's own environment]

Duplicator Masters, page 9
Skill Masters, page 9

PAGES 46–47

Objective

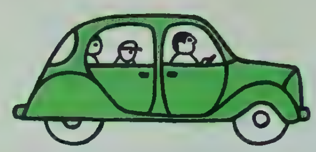
Given appropriate data and related word problems, the child will be able to use larger numerals and make comparisons involving place-value concepts to solve word problems.

Preparation

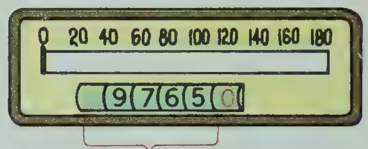
Review inequalities by asking each child to think of a 3-, 4-, or 5-digit numeral. Then call on pairs of children to write their numerals on the chalkboard. Let the class compare the two numerals and have a volunteer put the proper symbol between each pair. If you prefer, you may choose the pairs of numerals yourself, including several 5-digit numerals.

Solving Story Problems Travel Fun

Jim liked to watch the odometer on his father's car when he and his family were on vacation last summer. When they started the trip the odometer looked like this:



The odometer counts kilometres travelled.



This part tells the number of kilometres.

1. How far had Jim's family travelled when the odometer looked like this? → 9 7 8 5 20km

2. Jim saw this sign.

He looked at the odometer and saw this. → 9 7 9 8

What did the odometer show when they reached Redwood? 9898

3. At Wood City the odometer looked like this. → 1 0 7 8 4

At River City the odometer looked like this. → 1 0 7 9 4

How far is it from Wood City to River City? 10km

4. Logtown is 100 kilometres farther from Jim's house than Fish Hook is.

The odometer looked like this at Fish Hook. → 1 1 0 6 5

What do you think the odometer read at Logtown? 11 165

46

Discussion

Read and discuss the top of page 46 with the children. Stress in particular the word *odometer* and its accompanying diagram. You might draw a model odometer to show how it illustrates place value as it records kilometres. Explain that in the diagram in the book the tenths are shown on the odometer but that the bracket indicates the part of the odometer which shows the number of kilometres in whole numbers. In this first diagram the tenths' place is included because many of the children are used to seeing this on the odometers of cars they ride in, but in the other odometer diagrams the tenths' place is not included.

Have the children try to work the exercises on page 46 independently. If the class seems to have difficulty in reading and interpreting the exercises, discuss the problems and how the children's knowledge of place value can help to solve them. Then let the children proceed independently.

Assignments (page 46)

Minimum: 1–4, oral. Average: 1–4. Maximum: 1–4.

Mountain Peaks in North America

The table gives the heights of some of the highest mountains in North America. The first two mountains in the table are the highest in North America.

NAME	PLACE	METRES
McKinley	Alaska	6096
Logan	Canada	5955
Citlaltepetl	Mexico	5610
King	Canada	5139
Steele	Canada	4932
Bona	Alaska	4926
Wood	Canada	4765
Bear	Alaska	4455
Whitney	California	4348
Elbert	Colorado	4329
Rainier	Washington	4323
Lincoln	Colorado	4285

1. What mountain peak in the table is more than 5100 metres and less than 5400 metres ? King
2. What mountain peak in the table is between 4500 metres and 4800 metres ? Wood
3. How many peaks in the table are less than 4800 metres ? 6
4. How many peaks in the table are more than 4500 metres ? 7 How many are more than 4200 metres ? 12
5. If an airplane flies at 5100 metres, how many of these peaks could it fly over ? 8
6. Which mountain peaks are more than 4320 metres and less than 4350 metres ? Whitney, Elbert, and Rainier
7. How much higher is Mount Steele than Mount Bona ? 6 m
8. How much higher is Mount Elbert than Mount Rainier ? 6 m
9. Mount Hubbard (not listed in the table) is 30 metres higher than Mount Bear. How high is Mount Hubbard ? 4485 m
10. How much higher is Mount Wood than Mount Bear ? 310 m

47

Using the Exercises

Before discussing page 47 with the children, give them an opportunity to read and study the material on their own. They will probably want to read the names of the mountains and attempt to pronounce them aloud. Encourage them to observe that heights in the table are arranged from highest to lowest. It would be worthwhile practice for the children to read the heights aloud before tackling the word problems independently. When they have finished, discuss each problem. Emphasize the problems that focus attention on place-value concepts. Note in problem 8 that the altitudes of Mount Rainier and Mount Elbert differ only in the units' place. Use examples like this to strengthen the children's understanding of place value.

Assignments (page 47)

Minimum: 1–10, oral. Average: 1–10. Maximum: 1–10.

Follow-up

To give more practice in comparing numbers using place-value concepts, instruct the children to order groups of four numbers. Write the groups on the chalkboard and ask the children to copy and arrange them in proper order, or prepare and duplicate a worksheet similar to the following.

Rearrange each set of numbers from least to greatest.

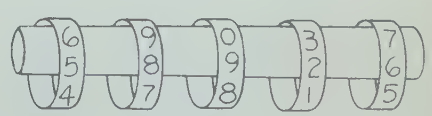
1. 689 52 1683 100
2. 5654 5454 5154 5954
3. 529 5280 592 2085
4. 7920 7990 7290 7092
5. 13 010 13 100 13 001 11 300

To challenge more capable children, include some exercises similar to the following.

Think about the number 8253. Now answer these questions.

1. What number is 100 and 10 more than 8253?
2. What number is 1000 and 3 less than 8253?
3. What number is 253 less than 8253?
4. What number is 8000 and 50 less than 8253?

The children might enjoy making an odometer and using it to show different numerals to a partner. Materials needed are a cardboard tube such as a paper-towel roller, 5 strips of paper (approximately 2 cm by 12 cm), and some tape. Have the children number the strips of paper from 0 to 9 and then loosely fit them around the tube. By rotating the strips, they can show any 5-digit numeral on the tube.



Resources for Active Learning

Mathex: Numeration No. 2, "Order Relations—Activity 7," p. 21, Encyclopaedia Britannica Publications Ltd.

Workbook, page 15

PAGES 48–49

Objectives

The child will demonstrate his ability to work with the concepts introduced in this chapter.

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

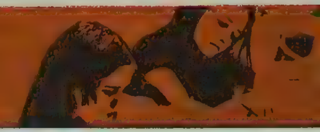
Preparation

Materials

centimetre rulers

Write several large numerals on the chalkboard and ask the children to list them in order. To focus attention on the place-value concepts shown, have the children read the number of ones, tens, hundreds, and so on. Then, circle a particular digit from each numeral with colored chalk, and ask one of the children to say the number which that digit stands for. You might vary the activity by saying, "I'm thinking of a numeral with 7 in the tens' place. Which numeral on the chalkboard am I thinking of?" Occasionally specify a digit that appears in more than one numeral in the list.

Reviewing the Ideas



1. Give the number for each set of sticks.

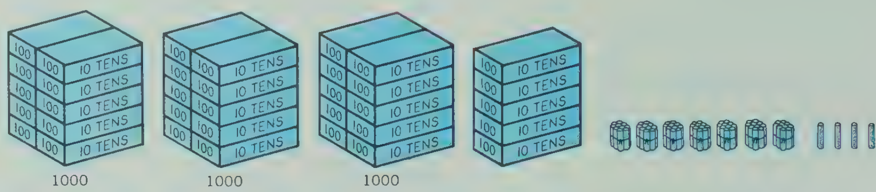
A 32 

B 25 

C 234 

2. Write the numeral for each of these.
   - A 7 tens and 6 76
   - B 3 tens and 0 30
   - C 5 tens and 2 52
   - D 6 tens and 9 69
   - E 8 hundreds, 6 tens, and 5 865
   - F 7 hundreds, 1 ten, and 2 712
   - G 1 hundred, 0 tens, and 9 109
   - H 9 hundreds, 9 tens, and 9 999

3. How many sticks in this set? 3574



4. Write the numeral for each of these.
   - A seventy-five 75
   - B forty-six 46
   - C two hundred eighty-seven 287
   - D four hundred ninety-three 493
   - E six thousand, four hundred thirty-two 6432
   - F twenty-three thousand, one hundred sixty-nine 23 169
   - G four hundred sixty-eight thousand, two hundred twenty-one 468 221

5. Give the next three numbers in each counting sequence.
   - A 16, 17, 18, . . . 19, 20, 21
   - B 55, 56, 57, . . . 58, 59, 60
   - C 96, 97, 98, . . . 99, 100, 101
   - D 127, 128, 129, . . . 130, 131, 132
   - E 397, 398, 399, . . . 400, 401, 402
   - F 996, 997, 998, . . . 999, 1000, 1001

6. Give the correct sign > or < for each ▒.
   - A 68 ▒ 78 <
   - B 43 ▒ 33 >
   - C 127 ▒ 327 <
   - D 546 ▒ 526 >
   - E 3285 ▒ 3286 <
   - F 9463 ▒ 9763 <

48

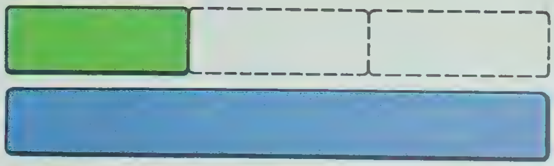
Discussion

You may treat page 48 either as a chapter review or as a test of the children's achievement for the chapter. If you use it as a review, you may also want to use the exercises as a guide in designing an evaluation instrument.

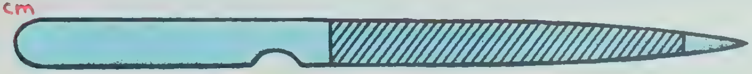
Some children may need to have you read the directions for certain exercises. Wait for them to write down answers before going on to the next section. A previously duplicated answer sheet may help these children organize their work so that you get a better evaluation of their mathematical skills.

Keeping in touch with Counting Measurement Fractions

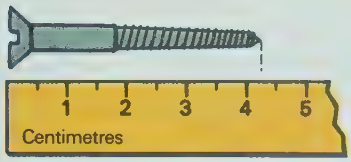
1. How many green strips long is the blue strip? 3



2. Use your ruler to measure the nail file to the nearest centimetre. 12 cm

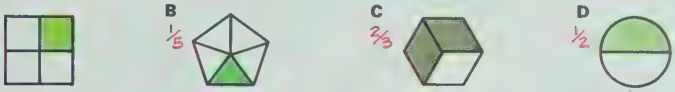


3. A Is the tip of the screw nearer to 4 centimetres or to $4\frac{1}{2}$ centimetres? $4\frac{1}{2}$
   B To the nearest half centimetre, the length is ▥ centimetres. $4\frac{1}{2}$



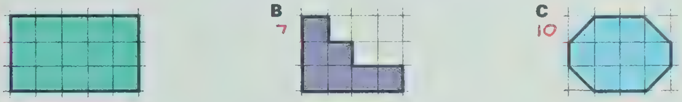
4. Give the fraction that tells what part of each region is colored.

A $\frac{1}{4}$ B $\frac{1}{5}$ C $\frac{2}{3}$ D $\frac{1}{2}$



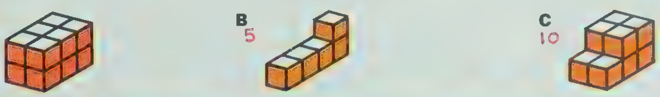
5. Find the area of each shaded region. Each small square is a unit.

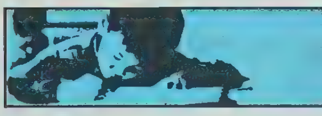
A 15 B 7 C 10



6. Give the volume of each figure below.

A 12 B 5 C 10





You are invited to explore

ACTIVITY CARD 1
Page 309

49

Using the Exercises

Page 49 may be assigned as independent work or used as a group review. Remind the children that, in order to measure to the nearest centimetre, they should use the ½-centimetre marks as guides in making their measure. Estimation is necessary for measurement to the nearest half-centimetre. For exercise 7, some children might want to use some cubes to build the actual shapes illustrated.

Follow-up

Let the children play the Place-Value Card Game suggested on page 41. Give them several numerals with 5 or 6 digits.

You might also prepare a worksheet which includes the following types of exercises:

> Count and write in the missing numerals.
> 97, 98, ___, ___, ___, ___, ___, 104
> 578, 579, ___, ___, ___, ___, ___
> 4097, 4098, ___, ___, ___, ___

> Use the correct symbol to compare these pairs of numerals.
> 752 ○ 725   41 857 ○ 42 310
> 3919 ○ 1999   2 357 112 ○ 999 999
> 1021 ○ 1111   8979 ○ 8797
> 425 ○ 245   101 090 ○ 100 990

> Give the numeral for each of the following.
> two hundred eighty-nine
> one thousand three hundred
> six thousand four hundred ninety-seven
> twenty-five thousand, five hundred four
> five million, three hundred twelve thousand, two hundred forty-nine

Workbook, page 16

Pages 50–73

General Objectives

To focus attention on the concepts of addition and subtraction
To provide experiences in working with equations
To improve skills with addition and subtraction facts for sums through 18
To provide experiences in working with word problems
To focus attention on the inverse relationship between addition and subtraction

This chapter is devoted primarily to exploring addition and subtraction concepts, but there is sufficient practice material for a review of addition and subtraction facts.

One of the main points of emphasis in the chapter is the relationship between addition and subtraction. Children are led to see that finding a difference is equivalent to finding a missing addend. Mastery of addition is given more attention than mastery of subtraction facts, since children should be able to arrive at subtraction facts if they have a good knowledge of addition facts.

Next, the commutative (order) and associative (grouping) principles for addition are introduced. The two principles are quickly generalized to express the idea that we can rearrange the addends in any convenient way when adding three or more numbers. The associative principle is used as the basis for finding sums greater than ten. For example, to find the sum $7 + 5$, the 5 is broken into $3 + 2$. Then, using the associative principle, 7 and 3 are grouped to give the sum $10 + 2$, or 12.

Mathematics

The sum of two cardinal numbers is introduced through the union of disjoint sets. Since disjoint sets have no common elements, they illustrate the idea well. The union of sets A and B, denoted by $A \cup B$, is the set containing those elements that are in A, or in B, or in both A and B. The following examples illustrate this definition.

Example 1. Consider sets A and B that are not disjoint:

$A = \{m, n, o, p\}$, $B = \{p, q, r\}$,

then

$A \cup B = \{m, n, o, p, q, r\}$.

Example 2. Consider two disjoint sets R and S as follows:

$R = \{1, 2, 3\}$, $S = \{15, 16\}$,

then

$R \cup S = \{1, 2, 3, 15, 16\}$.

Note that in example 1, set A contains 4 elements and set B contains 3 elements. The sets are not disjoint because element p belongs to both. The union $A \cup B$ contains 6 elements because p is listed only once. Thus, example 1 does not illustrate the addition concept. Note that in example 2, set R contains 3 elements and set S contains 2 elements. These sets are disjoint, and therefore, $R \cup S$ contains 5 elements, which does illustrate addition. These examples emphasize the importance of the word disjoint in the definition for the sum of two cardinal numbers:

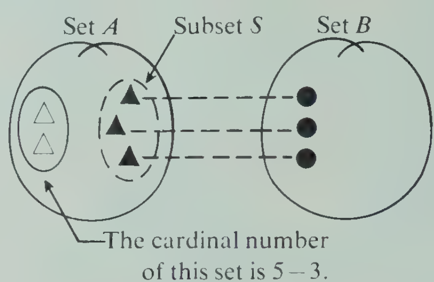
> Consider cardinal numbers, a and b, and disjoint sets, A and B, from these cardinal numbers. The cardinal number of the union of sets A and B is the sum of the cardinal numbers a and b (written $a + b$).

The sets children put together usually have no common elements, and forming the union of two sets merely requires pushing the objects together into one group.

We define subtraction in two ways: first in terms of a comparison of sets and then in terms of addition. Removing objects from a given set is a clear, concrete method for introducing subtraction. Closely related to this "take-away" idea is the "comparison" idea of subtraction. In fact, careful analysis of the definition below will show that take-away is a particular case of the more general comparison of sets.

> Let a and b be any two cardinal numbers such that $a > b$ or $a = b$. Choose sets A and B from a and b, respectively. There is a subset S of A that is equivalent to B. The difference of a and b (written $a - b$) is the cardinal number of the set of objects in A other than those in S.

The following diagram illustrates the definition for cardinal numbers 5 and 3. The associated subtraction equation is $5 - 3 = 2$.



The definition and diagram show how subtraction is used to compare two sets. The following paragraph explains how removing objects from a single set relates to this definition.

If you tell a child to take away 3 objects in set A, he will match 3 objects in set A one-to-one with the set "one, two, three" in his mind and then remove the 3 objects. Actually, he is imagining his own set of 3 and matching it with a subset of A, just as set B in the illustration is matched with set S. For an example using larger numbers, put 25 objects on a table and ask a child to remove 17 of them. Thus, take-away is a special case of the general method of subtraction, comparison of sets.

Now, let us investigate a definition of subtraction in terms of addition.

> Let a, b, and c be whole numbers such that $a + b = c$.
> The number a is the difference

$c - b$, and the number b is the difference $c - a$. In symbols,

$$a = c - b \quad \text{and} \quad b = c - a.$$

According to this definition of subtraction, two subtraction equations are associated with each addition equation. Sometimes we wish to associate only one subtraction equation with a given addition equation. For example, we can think of $5 + 3$ to show adding 3 to 5, and $8 - 3$ to show subtracting 3 from 8. It is useful to link the two equations,

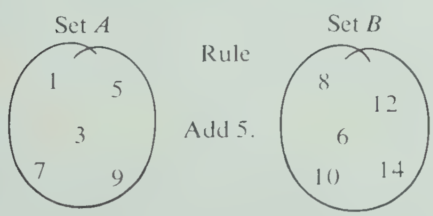
$$5 + 3 = 8 \text{ and } 8 - 3 = 5.$$

We say that adding 3 and subtracting 3 undo each other: We start with 5, add 3, and get 8; then we subtract 3 from 8, and return to 5 again. In general, if a, b, and c are whole numbers such that $a + b = c$, we link the two equations,

$$a + b = c \quad \text{and} \quad c - b = a.$$

This is the inverse relation for addition and subtraction.

The concept of a function, one of the most important ideas in mathematics, is presented to children on an intuitive basis. Stress that each function consists of a set and a rule. When we apply the rule to an element of the set, we get just one answer, resulting in a set of ordered pairs for each function. For example:



The ordered pairs are:
(1, 6), (3, 8), (5, 10), (7, 12), (9, 14).

Teaching the Chapter

Materials

Centimetre ruler (1 per child)
Colored chalk
Colored strips (1 set per child)
Counters or a substitute such as bottle caps, toothpicks, etc.
Demonstration number line
Felt objects, symbols, and numerals
Flannelboard
Slips of paper (approximately 5-by-10 cm)

Vocabulary

add	matching lines
addend	minus
addition	number line
combination	order principle
compare	output
difference	plus
equals	set
equation	solution
function machine	solve
grouping principle	subtract
input	subtraction
matching	sum

Symbols

$+$ plus $\quad -$ minus

Some of the investigations give the child the opportunity of manipulating set materials, counters, etc. Let him do so as long as it is necessary, but encourage him to use the colored strips where appropriate and eventually to work without the aid of physical objects. For further information regarding use of the strips, refer to *Mathematical Awareness*, by John V. Trivett, Parts 1 and 2 (New York: Cuisenaire Company, 1963).

Possible activities are suggested to help slower children and to encourage more capable children. Though the *Think* problems are intended primarily for the more able pupils, some of them are fun for everyone to try. Urge those who solve these problems to share their solutions with the rest of the class.

Lesson Schedule

If the children have used Books 1 and 2 in this series, you should be able to cover this chapter in less than three weeks. However, if they have not, you may need four and a half or five weeks.

Evaluation of Progress

It is often difficult to judge accurately whether children understand the concepts of addition and subtraction. Daily observation is a good way to determine such understanding. Use the chapter review on text page 72 for evaluation purposes or as a guide in designing your own test.

Note that mastery of addition and subtraction facts is one of the objectives of this chapter. However, do not feel that you must dwell on this objective until *every* child knows the facts. Work toward understanding for most, and then continue to help slower children as you proceed through the text.

Resources for Active Learning

GENERAL ACTIVITIES

Developmental Math Cards F#2, Addison-Wesley
Dienes Multibase Arithmetic Blocks, Tasks and Manual, Cards 6A-12, Herder and Herder (Available from Methuen Publications)
Mathex: Operations No. 3, "Adding Machines," pupil pages 18–25, Encyclopaedia Britannica Publications Ltd.
Maths Mini Lab, Starter Activity Cards, Selective Educational Equipment
Math Workshop: Games and Enrichment Activities, "Cuisenaire Rods" (Games), pp. 38–49, Encyclopaedia Britannica Educational Corp.
Nuffield Project: *Computation and Structure 2*, "Operation of Addition," pp. 58–69, Wiley
Nuffield Project: *Computation and Structure 3*, "Addition," pp. 2–13, Wiley
Toward Improving Computation (Bean Sticks), pp. 49–54, Curriculum Development Associates

MANIPULATIVE DEVICES

Attribute Games and Problems (Selective Educational Equipment; Webster, McGraw-Hill)
Cuisenaire Rods (Cuisenaire Co.)
Dienes Multibase Arithmetic Blocks (Herder and Herder)
"Invicta" Math Balance (Math Media; Selective Educational Equipment)
SEE Calculator (Selective Educational Equipment)
Unifix Math Lab Kit (Educational Teaching Aids; Math Media; Responsive Environments Corp.)

COMMERCIAL GAMES

Club Dominoes (school supplier)
One, Two, Three, Think! (Selective Educational Equipment).
Setsplay (Selective Educational Equipment)
TUF (Creative Publications; Cuisenaire Co.; TUF).

PAGES 50–51

Objective

Given a simple addition problem, the child will be able to write an addition equation to describe it and a subtraction equation to describe the related subtraction process.

Preparation

Materials

counters or substitutes such as bottle caps, toothpicks, etc.

In the investigation the children will be asked to write addition and subtraction equations. If the children used Books 1 and 2 of this series, they should be able to complete the investigation successfully without any special preparation. However, you might wish to review equations briefly by writing some unrelated addition and subtraction equations on the chalkboard and having the children read them.

Investigation

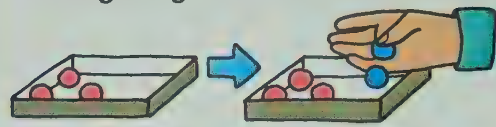
One of the main goals of this investigation is for children to gain an understanding of the relationship of addition to subtraction. It would be helpful for the children to work in groups of two or three and talk with each other about what they are doing.

Read the investigation section with the children, making sure that they read the equations properly; for instance, they should read $3 + 2 = 5$ as, "Three plus two equals five," and $5 - 2 = 3$ as, "Five minus two equals three." Give at least ten counters to each group. As they work, move from group to group making sure the children start with the specified number of counters, put in the specified number of counters, and then remove from the total the same number they put in. The children should record this addition and subtraction process in equation form. If for any reason you prefer not to use counters, have the children talk about the inverse process pictured in the text. At this level, they should have no difficulty imagining the adding and taking away processes, but most children would benefit from a concrete investigation of these concepts.

3 Addition and Subtraction

● *Are addition and subtraction related?*

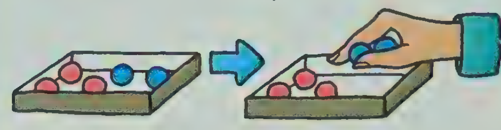
Investigating the Ideas



Start with 3 put in 2

$3 + 2 = 5$

and write an addition equation.



Now you have 5 take out 2

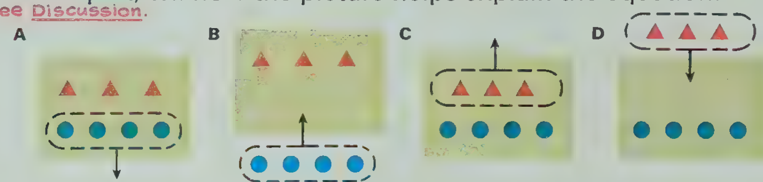
$5 - 2 = 3$

and write a subtraction equation.

? Can you do this using different numbers of counters?	Write the equations. See Investigation.

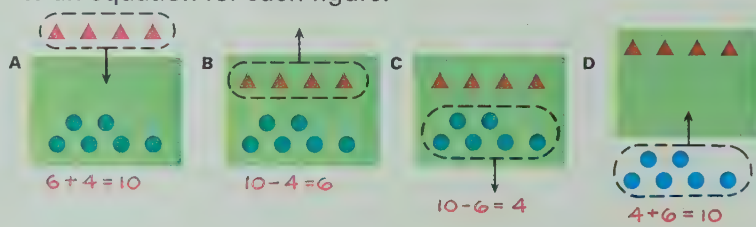
Discussing the Ideas

1. In each part, tell how the picture helps explain the equation. See Discussion.



$7 - 4 = 3$ $3 + 4 = 7$ $7 - 3 = 4$ $4 + 3 = 7$

2. Write an equation for each figure.



Discussion

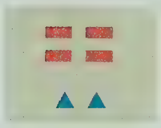
Before discussing the two exercises on page 50, ask some children to describe and show with counters the addition and subtraction processes they used for different numbers of counters. Ask them to tell you the pair of equations and write them on the chalkboard. Use the example to emphasize the inverse relationship, somewhat like this:

"If $4 + 5 = 9$ and $9 - 5 = 4$ are the equations, we begin with 4 and add 5, and then subtract 5 and return to 4."

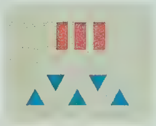
Use discussion exercises 1 and 2 not only to review the basic addition and subtraction processes but also to relate these processes by focussing on the given equations in pairs A and B and pairs C and D. According to class need, further develop the relation between addition and subtraction by using demonstrations showing the four equations associated with the breaking up of a given set.

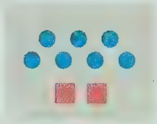
Using the Ideas

1. Write four different equations for each set.

Example: 

Answer:
$4 + 2 = 6$
$6 - 2 = 4$
$2 + 4 = 6$
$6 - 4 = 2$

A 
3+5=8, 8-5=3
5+3=8, 8-3=5

B 
7+2=9, 9-2=7
2+7=9, 9-7=2

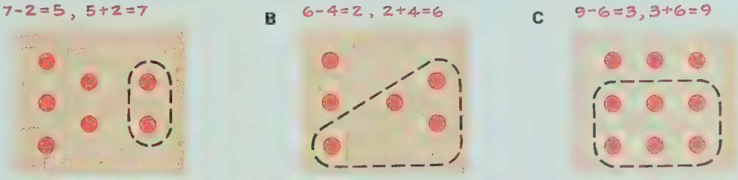
2. Answer the questions about the sets.

A How many dots in sets U and V together? 8
B How many dots in sets W and Y together? 6
C How many dots in sets V and Z together? 6
D How many dots in sets U and Y together? 7
E How many dots in sets W and X together? 2

3. Write an addition equation for each part of exercise 2.
A 3+5=8 B 2+4=6 C 5+1=6 D 3+4=7 E 2+0=2

4. Think about removing the dots inside the dotted ring and write a subtraction equation for each set. Then think about putting the dots back inside the dotted ring and write an addition equation for each set.

A 7-2=5, 5+2=7 B 6-4=2, 2+4=6 C 9-6=3, 3+6=9



5. Write two addition and two subtraction equations using the numbers 4, 5, and 9. 4+5=9, 5+4=9 9-5=4, 9-4=5

6. Use 5, 3, and one other number and write two addition and two subtraction equations. 5+3=8 3+5=8 8-5=3 8-3=5 or 2+3=5 3+2=5 5-2=3 5-3=2

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Using the Exercises

On page 51, work one part each of exercises 2, 3, and 4 with the children, if necessary. Then ask them to finish the page independently by writing the answers on their papers. Allow ample time for checking papers and answering questions.

Assignments (page 51)
Minimum: 1–4. Average: 1–6.
Maximum: 1–6.

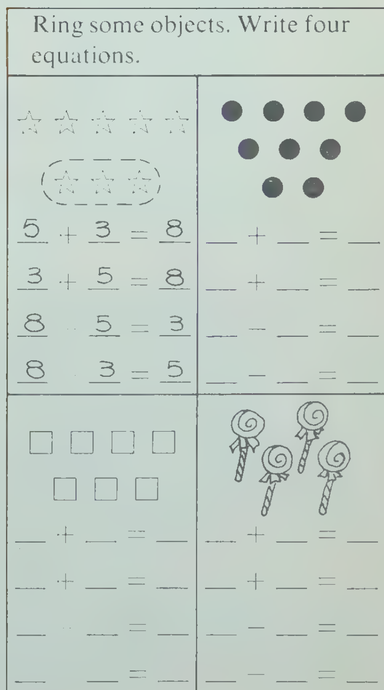
Mathematics

The discussion exercises illustrate the two addition and two subtraction equations which are associated with the breaking up of a given set into subsets. In the first exercise, arrows and dashed rings show the children how to think about the set in four different ways and arrive at four different equations. This demonstration is concerned with the more formal aspects of set union and subsets.

Our basic definition of addition in terms of sets makes no apparent distinction between $3 + 4$ and $4 + 3$, but this distinction can be made for the children if we show one set being put with another, or vice versa.

Follow-up/Equation Families

To provide children more practice in writing the equations associated with given subsets of a set, draw sets on the chalkboard or distribute worksheets, such as the one shown. Ask the pupils to separate each set into two parts by drawing a ring around any of the objects they choose. Then have them write two addition and two subtraction equations which correspond to the subsets they have made.



Workbook, page 17

PAGES 52–53

Objectives

Given appropriate addition and subtraction equations, the child will be able to use a number line to illustrate the equations. Given number-line representations of addition and subtraction problems, the child will be able to write related equations.

Preparation

Materials

centimetre ruler (1 per child); colored strips (1 set per child)

To prepare for this lesson, spend just a few minutes talking about things which are like a number line, such as a dress or shoe rack which lines up sizes, post office mail boxes, or street addresses. Then have the children study the edge of their ruler. Focus attention on the whole numbers and help them think of the edge of the ruler as a part of a number line for whole numbers.

Investigation

This investigation reintroduces the children to number-line activities by using the familiar centimetre ruler to represent the number line and the colored strips to indicate jumps along the number line.

Have the children individually use their ruler and colored strips to show the addition. After they have found the strips to illustrate $5 + 2$, encourage them to use their strips to find the other possible combinations of addends of 7, that is, $6 + 1$, $4 + 3$, $7 + 0$. (Note that for $7 + 0$ they should use only the single 7-strip.)

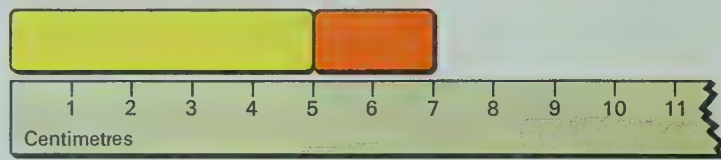
You might want to write a few more suggestions on the chalkboard, such as "How many different pairs of strips can you use to end at 6? at 8? at 9?" Ask the children to record each of their results in the form of an equation.

● *Can the number line help you with addition and subtraction?*

Investigating the Ideas

You can use your strips and your centimetre ruler to show addition.

$$5 + 2 = 7$$



? How many other pairs of your strips can you use to add to 7?

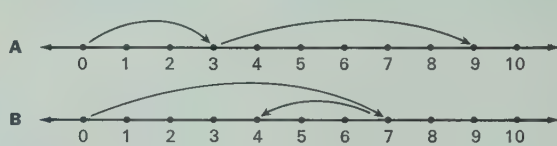
See Investigation.

Discussing the Ideas

1. How can you use your strips and ruler to show the subtraction equation? $7 - 2 = 5$

   Show 5-strip plus 2-strip and then remove the 2-strip.

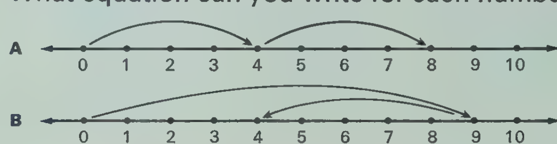
2. We can use **arrows** and a **number line** to show addition and subtraction instead of strips and a ruler. Give the missing number in the equation for each number line.



A $3 + 6 = n$ 9

B $7 - 3 = n$ 4

3. What equation can you write for each number-line picture?



A 4+4=8

B 9−5=4

52

Discussion

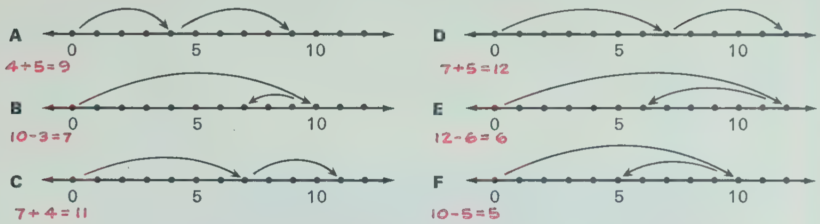
Devote special attention to the first discussion exercise. If a child has difficulty showing subtraction, have him repeat the addition problem and then remind him of how the subtraction equation is related. If need be, let him grapple with the problem until he realizes that the subtraction can be shown in this exercise by removing the red 2-strip, which is the equivalent of 2 jumps to the left on the number line.

If an overhead projector is available, you might want to show (or have the children show) some of the exercises and the correct results. In either case, always relate the number-line representation to a written equation. It would be best to have the children use two procedures: first, write equations associated with given jumps on the number line; second, show jumps for given equations.

Discussion exercises 2 and 3 demonstrate the former procedure. But the emphasis here is on the two operations, addition and subtraction. Stress the direction that is used for each: to the right for addition and to the left for subtraction. To avoid any confusion about the top arrow in the subtraction pictures, explain that it simply pictures where you are to begin.

Using the Ideas

1. Write an equation for each number-line picture.

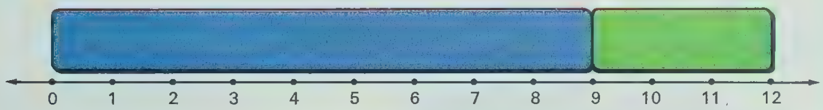


2. Draw a 12-centimetre line on your paper. Starting at the 0 dot, mark dots each 1 centimetre and label them as shown.

A Use arrows to show 5 + 6. B Show arrows for 11 − 9.

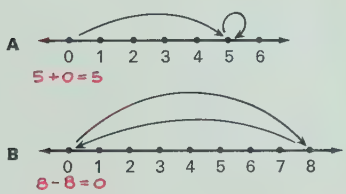
3. Write an equation for each part of exercise 2.
A 5+6=11 B 11−9=2

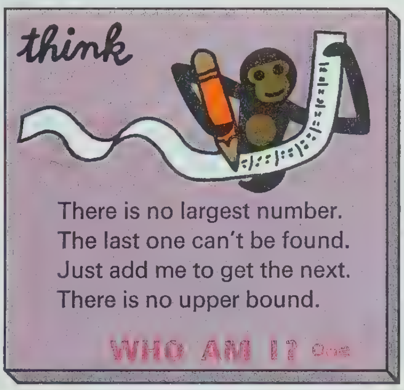
4. Write an addition equation for the picture. 9+3=12



5. Draw a number line and show the sum 3 + 3 + 3 + 3.

★ 6. Write an equation for each number-line picture.





53

Using the Exercises

Assign exercises 1 through 5 on page 53 for all the children. You might wish to consider giving each child a duplicated sheet with blank number lines for showing the jumps in exercises 2 and 5. This would allow children with less-developed motor skills to complete the mathematics of the page without getting bogged down in drawing and labelling number lines. You might also wish to give a few more, similar number-line exercises; these could be shown on the same worksheet. Allow time for discussion after the children have finished.

Note that, although exercise 6 is starred, many children will recall the effect of 0 in addition and should be able to work out the required equations: $5 + 0 = 5$ and $8 - 8 = 0$.

The *Think* problem is fairly easy, and everyone should be given an opportunity to consider the ideas presented. Be sure that no one gives the answer prematurely, spoiling the other children's chance to discover the correct response for themselves.

Assignments (page 53)
Minimum: 1–4. Average: 1–5.
Maximum: 1–6.

Mathematics

This lesson treats the ruler and then the number line primarily as physical devices for showing the order of the set of whole numbers. Below is a model of the number line.

Notice that certain points on the number line are labelled with whole numbers. The unit distance, the distance between each pair of points, is the same. (We can also find points on the number line which do not represent whole numbers—points which represent fractional or negative numbers; but the children are not yet ready to consider these.) Although the number line could point in any direction, it is conventional to place it in a horizontal position with the numbers, represented by points, progressing from left to right. The arrows indicate that the number line has no endpoints in either direction. Though there is an arrow to the left of zero, the class should not yet be concerned with negative numbers.

Addition and subtraction equations are shown by "jumps" on the number line. For example, in the "ruler number line" at the top of page 52, the combination $5 + 2$ is shown by a strip (jump) from 0 to 5, followed by a strip of 2 more units added to the *right*. In exercise 2B of the discussion section, the combination $7 - 3$ is shown by a jump from 0 to 7, followed by a jump of 3 units to the *left*.

Follow-up

The children might enjoy a game using a number line marked off on the floor with masking tape or adding machine tape. If you make the units about 15 centimetres, the children can be "Jumping Jacks" and jump (or step) out equations suggested by classmates.

Resources for Active Learning

Developmental Math Cards, D119, Addison-Wesley. [Writing number-line sentences]

Workbook, page 18

PAGES 54–55

Objective

Given a variety of pictorial and numerical representations of addition situations, the child will recognize and be able to perform the appropriate operations, and to identify addends and sums.

Preparation

Materials

colored strips (1 set per child)

Since this lesson is concerned primarily with reviewing ways of thinking about addition and no new concepts are presented, the investigation can be started without special preparation. Note the suggested "strip board" in the follow-up section.

Investigation

Although the investigation introduces the terms *addend* and *sum*, the new terms need not be emphasized. The children will experience little difficulty in using the terms correctly and naturally after the exposure provided by the activities, discussions, and exercises of this lesson. The introduction of the terms here is an incidental, albeit useful, adjunct to the main objectives of the lesson—to provide a review of the ways the children have learned to think about addition and to present a variety of experiences that will reinforce their understanding of these approaches while reviewing the simpler arithmetic facts.

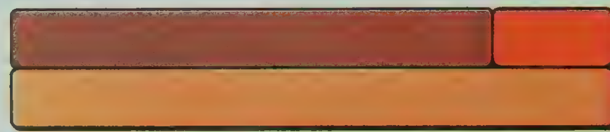
For the investigation, explain simply that it is convenient to have labels for the elements involved in addition situations. In the situation presented in the investigation, we could refer to the elements being added as "the numbers of the brown strip and the red strip" or "the numbers of the 8-strip and the 2-strip," and we could call our result "the number of the orange strip or 10-strip." Often, though, we want to talk about the parts of an addition equation without referring to specific numbers. Hence, it is helpful for us to be able to use the terms *addend* and *sum*.

Let's explore addends and sums.

Investigating the Ideas

8	+	2	=	10
Addend		Addend		Sum

	8	Addend
+	2	Addend
	10	Sum



? Can you use your strips to find other pairs of addends that will give a sum of 10? | Write an equation for each pair you find.

$10+0=10$
$9+1=10$
$7+3=10$
$6+4=10$
$5+5=10$

See Investigation.

Discussing the Ideas

1. Each picture suggests addition. Which numbers are addends? What is the sum?

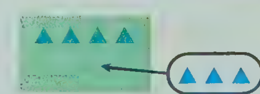
A

Addends: 2, 3
Sum: 5

B

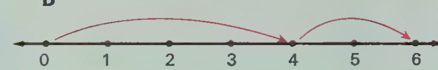
Addends: 6, 3 Sum: 9

C



Addends: 4, 3 Sum: 7

D



Addends: 4, 2 Sum: 6

2. The sum is 8. One addend is 5. The other addend is 3. Explain how to write an addition equation for these three numbers.
See Discussion.
3. Use 2 as an addend as many times as you need to write an equation with a sum of 10. $2+2+2+2+2=10$

54

Discussion

Allow adequate time for children to complete the investigation and discuss their results. Encourage them to use the words *addends* and *sums* when appropriate in talking about the pairs of strips and the results of their addition.

In the first exercise, the children are given further experience in using the terms *addend* and *sum*, and are reminded of the various ways they learned previously to think about addition.

As the children give their answers for exercise 2, encourage them to explain what is meant by the terms sum and addend. Then use this exercise to point out an example of another way of thinking about addition, namely, reasoning. For example, if we know 8 is the sum of 5 and 3, we could figure out that 5 plus 4 is 9. We added 1 to one of the addends, so we see that the sum must be 1 more also. Similarly, if we know that $3+3=6$, we should be able to solve $3+\square=7$. It would be helpful to use similar examples enabling children to see different ways of approaching a familiar addition fact.

Using the Ideas

1. The addends are given. Find the sums.

A	B	C	D	E	F	G
3 + 3 = 6	3 + 4 = 7	3 + 2 = 5	4 + 4 = 8	4 + 3 = 7	4 + 5 = 9	5 + 5 = 10

2. Solve the equations.

A $5 + 4 = n$ 9 C $2 + 6 = n$ 8 E $4 + 5 = n$ 9 G $6 + 2 = n$ 8
B $3 + 6 = n$ 9 D $5 + 5 = n$ 10 F $2 + 7 = n$ 9 H $8 + 2 = n$ 10

3. Give the missing numbers in the addition tables.

	Add 2	
	5	7
	3	5
A	6	8
B	4	6

	Add 4	
	4	8
C	5	9
D	3	7
E	6	10

	Add 3	
	7	10
F	5	8
G	6	9
H	4	7

	Add 5	
I	2	7
J	5	10
K	3	8
L	4	9

4. Copy each addition table and give the missing numbers.

A

+	5	4
3	8	7
2	7	6

B

+	4	6
4	8	10
3	7	9

C

+	5	0
1	6	1
4	9	4

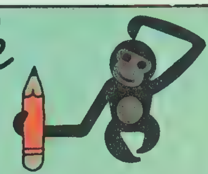
D

+	7	6
2	9	8
3	10	9

5. Give the missing numbers in the table.

	Addend	Addend	Sum
	3	2	5
A	5	3	8
B	4	2	6
C	7	2	9
D	2	3	5
E	3	5	8
F	6	4	10

think



Find two numbers so that
their sum is 9
and
their difference is 9.
9, 0

More practice, page A-7, Set 10

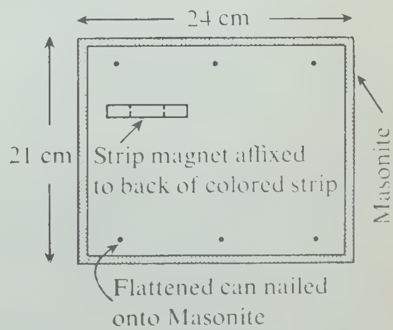
Mathematics

This lesson reviews three ways of thinking about sums. As mentioned in the mathematics section at the beginning of this chapter, addition may be defined in terms of the union of disjoint sets. Third graders need not know the set terminology, but they should be able to understand how the joining of two sets relates to addition. The number line provides a second approach; as explained in the previous lesson, addition is pictorially represented by jumps to the right on the number line. A third way of thinking about addition is with a "reasoning" approach. In this case known facts are used to yield other facts. Such reasoning employs the basic principles. For example, finding $5 + 6$ from a knowledge of $5 + 5$ uses the grouping principle.

$$\begin{aligned} 5 + 6 &= 5 + (5 + 1) \\ &= (5 + 5) + 1 \\ &= 10 + 1 \\ &= 11 \end{aligned}$$

Follow-up

To facilitate the use of the colored strips, you might want to make some "strip boards" for the class. A homemade model can be constructed by cutting both lids off a can, cutting along its vertical seam, flattening it, taping the edges, and tacking it to a piece of Masonite. Strip magnets can be purchased from your school supplier and glued onto the back of the colored strips.



A ruler may be drawn on the can.

Using the Exercises

On page 55, since exercises 1 and 2 lend themselves to the use of reasoning in addition, you might discuss some examples with the class. Exercise 3 may be treated orally or in writing. The children may need illustrations of how to use an addition table before you assign exercises 3 through 5. For example, draw a small addition table on the chalkboard, like the one below, and ask the children to help you fill in the missing numbers.

+	3	5
2		
4		

+	3	5
2		
4		

Explain that they should take a number, such as 2, from the left column, and add it to a number, such as 3, from the top row. The sum, 5, belongs in the top, left empty space.

Urge all the children who finish the exercises to try the *Think* problem. Remind them not to spoil the fun for others by giving away the answer prematurely.

Assignments (page 55)

Manimum: 1–3. Average: 1–4.
Maximum: 1–5.

Resources for Active Learning

Mathex: Operations No. 3, "A Magic Square Game – Game 4," p. 8 (Pupil page 12), Encyclopaedia Britannica Publications Ltd.

Duplicator Masters, page 10

PAGES 56–57

Objective

Given the sum and an addend from a familiar addition combination, the child will be able to supply the missing addend.

Preparation

You might want to review briefly the meaning of the terms addend and sum. For example, you could write an equation on the chalkboard, such as $6 + 2 = 8$, and remind the children that 6 and 2, the numbers to be added, are called *addends*, and the answer, 8, is called the *sum*.

Investigation

It would be helpful to read the directions for this investigation with the children. Then encourage them to work individually, recording their results. If some children write $4 + 3 + 2 = 9$, praise them for a correct equation but ask them to reread the directions. If necessary, point out that they are asked to find *two* addends, the third number being the sum.

As the quicker children complete their four equations for the first three numbers they selected, suggest that they continue similarly with another combination of three numbers from the set. If a child writes $2 + 2 = 4$, you may accept it but he will not be able to write four equations.

As the children finish, have volunteers write their three numbers on the chalkboard and, if space allows, the related four equations. You might have them write it in chart form, as shown.

A	A	S	Equations	
2	7	9	$2+7=9$	$9-7=2$
			$7+2=9$	$9-2=7$
5	4	9	$5+4=9$	$9-4=5$
			$4+5=9$	$9-5=4$
3	4	7	$3+4=7$	$7-4=3$
			$4+3=7$	$7-3=4$
5	2	7	$5+2=7$	$7-2=5$
			$2+5=7$	$7-5=2$
3	2	5	$3+2=5$	$5-2=3$
			$2+3=5$	$5-3=2$

● *Can you find differences by finding missing addends?*

Investigating the Ideas

Find three numbers from the set so that

two are **addends**
and
the other is their **sum**.

7 3 4 2 5 9

? Can you use your numbers to write two addition equations **and** two subtraction equations?

See Investigation.

Discussing the Ideas

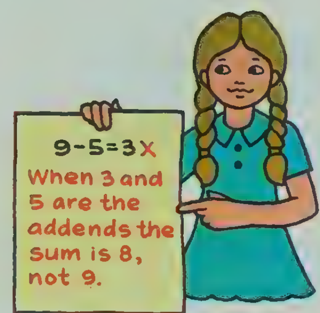
1. The letters A and S help you think about addends and their sum.

$$\begin{array}{ccc} A & & A & & S \\ 6 & + & 2 & = & 8 \end{array}$$

   A Write another addition equation using these numbers. 2+6=8

   B Write two subtraction equations using these numbers. 8-6=2, 8-2=6

2. The third-grade children checked their arithmetic papers. Kristy checked Jay's paper. Under one exercise, she wrote



   A Explain what Kristy was trying to tell Jay. We can use addition to check the accuracy of subtraction.

   B What do you think Kristy would say about these exercises?

$8 - 3 = 4$

$$\begin{array}{r} 9 \\ -\ 6 \\ \hline 2 \end{array}$$

"When 3 and 4 are the addends, the sum is 7, not 8."
"When 6 and 2 are the addends, the sum is 8, not 9."

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Discussion

The thrust of the investigation was to reinforce for the children the relationship of addends to sum. A discussion of exercise 1 would clarify this relationship. Emphasize particularly the position of the addends in the subtraction equations. For example, let volunteers write the equations for 1A and 1B on the chalkboard. Then discuss and label each number in the equations.

A		A		S		S		A		A
6	+	2	=	8		8	−	2	=	6
A		A		S		S		A		A
2	+	6	=	8		8	−	6	=	2

Develop other equations similarly.

Then write "$7 - 3 = ?$" This may be thought of in terms of addition as $? + 3 = 7$ (read "What number plus three equals seven?"). The familiar addition fact $4 + 3 = 7$ should come to mind and supply the missing addend. Continue with similar pairs of equations until the children see that they can find differences by thinking about the missing addend.

As you discuss exercise 2, emphasize the role of addition in checking subtraction. Writing the related addition equation for a given subtraction problem would enable the children to see this.

Using the Ideas

1. Find the missing addends.

A $n + 4 = 7$ 3	E $n + 4 = 8$ 4	I $n + 0 = 8$ 8
B $n + 3 = 10$ 7	F $n + 5 = 10$ 5	J $n + 7 = 10$ 3
C $n + 4 = 9$ 5	G $n + 6 = 9$ 3	K $n + 4 = 5$ 1
D $n + 6 = 10$ 4	H $n + 1 = 7$ 6	L $n + 3 = 9$ 6

2. Find the differences by thinking about missing addends.

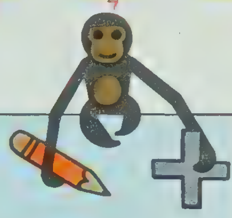
Think $? + 4 = 7$	Think $? + 3 = 10$	Think $? + 4 = 9$
A $7 - 4 = n$ 3	B $10 - 3 = n$ 7	C $9 - 4 = n$ 5
D $10 - 6 = n$ 4	E $9 - 6 = n$ 3	F $10 - 7 = n$ 3
G $8 - 4 = n$ 4	H $7 - 1 = n$ 6	I $5 - 4 = n$ 1
J $10 - 5 = n$ 5	K $8 - 0 = n$ 8	L $9 - 3 = n$ 6

3. Give the missing numbers. Check your answer.

7	−3	4	+2	6	−4	A 2	+8	B 10	−1	C 9	−5	4
2	+6	8	+2	10	−9	D 1	+4	E 5	+2	F 7	−7	0

4. Find the differences. Check your answers.

A $10 - 6 = $ 4	B $10 - 3 = $ 7	C $8 - 2 = $ 6
D $6 - 3 = $ 3	E $7 - 3 = $ 4	F $5 - 5 = $ 0
G $9 - 3 = $ 6	H $8 - 1 = $ 7	I $6 - 0 = $ 6
J $9 - 5 = $ 4	K $10 - 7 = $ 3	L $10 - 5 = $ 5



think

Addition is my dearest friend.
We never fuss or fight.
When I am done,
He comes and checks
To see that I am right.

Subtraction
WHO AM I?

More practice, page A-7, Set 11

Mathematics

This lesson stresses the idea that finding a difference is equivalent to finding a missing addend, an idea directly related to the definition of subtraction. In subtraction, we define the number $a - b$ as the number that adds to b to give a. That is, $(a - b) + b = a$. In order to find the difference $a - b$, we find the number that adds to b to give a.

Children have long been taught to check their subtraction problems by addition; however, the missing-addend method for finding differences, the approach used in this series, has proven successful. We intend to have children memorize the addition combinations, but not the subtraction combinations. If they have a thorough knowledge of addition combinations, they should know the corresponding subtraction combinations. Later, this method will be pursued with multiplication and division by treating quotients as missing factors.

Follow-up

To give children more practice in finding a pattern, pass out a worksheet like the following one. Direct the children to find the pattern and then to complete the table.

Study the chart. Fill in the missing numbers.

Start	9	7	5	8	6	9
Subtract	−3	−5	−2	−6	−1	−8
Missing Addend	6	?				
Addend	+3	+5	+2	+6	+1	+8
Sum	9	?				

Duplicator Masters, page 11
Skill Masters, page 11

Using the Exercises

On page 57, guide the children through one or two parts of exercises 1 and 2 and check their answers. In exercise 2, they should give the reason for their answer. In exercise 2A, for example, they should write "$7 - 4 = 3$ because $3 + 4 = 7$"; and for exercise 4A they should write:

$$\begin{array}{r} 10 \\ -\ 6 \\ \hline 4 \end{array} \quad \text{because} \quad \begin{array}{r} 4 \\ +6 \\ \hline 10 \end{array}$$

In exercise 3, help the children see that each answer combines with the number on its right to form a new problem. You may find it helpful to treat this exercise as a class activity for some groups.

Since many of the children will be able to answer the *Think* problem, caution them to keep the answer a secret until everyone has had a chance to work on it. Then allow time for discussion.

Assignments (page 57)

Minimum: 1–2. Average: 1–4. Maximum: 1–4.

PAGES 58–59

Objective

Given two sets to compare, the child will be able to write and solve a subtraction equation to express a numerical comparison of the sets.

Preparation

Materials

counters (approximately 22 per child)

Since the investigation should result in a variety of responses and a specific preparation would limit the range of possibilities, it is suggested that you begin immediately with the investigation.

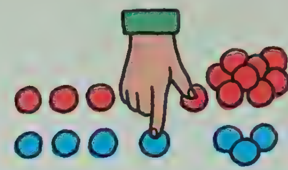
Investigation

Read the directions with the class, making sure the children understand the phrase "one-to-one." In this investigation, the children are challenged to express the comparison of sets with a subtraction equation. Remind the children that the problem is not just to find out how many counters are not matched, but also to be able to write an equation to express how many counters are not matched. You may prefer that they work in groups of two or three so that they can talk about the investigation. Encourage the more capable children to select other sets and do the same. You may wish to extend the investigation by suggesting other sets, including equal sets, for the children to match and describe by subtraction equations.

● *Can subtraction be used to compare sets?*

Investigating the Ideas

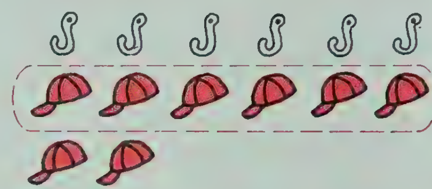
Choose one pile of 12 counters and a second pile of 7 counters. Match the counters from the piles one-to-one until one pile is used up.



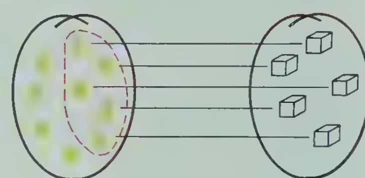
? Can you copy and complete this equation about the piles? It tells how many counters were not matched. $12 - 7 = ?$ 5
see Investigation.

Discussing the Ideas

1. How many more caps are there than hooks? By matching we see there 2 are ▒ more caps than hooks. Here is the subtraction equation: $8 - 6 = n$



2. How many more balls are there than blocks? The matching lines show there 4 are ▒ more balls than blocks. Also, you can use subtraction. $9 - 5 = n$

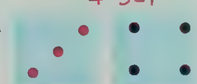


3. For each pair of sets, there are more black dots than red dots. Write a subtraction equation for each pair to tell how many more black dots.

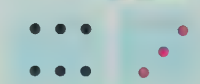
Example:

Equation: $6 - 4 = 2$

A 4-3=1



B

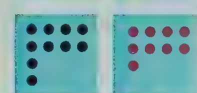


6-3=3

C

3-0=3

D



10-9=1

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Discussion

When most children have written at least one subtraction equation to show the comparison, have one or two explain their equation by showing the comparison with the counters. Or you may choose to do this as a demonstration. For example, select a set of 5 and a set of 7. Have a child match them one-to-one. Have him show the 2 left-over counters to the class and write the equation $7 - 5 = 2$ on the chalkboard. Then proceed to develop this comparison concept through the discussion exercises.

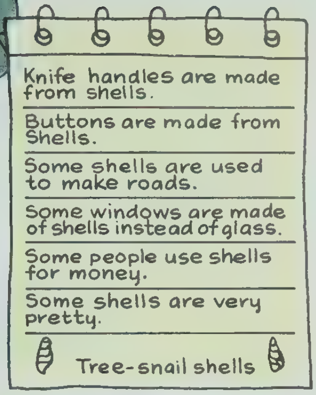
As you discuss the first two exercises, call attention to the lines that match some objects of one set with all of the objects in the other set. Point out the dashed ring around the matched objects and explain that it suggests subtraction by separating a total set into two subsets—the set of the objects that match one-to-one with the other set, and the set of the objects that are left over.

You might have the children do exercise 3 by themselves and then discuss their equations.

Collecting Shells

Using the Ideas

Nan collects shells. She also
collects facts about shells.
Page 1 in her notebook looked like this.



1. Nan collected 5 tree-snail shells and 3 clam shells. How many more tree-snail shells does she have than clam shells? 2

2. Nan and Jill went to the beach to look for shells. Nan placed her shells in a row like this. Jill placed her shells like this.

   A Who had more shells? Nan B How many more? 2

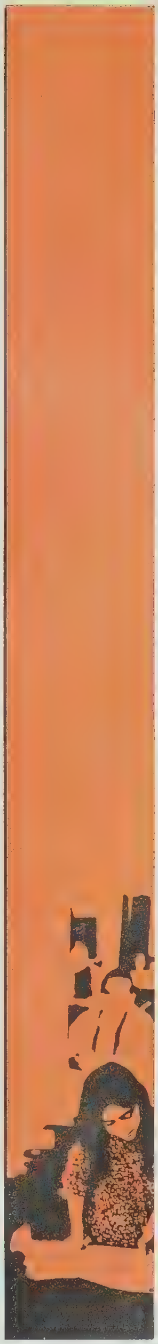
3. Nan found 9 mussel shells beside a river. She gave 4 of them to Jill. How many did Nan have left? 5

4. Nan wants pictures of her shells. She has 9 different snail shells. She has pictures of 7 of them. How many more pictures does she need? 2

5. Nan took 8 colored shells to school. She had 3 colored shells at home. She had fewer colored shells at home than at school. How many fewer? 5

6. Jill had 4 blue shells and 3 white shells in a box. She gave Nan 5 of these shells. How many shells are left? 2

More practice, page A-8, Set 12



Using the Exercises

Read with the children the first paragraph and the notebook page shown on page 59. Then ask the children to read and try the first exercise. When they have finished, choose someone to write the proper subtraction equation on the chalkboard. Encourage a discussion of the way this equation relates to exercise 1. If you think it necessary, do exercise 2 as a class activity also. Then, have the children complete the page independently, giving help only if they have reading problems. Exercise 6 might be considered enrichment, since it is a two-step problem.

Assignments (page 59)
Minimum: 1–6, oral. Average: 1–6.
Maximum: 1–6.

Mathematics

In this lesson, subtraction is directly related to the definition of subtraction in terms of sets. The comparison of sets by matching objects one-to-one demonstrates that subtraction is used to find how many more or how many less one set has than another. Matching lines enable the children to determine by quick examination that one set has more than another set.

Follow-up

At this level, children need continued practice on combinations, but it is important to vary the approach and pace to provide practice without boredom. A relay race is a competitive approach to drill. Select two evenly matched teams, line them up facing the chalkboard, and provide the leader of each team with a felt pen in the chosen team color, such as green or gold. In two different places on the chalkboard, tape charts bearing addition tables like those illustrated and assign one chart to each team. (If you prefer, you could draw the charts directly on the chalkboard and have children use colored chalk.) Instruct

Green Team

Add 2	
3	
0	
4	
2	
7	

Gold Team

Add 2	
5	
3	
6	
0	
1	

the team leaders to go to the chalkboard, at your signal and fill in the first sum of their respective tables. Then each team leader gives the pen or chalk to the next person on his team and goes to the end of the line. Play should continue until each child has had at least one turn. A player may correct an error on his team's table. Award two points to the first team finished and two points for each correct sum.

Resources for Active Learning

Mathex: Operations No. 3, "Subtraction by Matching," p. 4, Encyclopaedia Britannica Publications Ltd.

Workbook, page 19

PAGES 60–61

Objectives

Given two addends, the child will demonstrate his understanding of the order principle by showing that the order of the addends can be changed without changing the sum.

Given three addends, the child will demonstrate his understanding of the grouping principle by showing that the grouping of the addends can be changed without changing the sum.

Preparation

Materials

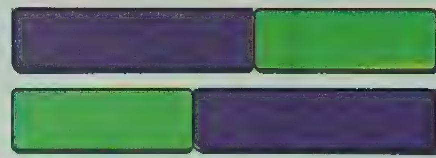
sets of colored strips

The preparation for this lesson can be brief: simply remind the children of the different uses they have made of their colored strips and explain that they will use them again in this lesson.

What are the order and grouping principles?

Discussing the Ideas

1. Follow a 4-strip with a 3-strip. Then follow a 3-strip with a 4-strip.



A Do the trains match? Yes

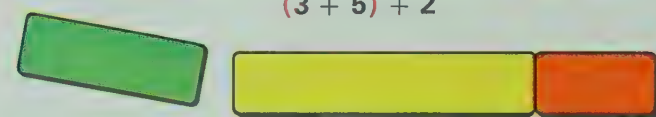
B Try this again using other strips. What do you find? (See Discussion.)

C Complete this sentence about the order principle.

> When we change the ? (order) of the ? (addends), we get the same sum.

2. A Do the two trains below match? Explain the grouping. Yes (See Discussion.)

$(3 + 5) + 2$



$3 + (5 + 2)$

B Addends: 4, 5, 7. The arrows tell what to add first.

$$(4 + 5) + 7 \stackrel{?}{=} 4 + (5 + 7)$$

Are the two sums equal? Yes

C Complete this sentence about the grouping principle.

> When we change the ? (grouping) of the ? (addends), we get the same sum.

Discussion

Read the directions for exercise 1 with the children and have them use their colored strips as specified. Be sure that they reverse the order of the strips in making the second train.

After the children have had a chance to make several combinations of matching trains for part B, ask them to explain what they found. Then use the chalkboard or an overhead projector to illustrate the order principle with a column of equations like the following.

$$3 + 4 = 4 + n$$
$$12 + 36 = n + 12$$
$$124 + 59 = 59 + n$$

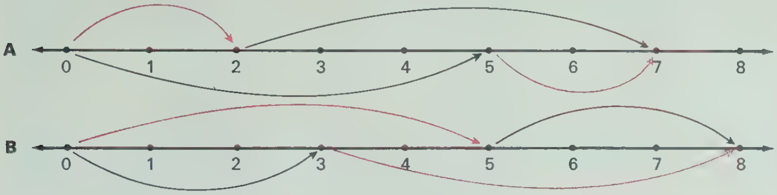
Make the last few sums difficult enough that the children will have to use the order principle to find n. Then direct the children to write the answer for part C, and have a volunteer write the complete statement on the chalkboard.

Develop parts A, B, and C of exercise 2 in the same manner. Then write some different equations to illustrate the grouping principle. For example, you could write $(4 + 3) + 2 = n$ and $4 + (3 + 2) = n$ and solve each equation, with the solutions side by side for comparison:

$7 + 2 = n$	$4 + 5 = n$
$9 = n$	$9 = n$

Using the Ideas

1. The number line in part A shows that $2 + 5 = 5 + 2$. What does the number line in part B show? $5+3=3+5$



2. Solve the equations.
   - A $4 + 6 = 6 + n$ 4
   - B $5 + n = 3 + 5$ 3
   - C $n + 8 = 8 + 9$ 9
   - D $7 + 9 = n + 7$ 9
   - E $327 + n = 22 + 327$ 22
   - F $9856 + 6542 = n + 9856$ 6542

3. Give the missing number.

IF	THEN
$137 + 387 = 524$	$387 + 137 = n$

4. Find the sums. Use the grouping shown by arrows.
   - A {1, 5, 2} 8
   - B {3, 4, 2} 9
   - C {1, 5, 2} 8
   - D {3, 4, 2} 9
   - E {2, 4, 1} 7
   - F {5, 3, 2} 10
   - G {2, 4, 1} 7
   - H {5, 3, 2} 10

5. The last two addends are grouped in the problems below. By changing the grouping, you can find the same sum more easily. Do this.
   - A $7 + (3 + 5)$ 15
   - B $78 + (2 + 6)$ 86
   - C $999 + (1 + 235)$ 1235
   - D $99 + (1 + 532)$ 632
   - E $6 + (4 + 8321)$ 8331
   - F $7 + (3 + 8267)$ 8277

6. Find the sums.

   A

IF	THEN
$(35 + 27) + 68 = 130$	$35 + (27 + 68) = n.$

   B

IF	THEN
$(56 + 39) + 28 = 123$	$56 + (39 + 28) = n.$

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Develop other equations and then have the children complete the statement of the grouping principle as given in part C.

Help the children see that, in the order principle, position of the addends is changed. In the grouping principle, on the other hand, the position of the addends remains the same but the way they are grouped is changed: the middle addend may be grouped with the first, or the middle addend may be grouped with the third. Stress that, if either the order of addends or the grouping of addends is changed, the sum is still the same.

Using the Exercises

Assign the exercises on page 61 after you have discussed both the order principle and the grouping principle. It would be helpful to discuss exercise 1 to be sure that the children properly interpret how the number lines are used to show addends and sums.

When the children have finished, check the answers with them and reemphasize the statements of the order principle and the grouping principle.

Assignments (page 61)

Minimum: 4. Average: 1–6.
Maximum: 1–6.

Mathematics

Two useful principles for addition of whole numbers are the commutative (order) principle,

For any whole number a and b,
$$a + b = b + a,$$

and the associative (grouping) principle,

For any whole numbers a, b, and c,
$$(a + b) + c = a + (b + c).$$

These two addition principles are presented through numerical examples, enabling the children to generalize the main concepts. They might state the ideas this way: "We can change the order of two addends and still get the same sum," and "We can change the grouping of three addends and still get the same sum."

Follow-up/Three-team Relay

For a practice game organize the class into three closely matched teams. Ask them to line up facing the chalkboard, and give each team leader a different-colored piece of chalk. Uncover addition tables that you have previously put on the chalkboard. Use the same numerals (but in varying order) on tables in three locations. Instruct the leaders to go to the board, write in the first answer, return the chalk to the next teammate, and go to the end of the line. Award two points to the first team back in position and one point for each correct answer. Use addition tables like the following. (The tables should include one blank for each team member.)

Add 7		Add 8		Add 9	
3		7		5	
4		5		2	
5		3		7	
9		9		6	

Resources for Active Learning

Discovery, Section 1, Activity 3, pp. 3–4, Encyclopaedia Britannica Educational Corp.

Mathex: Operations No 3, "Basic Games—Games 1 and 2," p. 8, Encyclopaedia Britannica Publications Ltd.

PAGES 62–63

Objective

Given three or more addends, the child will use generalizations of the order and grouping principles to rearrange the addends and find the sum.

Preparation

Materials

5-by-10 centimetre slips of paper (3 per child)

To prepare for this lesson, you might briefly review the order and the grouping principles studied in the previous lesson. You could write some equations like the following on the chalkboard and ask children what principle is used in each one.

$$2 + (7 + 1) = (2 + 7) + 1$$
$$3 + 4 = 4 + 3$$
$$(1 + 5) + 7 = 1 + (5 + 7)$$
$$8 + 3 = 3 + 8$$

After the review, explain that in this lesson the children see how using both of these principles on the same problem can make adding easier.

Investigation

Since this investigation is relatively simple, you might let the children do it completely on their own. That is, after you've distributed the slips of paper, let them read the directions independently and encourage them to follow the instructions exactly. Note that the directions do not tell the children to record the five sets of addends that they pick, but praise any child who does so. If some finish very quickly you might have them do the same investigation with three other numbers or with four numbers. As you move around the room observing the children, make sure that the major effort is directed toward finding out if the sum is the same.

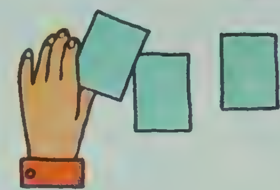
Let's explore rearranging addends.

Investigating the Ideas

Make three slips of paper like these. Then turn them over and mix them up.

2 3 4

Pick any two slips and add the numbers on them. Then add the number on the other slip.



? If you do this five times, will you get the same sum each time? Yes (See Investigation.)

Discussing the Ideas

1. In the Investigation you might have picked the pair $\boxed{2}$, $\boxed{4}$ first. What other pairs might you have picked? 2,3 ; 3,4

2. With the three addends 2, 3, and 4, we could

A add these first.	B add these first.	C add these first.
2 3 4	2 3 4	2 3 4
$(2 + 3) + 4$	$2 + (3 + 4)$	$(2 + 4) + 3$

Answer these questions for A, B, and C:
Which two addends are grouped together? A: 2,3 B: 3,4 C: 2,4
What is their sum? What is the total sum? A: 5,9 B: 7,9 C: 6,9

3. We can add any two numbers first. Which two would you add first in the problem $6 + 8 + 4$? Why?
6 and 4, because they give a sum of 10, to which it is easy to add the third addend.

62

Discussion

Although the rearrangement of addends technically depends upon the various uses of the order and grouping principles, this lesson stresses the resulting generalization that addends may be reordered or regrouped in any way without changing the sum.

In exercise 1, the possible pairs are 2, 4; 4, 3; 2, 3. This gives a total of 12 equations.

2 3 4	2 3 4	2 3 4
A $(2 + 3) + 4$	E $2 + (3 + 4)$	I $(2 + 4) + 3$
B $(4 + 2) + 3$	F $2 + (4 + 3)$	J $4 + (2 + 3)$
C $3 + (2 + 4)$	G $(4 + 3) + 2$	K $(3 + 2) + 4$
D $3 + (4 + 2)$	H $(3 + 4) + 2$	L $4 + (3 + 2)$

As the children answer exercise 2, it would be helpful for you to work through the three equations side by side on the chalkboard.

2 3 4	2 3 4
$(2 + 3) + 4 = n$	$2 + (3 + 4) = n$
$5 + 4 = n$	$2 + 7 = n$
$9 = n$	$9 = n$

Using the Ideas

1. Solve these equations for addends 2 3 5
   - A $(2+3)+5=n$ 10
   - B $(5+2)+3=n$ 10
   - C $(3+5)+2=n$ 10
   - D $(2+5)+3=n$ 10

2. However we order or group,
   - A when the addends are 2 4 3, the sum is ▥. 9
   - B when the addends are 4 1 2, the sum is ▥. 7
   - C when the addends are 2 5 0, the sum is ▥. 7

3. Find the sums. Look for tens.

A $7+3+2$ 12	E $5+6+5$ 15	I $6+3+1$ 10	M $10+0+8$ 18
B $7+2+3$ 12	F $4+6+9$ 19	J $5+2+8$ 15	N $2+5+2$ 9
C $9+8+1$ 18	G $7+8+2$ 17	K $8+5+2$ 15	O $3+5+5$ 13
D $6+2+4$ 12	H $5+7+3$ 15	L $9+5+5$ 19	P $1+6+9$ 16

4. Find the sums. Look for tens.

A	B	C	D	E	F	G	H
8	4	4	7	9	8	6	4
2	2	8	5	9	2	0	6
+4	+8	+2	+3	+1	+8	+4	+9
14	14	14	15	19	18	10	19

5. Find the sums. Look for tens.

A $2+8+3+4$ 17	D $2+3+4+8$ 17	G $4+3+3+6$ 16
B $2+3+8+4$ 17	E $4+6+3+3$ 16	H $7+4+3+5$ 19
C $9+5+1+5$ 20	F $2+4+8+2$ 16	I $7+6+2+4$ 19

6. Find the sums. Look for tens.

A	B	C	D	E	F	G
9	8	5	6	6	7	6
7	2	7	4	7	5	5
1	3	3	7	4	3	4
+2	+1	+2	+3	+3	+3	+5
19	14	17	20	20	18	20

More practice, page A-8, Set 13

63

2 3 4

$(2+4)+3=n$

$6+3=n$

$9=n$

In exercise 3, help the children see the benefit of adding the 6 and 4 first. Stress this idea of looking for tens with a few examples, using both horizontal and vertical notation, such as,

$2+4+8$	6	9
	5	3
$3+7+5$	+4	+1

Using the Exercises

As you assign the exercises on page 63, remind the children that in exercise 1 they should do the work in the parentheses first. In exercises 3 through 6, stress the idea of looking for tens. (You may wish to do a few of these as a class activity. When everyone has finished, check the answers and ask some children to explain *how* they rearranged the addends for a few of the problems.

Assignments (page 63)

Minimum: 1–4. Average: 1–6.
Maximum: 1–6.

Mathematics

This lesson involves a simple but important idea in the mathematics of the third-grade program: Changing the order and the grouping of numbers in addition permits any rearrangement that is convenient for three or more addends.

Since three numbers can be ordered six ways, and since each of these orderings can be grouped in two ways, three addends a, b, and c can be ordered and grouped in 12 ways.

1. $(a+b)+c$	7. $c+(b+a)$
2. $a+(b+c)$	8. $(c+b)+a$
3. $a+(c+b)$	9. $(b+c)+a$
4. $(a+c)+b$	10. $b+(c+a)$
5. $(c+a)+b$	11. $b+(a+c)$
6. $c+(a+b)$	12. $(b+a)+c$

We can proceed down this list and verify the equality of the first expression to the second, the second to the third, and so on, by alternately applying the grouping and order principles for addition. We can also prove any two of these expressions equal by using either the order principle, the grouping principle, or a combination of both.

Another important idea in this lesson is that the grouping principle allows the omission of parentheses from expressions involving the sum of three numbers. Since we can group these numbers in any convenient way, the expression will be equally correct without parentheses.

Follow-up

Children at this level will benefit from a variety of ways to practice basic combinations. For this purpose, you might have the children play "How Many Names?" Put a number such as 17 on the chalkboard and have the children write as many names for 17 as they can within a time limit.

Resources for Active Learning

Developmental Math Cards, F15, Addison-Wesley. [A riddle card]

Mathex: Operations No. 3, "A Game," pupil page 6, Encyclopaedia Britannica Publications Ltd.

Duplicator Masters, page 12
Workbook, page 20
Skill Masters, page 12

Objective

Given two or more single-digit addends, such as 7 and 5, the child will be able to find their sum by "making 10," that is, by adding $(7 + 3) + 2$.

Preparation

Materials

sets of colored strips

The children should be fairly familiar with the colored strips, but it might be helpful to identify each strip by its proper color name: white, red, light green, purple, yellow, dark green, black, brown, blue, and orange. Also explain to the children that each strip is 1 centimetre wide and that the white one is 1 centimetre long as well. It would be helpful to review the meaning of "rectangle": a shape that has four "square" corners and whose opposite sides have the same length.

Investigation

In this investigation, the children concretely investigate the basic combinations with a sum of 10. Read the questions with the class and direct the children to take from their set of strips one of each color plus an extra yellow. Encourage independent work but take care that each child knows what he is to do. If some remain confused about putting the strips in rectangular form, use the 10-strip and another combination pair to start the child working. Two possible arrangements are shown below.

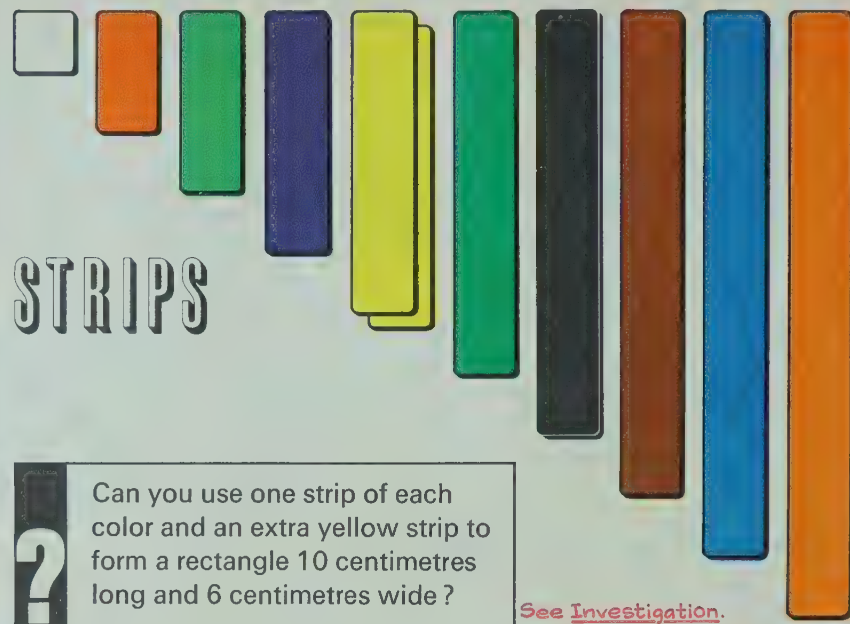
yellow	yellow
purple	dk. green
green	black
red	brown
w	blue
orange	

orange	
brown	red
blue	w
black	green
yellow	yellow
purple	dk. green

If some children finish quickly, have them find an arrangement other than their first. This should help them realize that, although they can change the order of two strips and the position of two strips, the same two strips must always be paired.

● *Let's explore ways to think about sums.*

Investigating the Ideas



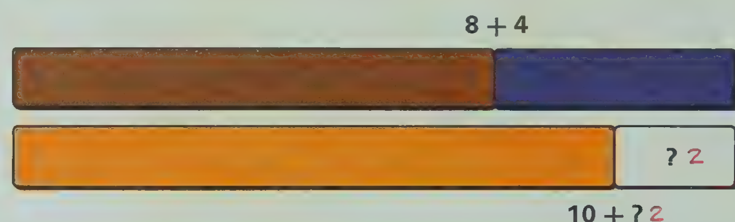
? Can you use one strip of each color and an extra yellow strip to form a rectangle 10 centimetres long and 6 centimetres wide?

See Investigation.

Discussing the Ideas

1. Give some equations suggested by rows of the rectangle.
See Discussion.

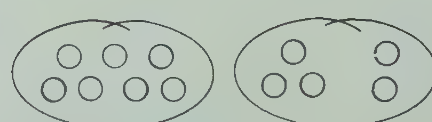
2. A Give the missing number for the picture below.



B Can you show some other "larger facts" using your strips?

Discussion

In discussing exercise 1, relate the colored strips to the numbers they represent. For example, brown and red should be recognized as representing the equation $8 + 2$. It would be helpful to write the equations for the 10 combinations on the chalkboard. Then, as you discuss exercise 2, help the children see how to "break apart" one addend and use part of it with the other addend to make ten. For example, in exercise 2A we break the 4 into 2 and 2 because $8 + 2 = 10$. In discussing exercise 2B, have someone draw an appropriate picture on the chalkboard, such as:



Then provide an explanation similar to this: "We want to add $7 + 5$. It is easy to add when 10 is an addend, so think of 5 as $3 + 2$. Now group the 3 with the 7 and add $(7 + 3) + 2$ or $10 + 2 = 12$."

Using the Ideas

1. Find the missing numbers. Then give the sums.

A $8 + 6 = 10 + n$ 4 14	D $7 + 5 = 10 + n$ 2 12	G $6 + 5 = 10 + n$ 1 11
B $9 + 4 = 10 + n$ 3 13	E $9 + 6 = 10 + n$ 5 15	H $6 + 6 = 10 + n$ 2 12
C $8 + 5 = 10 + n$ 3 13	F $9 + 3 = 10 + n$ 2 12	I $6 + 7 = 10 + n$ 3 13

2. The **double** of 9 is 18 because 9 + 9 = 18.
Find the double of each number.
A 4 8 B 6 12 C 3 6 D 7 14 E 5 10 F 2 4 G 8 16

3. What number doubled gives each of these numbers?
A 10 5 B 6 3 C 14 7 D 8 4 E 18 9 F 16 8 G 2 1 H 0 0

4. Read each exercise carefully. Then give the sum.
A Because 4 + 4 = 8, we know that 4 + 5 = n. 9
B Because 7 + 7 = 14, we know that 7 + 6 = n. 13
C Because 5 + 5 = 10, we know that 5 + 6 = n. 11
D Because 6 + 6 = 12, we know that 6 + 7 = n. 13
E Because 8 + 8 = 16, we know that 8 + 9 = n. 17

5. Find the sums.

A	B	C	D
2	6	9	7
+8	+3	+4	+6
10	9	13	13

E	F	G	H
8	4	7	9
+1	+6	+2	+8
9	10	9	17

I	J	K	L
7	2	8	7
+7	+7	+3	+8
14	9	11	15

6. Give the sums.

A	B	C	D
3	5	5	5
4	3	1	4
+5	+7	+9	+6
12	15	15	15

I'm quite a tiny number.
So very small indeed.
If you add me to another,
A change you cannot read.

WHO AM I?
Zero

More practice, page A-9, Set 14

Using the Exercises

If the children are capable, direct them to complete the exercises on page 65 independently. However, you may wish to have them work through exercises 2 and 3 orally and write out exercises 4 through 6. In exercise 4, the children should see that knowing the doubles in the first equation will help them find the solution to the second equation. This method is particularly important, since later it will aid them in learning multiplication combinations.

Encourage all the children to attempt the *Think* problem. You may wish to use several examples of adding zero to 1-, 2-, and 3-digit numbers to help some children recognize that zero is the correct answer to the riddle.

Assignments (page 65)
Minimum: 6. Average: 1–6.
Maximum: 1–6.

Mathematics

Below is an example of how the grouping principle may be used to compute sums greater than ten. In this case, notice that we need to use only the grouping principle and the concept of place value to find the sum.

$$\begin{aligned} 7 + 5 &= 7 + (3 + 2) \\ &= (7 + 3) + 2 \\ &= 10 + 2 \\ &= 12 \end{aligned}$$

Follow-up

For continued practice with combinations greater than ten, develop a worksheet like the one below.

Match the sums on the left with those on the right.

9 + 3	
4 + 7	10 + 1
5 + 8	
6 + 5	10 + 2
8 + 7	
4 + 9	10 + 3
3 + 8	
9 + 5	10 + 4
6 + 7	
7 + 5	10 + 5

Resources for Active Learning

Mathex: Operations No. 3, "Operation Big Ten," pp. 16–20, Encyclopaedia Britannica Publications Ltd.

Maths Mini-Lab, Starter Activity Cards, N5/1–25, Selective Educational Equipment.

Duplicator Masters, page 13
Workbook, page 21
Skill Masters, page 13

PAGES 66–67

Objective

Given a simple subtraction equation, the child will be able to find the difference by thinking of the missing addend.

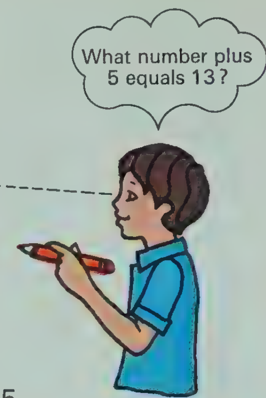
Preparation

To prepare for this lesson, review the terms addend and sum in both subtraction and addition equations. For example, write $7 + 8 = 15$ and $15 - 8 = 7$ on the chalkboard and have the children label the sum and addends.

How can addition help you find differences?

Discussing the Ideas

You can find
differences
if you can find
missing addends.



S A A
$13 - 5 = n$

1. When the sum is 13 and one addend is 5, how can you find the other addend? Subtract (See Discussion.)

2. Read each sentence aloud and give the difference.

   A A S / S A A
   - A Because $6 + 7 = 13$, we know that $13 - 7 = n$. 6
   - B Because $8 + 7 = 15$, we know that $15 - 7 = n$. 8
   - C Because $9 + 5 = 14$, we know that $14 - 5 = n$. 9
   - D Because $8 + 6 = 14$, we know that $14 - 6 = n$. 8

3. Give the difference and explain how you know it is correct.
   - A Because $48 + 37 = 85$, we know that $85 - 37 = n$. 48
   - B Because $76 + 88 = 164$, we know that $164 - 88 = n$. 76
   - C Because $57 + 19 = 76$, we know that $76 - 19 = n$. 57

4. Give the missing number and explain how it helps you find the difference.
   - A To find $15 - 7$, it helps to think $n + 7 = 15$. 8
   - B To find $17 - 9$, it helps to think $n + 9 = 17$. 8
   - C To find $14 - 6$, it helps to think $n + 6 = 14$. 8
   - D To find $12 - 7$, it helps to think $n + 7 = 12$. 5

66

Discussion

One of the main points of this lesson is to help the child see that by knowing an addition fact he is able to find the difference of a related subtraction equation. In exercise 1, stress the point that $8 + 5 = 13$ must be known if we are going to solve $13 - 5 = n$. As you discuss exercises 2 and 3, help the children formulate a question for each equation like the one given by the boy in the text. For example in exercise 2A, after a child reads the sentence, he should ask the question "What number plus 7 equals 13?" and then find the answer in the given addition equation.

Exercise 4 not only continues to develop an understanding of how addition helps one find the difference; it also points out the importance of knowing the basic addition facts. If children do not have a fair command of the addition facts, spend five to ten minutes each day working on them. Games and worksheets using a variety of approaches will help to keep the practice interesting.

Since the exercises on page 66 are to be treated orally, it would be helpful to add examples of your own following the pattern in the text. To work into the independent study section, you might give the class regular subtraction problems and work through them together, guiding the children with questions such as, "If you changed the equation to an addition equation, what missing addend would you need in order to get this sum?"

Using the Ideas

1. Find the differences.

A $12 - 4 = n$ 8	D $14 - 5 = n$ 9	G $16 - 9 = n$ 7
B $11 - 5 = n$ 6	E $14 - 7 = n$ 7	H $12 - 6 = n$ 6
C $13 - 6 = n$ 7	F $15 - 8 = n$ 7	I $11 - 8 = n$ 3

2. Find the sums and differences. Use any method you choose.

A $8 + 2$ = 10	B $9 - 6$ = 3	C $8 + 7$ = 15	D $7 + 6$ = 13	E $12 - 3$ = 9	F $14 - 6$ = 8	G $7 + 8$ = 15	H $8 + 0$ = 8
I $13 - 4$ = 9	J $13 - 5$ = 8	K $6 + 6$ = 12	L $17 - 7$ = 10	M $18 - 9$ = 9	N $6 + 8$ = 14	O $5 + 5$ = 10	P $7 + 3$ = 10
Q $10 - 4$ = 6	R $10 + 8$ = 18	S $9 + 7$ = 16	T $15 - 7$ = 8	U $8 + 8$ = 16	V $9 + 9$ = 18	W $16 - 6$ = 10	X $14 - 9$ = 5

Short Stories

1 5 merit badges.
Need 14 in all.
How many more needed? 9

2 13 balloons. Stick 8 with a pin. How many left? 5

3 16 flies.
7 frogs.
Each frog gets a fly.
How many flies are left? 9

4 12 peanuts. Ate 2 and drank milk.
Ate 5 more. How many left? 5

5 Caught 17 fish.
9 too small so threw them back.
How many left? 8

6 Caught 12 butterflies.
7 got away.
How many left? 5

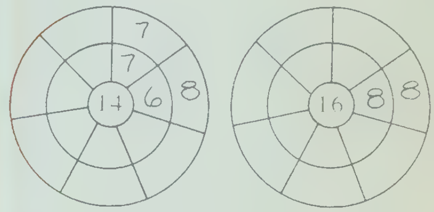
7 17 papers. Want to sell all but 10. How many must be sold? 7

8 Gave 8 valentines.
Received 17 valentines.
Gave how many fewer than received? 9

More practice, page A-10, Set 15

Follow-up/"Number Wheels"
This activity will help children with the basic addition combinations.

"Number Wheels" is similar to the "How Many Names?" suggested on page 63. Both are good practice activities. For Number Wheels, reproduce number wheels like those shown below. Direct the children to write combinations totalling the centre number.



Resources for Active Learning
Mathex: Operations No. 3, "Basic Games – Game 3," p. 8, Encyclopaedia Britannica Publications Ltd.

Duplicator Masters, page 14
Workbook, pages 22, 23, 24
Skill Masters, page 14

Using the Exercises
Before assigning page 67 as independent work, treat a few of the short story problems orally. Then let the children work independently both on the exercises at the top of the page and on the short stories. When the children finish, check the answers and discuss any exercises which caused difficulty. You might have volunteers explain what method they used in exercise 2. Short story problem 6 may deserve discussion also.

Assignments (page 67)
Minimum: 1–2, odd-numbered short stories. Average: 1–2, short stories. Maximum: 1–2, short stories.

PAGES 68–69

Objective

Given a set of addends and a corresponding set of sums formed by a particular rule, the child will be able to state the missing rule, such as "Add 4" or "Add 1."

Preparation

This lesson is built around a function game to help children discover patterns and practice the addition facts. You may choose to begin immediately with the investigation by explaining what a "rule" is. Stress the idea that a rule is a kind of guide for doing things, or pattern, which we want everyone to follow.

This lesson lays the foundation for the introduction of the function machine in the lesson immediately following.

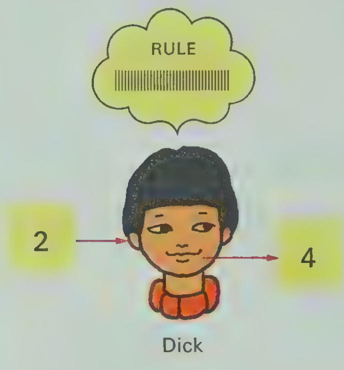
Investigation

Explain to the children that in this section they will investigate different patterns or rules that were made up by the children mentioned in the text. Direct the children to study the investigation section individually. After a few minutes, assign partners or small groups to work together. Encourage them to let each child have at least one turn at making up a rule. If anyone has difficulty thinking of a rule, you might suggest that he look at the tables on page 69 for a hint. As long as the children are guessing each others' rules allow them to continue in this manner. Then lead them into discussion.

Let's play "What's My Rule."

Investigating the Ideas

Tom thought of a rule.
When Jane said 3, Tom answered 5.
When Jane said 8, Tom answered 10.
When Jane said 5, Tom answered 7.
When Jane said 2, Tom answered 4.
Jane tried to figure out what rule Tom was using.



Dick thought of a rule.
When Sue said 2, Dick answered 4.
When Sue said 5, Dick answered 10.
When Sue said 8, Dick answered 16.
Sue said 9. Dick answered 18.
Sue thought, "What did Dick do to my number to get his number?"

What rules were the boys using? Tom-Add 2 Dick-Multiply by 2 See Investigation.

? Make up a rule of your own and play "What's My Rule" with a classmate. Can you guess each other's rules?

Discussing the Ideas

1. How would you keep a record of what happened in a game of What's My Rule? See Discussion.

2. Show a record of an imaginary game, but leave out the last answer. Can your classmates give the answer?

68

Discussion

You might let a few children give one of their rules to the class and let everyone try to guess it. Then relate the first discussion question to this game and talk about the different ways of recording results of a game. Some suggestions might be:

Listing the numbers

$6 \to 8,\ 7 \to 9,\ 3 \to 5,\ 4 \to 6,$

Pairing them

(2, 0), (4, 2), (5, 3), (7, 5), (8, 6)

Using rows and columns

a	8	3	4	5	7	9	6
b	11	6	7	8	?	?	?

Making a table like those on page 69

3	7
9	13
?	14
5	?

Since this type of table is the method of recording used in the text, encourage children to use it, but allow other ways to be used as long as they can be understood. Have the children make up a rule and make a table to describe it. Then let as many of their classmates as possible show and explain

Using the Ideas

Study the tables carefully. Guess the rule, then give what you think should go in each ▥.

1.

	Carol's number	Jill's answer
	4	14
	6	16
	2	12
A	7	▥ 17
B	9	▥ 19
C	▥ 3	13
D	24	▥ 34

Add 10

2.

	Cindy's number	Nan's answer
	8	1
	7	0
	10	3
A	9	▥ 2
B	▥ 13	6
C	▥ 15	8
D	▥ 17	10

Subtract 7

★ **3.**

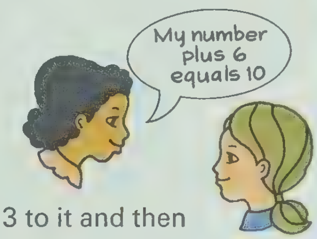
	Cathy's number	Susan's answer
	2	0
	4	0
	3	1
	5	1
	8	0
A	6	▥ 0
B	11	▥ 1

0 if it's even, 1 if it's odd

4. Choose a rule and make a table to show what might happen when you play What's My Rule.

5. Jane and Brenda played a game. Try to solve their puzzles.

A I'm thinking of a number. If you add 6 to it, you get 10. What is the number? 4



B I'm thinking of a number. If you add it to 4, you get 8. What is the number? 4

C I'm thinking of a number. If you add 3 to it and then add 2, you get 10. What is the number? 5

D I'm thinking of a number. If you subtract 5 from it, you get 10. What is the number? 15

E If you subtract 6 from a number and then subtract 2, you get 8. What is the number? 16

★ F I'm thinking of a number. If you add it to itself and then add 4, you get 10. What is the number? 3

69

the record of their imaginary game, and ask others in the class to supply the missing numbers.

Using the Exercises
Assign the exercises on page 69 as independent work. Point out the starred exercises, explaining that they might be a little more difficult than the others. Also explain how important it is to keep discovery of a rule a secret so that all can have a chance to discover it on their own. When the children have finished, discuss their answers and have them explain the rule for each exercise.

Assignments (page 69)
Minimum: 1–2, 4. Average: 1–2, 4–5E. Maximum: 1–5.

Follow-up
You may play the "What's My Rule" game in a variety of ways. For instance, ask the children to give you a number less than 10. Respond to each number suggested with another number, according to your rule. For example, if "Add 4" is your rule, your answer to "7" would be "11." When a child thinks he knows your rule, he should fold his arms and not tell anyone. When you call on a person with his arms folded, he should demonstrate that he knows the rule by asking you for a number less than 10 and then responding with a number that follows your rule. Continue the game until nearly all students have their arms crossed, signifying that they know your rule.

For variety, you might play the game as above and record the answers. For example, draw a grid on the chalkboard, and fill in one row, row *a*, with numbers 10 or less than 10. Form row *b* by using a rule on the numbers in row *a*. To form row *c*, use a rule with the numbers in either row *a* or row *b*, and so on. This often results in rows that may be formed by more than one pattern.

c	7	11	9	10	6	?	?	?
a	5	9	7	8	4	10	6	3
b	9	13	11	12	8	?	?	?
d	2	6	4	5	?	?	?	?

The different rules are summarized as follows:

$b = a + 4$

$c = a + 2$ or $b - 2$

$d = a - 3$ or $b - 7$ or $c - 5$

Resources for Active Learning
Developmental Math Cards, D[1]15, Addison-Wesley. [Pattern-hunting puzzles]

PAGES 70–71

Objective

Given addition and subtraction patterns presented in function machine notation, the child will be able to give the one missing element: input, function rule, or output.

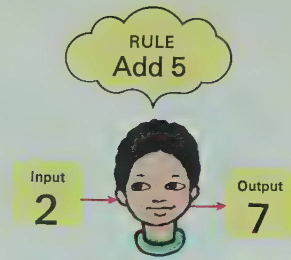
Preparation

Since the "What's My Rule" game is so closely related to the function machine concept, it would be appropriate to play a short warm-up game. For example, think of a rule; then ask for a number less than 100, mentally apply the rule, and give the correct response. Do not state the rule itself; pupils should fold their arms to signal that they have guessed it. When you call on a child with folded arms, he should ask you for a number less than 100; if he knows your rule, he will be able to reply correctly. When most of the children have their arms folded, announce a new game and make a new rule. To use subtraction, limit the range of numbers to those greater than the one you choose to subtract.

How does the function machine work?

Discussing the Ideas

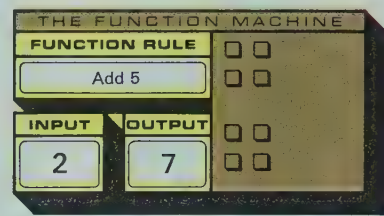
1. Study the pictures and explain how you think the function machine works. A record of the operations of the function machine is shown below. What are the missing numbers?



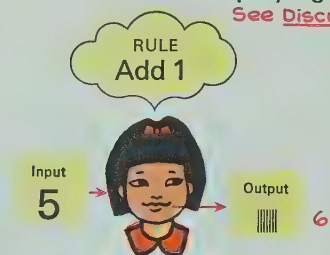
Function Rule

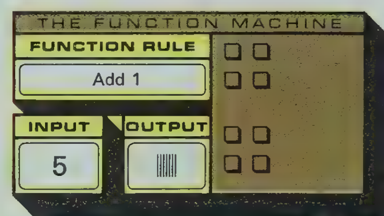
Add 5

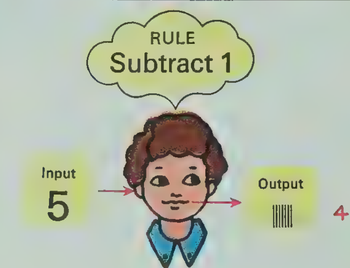
Input	Output
2	7
4	9
3	8
6	11

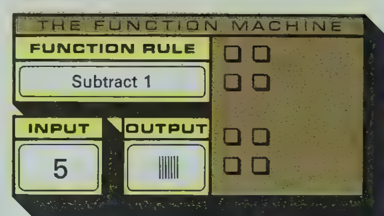


2. Find each output number. Explain how the function machine is like the student playing the What's My Rule game. See Discussion.









70

Discussion

Besides providing the child with an opportunity to discover patterns and practice basic combinations, this lesson provides an informal introduction to the concept of a function. As you read the discussion section with the class, point out the relationship between the child in the picture and the function machine. Explain that the function machine is simply a means of describing what we do when we work with these ideas. The input corresponds to our thinking of a number, and the output corresponds to the result we get when we apply the function rule. Help the children see that this process is what they do when they play "What's My Rule." You might also mention that for any given number and a function rule there is only one correct answer. If you use the chalkboard to display some input numbers and a function rule, the children will see this. For example, you might develop one of the rules on page 70 by suggesting other input numbers.

Using the Ideas

Think about the function machine and tell what you think should go in each gray space.

1. Function Rule: Add 5

	Input	Output
	2	7
	4	9
A	1	6
B	5	10
C	3	8

2. Function Rule: Subtract 3

	Input	Output
A	6	3
B	8	5
C	10	7
D	3	0
E	13	10

3. Function Rule: Add 4

	Input	Output
A	2	6
B	4	8
C	7	11
D	8	12
E	9	13

4. Function Rule: Subtract 2

	Input	Output
	3	1
A	7	5
B	5	3
C	10	8
D	12	10

5. Function Rule: Subtract 8

	Input	Output
	10	2
A	13	5
B	15	6
C	11	3
D	17	9

6. Function Rule: If odd, subt. 1 If even, add 0

	Input	Output
	3	2
	5	4
A	9	8
B	8	8

7. Function Rule: A Subtract 6

	Input	Output
	7	1
	10	4
	6	0
B	9	3
C	14	8

8. Function Rule: A Add 8

	Input	Output
	2	10
	5	13
	7	15
B	3	11
C	9	17

9. Function Rule: A Add 9

	Input	Output
	0	9
	1	10
	9	18
B	7	16
C	5	14

71

Using the Exercises
After the children read the directions, you might want to work through the first exercise with them, clarifying any questions. Before you assign the page as independent work, draw attention to exercises 7, 8, and 9. Help the children see that in these exercises they must discover the rule themselves and state it as the rule is stated in exercises 1 through 5. When the children have finished the exercises independently, check their answers and discuss any which caused difficulty.

Assignments (page 71)
Minimum: 1–5. Average: 1–9.
Maximum: 1–9.

Mathematics
The concept of a function is one of the most important ideas in mathematics, yet it can be presented on an intuitive basis very early in the child's experiences. Rather than giving a precise mathematical definition of *function*, we present examples and point to some significant features.

Example 1

Set A: 3, 5, 7, 40, 12 — *Rule* Multiply by 2 and add 1. — *Set B*: 7, 11, 15, 81, 25

If we apply the rule to each number in set *A*, we get exactly one number in set *B*. For example, applying the rule to 3 from set *A*, we get 7 in set *B*. Hence, we get the set of pairs (3, 7), (7, 15), (5, 11), (12, 25), *and* (40, 81). One vital feature of these pairs is that *for each first number, there is only one second number.*

Example 2

Set A: All whole numbers — *Rule* Multiply by 0 and add 3. — *Set B*: 3

Some of the pairs of numbers are (17, 3), (2, 3), (1, 3), (0, 3), (285, 3), . . . Though every second number is 3, it is still true that *given any first number, we get only one second number.* In summary, each function yields a set of ordered pairs, no two of which have the same first number.

Follow-up
Since the "What's My Rule" game can offer such variety, you might continue using it as a follow-up. However, if you think the children need a change of pace, you might use the follow-up activity suggested on page 61.

Workbook, page 25

PAGES 72–73

Objectives

The child will demonstrate his ability to work with the concepts presented in this chapter.

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

An oral review of the basic addition and subtraction combinations is a good preparation for page 72.

If you treat page 73 in a separate lesson, you might choose a key concept to review, preferably concentrating on one that has been troublesome. For example, you might write large numerals on the chalkboard and have children read them and compare a few using appropriate inequality symbols.

Reviewing the Ideas

1. Write 2 addition and 2 subtraction equations for each set.

A
4+3=7
3+4=7
7-4=3
7-3=4

B
7+2=9
2+7=9
9-7=2
9-2=7

C
5+4=9
4+5=9
9-5=4
9-4=5

2. Find the sums and differences.

A $8 + 2$ = 10	B $6 + 3$ = 9	C $9 - 4$ = 5	D $2 + 5$ = 7	E $10 - 4$ = 6	F $7 - 3$ = 4	G $6 + 4$ = 10	H $4 + 4$ = 8
I $7 - 5$ = 2	J $8 - 5$ = 3	K $0 + 6$ = 6	L $8 - 2$ = 6	M $10 - 6$ = 4	N $4 + 5$ = 9	O $10 - 3$ = 7	P $3 + 5$ = 8

3. Find the differences.

A $7 - 3 = n$ 4 E $12 - 9 = n$ 3
B $13 - 9 = n$ 4 F $16 - 8 = n$ 8
C $11 - 3 = n$ 8 G $13 - 6 = n$ 7
D $15 - 7 = n$ 8 H $14 - 7 = n$ 7

4. Find the sums. Look for a sum of 10.

A $6 + 4 + 7$ 17 E $4 + 6 + 5$ 15
B $8 + 7 + 3$ 18 F $7 + 8 + 3$ 18
C $8 + 9 + 2$ 19 G $6 + 2 + 8$ 16
D $5 + 7 + 5$ 17 H $9 + 8 + 1$ 18

5. Find the missing numbers.

A If $47 + 38 = 85$, then $85 - 38 = n$. 47
B If $92 - 18 = 74$, then $74 + 18 = n$. 92
C $27 + 68 = 68 + n$. 27
D $(39 + 27) + 68 = 39 + (n + 68)$ 27

6. Use the number line to help you solve each equation.

A

$7 + n + 2 = 12$
3

B

$13 - n = 6$
7

think

Increase me by 5.
Then take away 7.
When you are done,
You should have 11.

WHO AM I? 13

72

Discussion

You may treat page 72 as an evaluation page or as a chapter review. In either case, after the page has been completed, go over it carefully with the children, stressing any points with which they had difficulty. The focal point of this review is knowledge of the basic combinations. It is quite acceptable for children to learn only the addition combinations, if they can figure out the subtraction combinations by thinking about missing addends.

The *Think* problem on page 72 is intended as enrichment material. However, all the children should benefit from hearing those who solve the problems tell how they did so.

Page 73 provides a cumulative review. You might have the children do the exercises independently and, afterward, discuss the concepts according to the needs of the children. For example, any concept which troubled most of the children should be reviewed with the whole class, and concepts that caused difficulties for individuals may be reviewed with them while the children who do not need individual help work on projects as suggested in the follow-up.

Keeping in Touch with Addition, Place value, Inequalities, Fractions

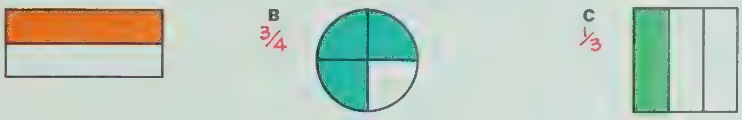
1. Write the missing numerals.
   - A For 3 tens and 6, we write ▯. 36
   - B For 5 tens and 0, we write ▯. 50
   - C For 7 tens and 4, we write ▯. 74
   - D 46 means ▯ tens and 6. 4

2. Give the missing words.
   - A In 1847, the 4 means four __?__. tens
   - B In 6253, the 6 means six __?__. thousands
   - C In 2584, the 5 means five __?__. hundreds
   - D In 3475, the 7 means seven __?__. tens

3. Solve the equations.
   - A $63 = 60 + n$ 3
   - B $10 + 7 = n$ 17
   - C $18 = n + 8$ 10
   - D $70 = 70 + n$ 0
   - E $80 + 0 = n$ 80
   - F $30 + 4 = n$ 34

4. Write a fraction for the part of each region that is shaded.

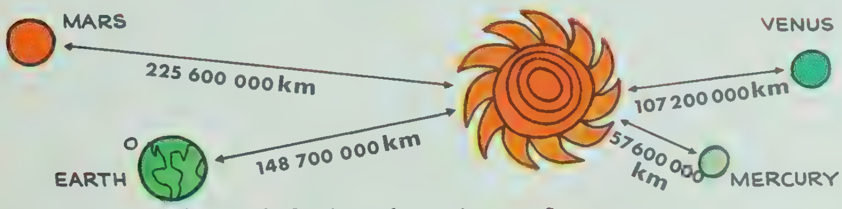
A $\frac{1}{2}$ B $\frac{3}{4}$ C $\frac{1}{3}$



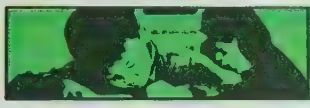
5. In each pair, which number is larger?

A	B	C	D	E
624	283	4284	9999	26,354
621	300	4384	10,000	26,364

★ 6.



- A Which of these planets is farthest from the sun? Mars
- B Which of these planets is closest to the sun? Mercury



You are invited to explore

ACTIVITY CARD 2
Page 310

73

Follow-up

Some children would enjoy making a function machine. You might let part of the challenge be to figure out how to make one. They would probably need materials such as a large cardboard box, tape, pieces of cardboard, scissors, strips of paper, tubes on which to roll the paper, and numbered discs for answers. You might limit the range of numbers they use so that it will be possible for them to show the answer.

Other children may print equations on a number line to display around the chalkboard. Still others may make display-size tables for the "What's My Rule" game.

If you prefer to save such projects as the above for use with all the children, you may want to distribute to the more capable children a worksheet such as the sample below.

1. Complete the blank spaces.

8	−3	5	+2		−1		−5		+8	9	
	−4	5	+3		−1		+2		−6	3	
	+3	11	−4		+5		−3		+6	−7	8

2. Can you find the rules and fill in the blank spaces?

3	9	4	6	8	5	7
12	18	13	?	?	?	?
6	12	7	?	?	?	?
13	19	14	?	?	?	?

Resources for Active Learning

Mathex: Operations No. 3, "An Addition Game" (pupil pages 26–27), Encyclopaedia Britannica Publications Ltd. [Two games for review]

Mathex: Operations and Problem Solving No. 8, "Rule Rummy," pp. 1–2, Encyclopaedia Britannica Publications Ltd.

Workbook, page 26

CHAPTER 4 | Geometry

Pages 74–89

General Objectives

To provide an informal introduction to the study of geometry

To introduce some of the basic figures of geometry, such as points, lines, line segments, rays, angles, and triangles

To provide intuitive experiences with the sum of the angles of a triangle

To introduce right angles, squares, and rectangles

To provide an informal introduction to the Pythagorean Theorem

The primary purpose of this chapter is to provide exciting experiences in working with key geometric ideas. The children are not expected to master all the facts presented here; however, early exposure will have some carry-over to a more formal presentation later. These lessons should be presented on an intuitive basis, with the children having an opportunity to discover the relationships for themselves. Since the children are familiar with the three-dimensional world, the first lesson draws on this experience to prepare for the study of one- and two-dimensional figures. Reference to the vertex of a cube as a point, the edge of a cube as a line segment, and the face of a cube as a plane should help the children gain correct intuitive notions of the basic geometric figures. Angles and right angles are also developed intuitively, and children are given the opportunity to construct them. Sets of 3 strips are used to help children discover what a triangle is. In the concluding lessons, angles of a triangle and right triangles are investigated, and the Pythagorean Theorem is presented, on an informal basis.

Mathematics

From our point of view, there are two kinds of geometry: physical and mathematical. Physical geometry is the geometry of the drawing board, or of the physical world. Mathematical geometry is an abstract mathematical system that unfolds from a set of undefined objects. While, in theory, we could completely separate mathematical geometry from any dependence upon the physical world, such a development would hardly be practical, at least not in an early study of geometry.

The mathematical geometry that is related to the geometry investigated by the children includes some fundamental geometric concepts. In this chapter, the concepts of point, line, segment, and plane are introduced by providing children with physical experiences involving the cube. The vertices, faces, and edges of the cube are related to points, planes, and segments, respectively. Thus, the children experience physically these more abstract mathematical concepts. Ensuing lessons provide intuitive experiences with such geometric figures as rays, angles, and triangles.

The chapter concludes with two lessons involving relatively sophisticated geometric theorems. First the children are provided with the physical experience relating to the fact that the sum of the angles of a triangle is 180 degrees. Then the Pythagorean Theorem is introduced through a special case involving an isosceles right triangle. As with the other experiences in geometry, these two basic geometric theorems are introduced primarily through a type of physical experience in which the children can actually participate.

Teaching the Chapter

Materials

Colored strips
Crayons or colored pencils (4 colors per child)
Cubes (1 per child)
Geoboard (1 per child if available)
Models of geometric shapes—cones, spheres, prisms, etc.
Paper clips (1 per child)
Paper suitable for folding
Paste
Ruler or straightedge (1 per child)
Scissors
Tracing paper
Unlined paper

Vocabulary

angle	line
cube	line segment
curved	outside
diagonal	point
edge	ray
face	right angle
figure	right triangle
flat	straight
geometry	surface
hypotenuse	triangle
inside	vertex
legs of a right triangle	

This chapter should afford fun and excitement for both teacher and children. The principal purpose is to introduce basic geometric concepts in such a way that the child intuitively grasps their meaning. The investigations that the lessons provide should inspire children to try other investigations. Try to engender and maintain a classroom mood of excitement in the discovery of new things. Encourage each child to investigate and discover for himself; help the children to respect each other as learners even though abilities and learning rates may differ widely. Try to provide stimulating activities for those who may profit from work beyond that presented in the text; the suggestions in the follow-up sections should prompt additional ideas of your own. If children respond especially well to any particular lesson, plan a class project to develop it further. The children will have a better understanding of

the terms and concepts if they use them while working together on a class project. Some project ideas are included in the follow-up section of the last lesson in this chapter. Also, refer to the materials suggested under "Resources for Active Learning."

Lesson Schedule

The amount of time you spend on this lesson will depend on how many lessons you enrich with class or individual projects. Plan to spend a maximum of two weeks, unless one of the lessons develops into a valid extended-learning experience.

Evaluation of Progress

As stated earlier, the main purpose of this chapter is to introduce basic geometric concepts. The ability to recognize by name the simple geometric figures might be expected from the average third grader, but he should not be expected to retain the more difficult ideas or terms, such as hypotenuse. One criterion of progress you might observe but not try to measure is a child's ability to follow written directions, undertake an investigation with confidence, and think independently. All of these qualities are characteristic of a certain maturity level in a learner.

Resources for Active Learning

General Activities

Franklin Series: *Learning About Measurement,* "Rays, Angles, and Triangles," pp. 39–48, Lyons and Carnahan (Available from McGraw-Hill Ryerson)

Manipulative Devices

ETA Discovery Blocks (Educational Teaching Aids)

Geo Blocks (Selective Educational Equipment; McGraw-Hill)

Geo-boards (Addison-Wesley Publishing Co.)

Geo-D-Stix (Childcraft; Edmund Scientific)

Geo Strips (Math Media; Selective Educational Equipment)

Pattern Blocks (Selective Educational Equipment; Webster, McGraw-Hill)

Sage Kit (LaPine Scientific) Clear plastic geometric solids

Soma Puzzle (Creative Publications; Cuisenaire Co.; Edmund Scientific)

Commercial Games

Geometric and Shape Dominoes (Selective Educational Equipment)

Madagascar Madness (Creative Publications; Math Media) A game related to the Pythagorean Theorem

Spot the Set (Selective Educational Equipment)

PAGES 74–75

Objective

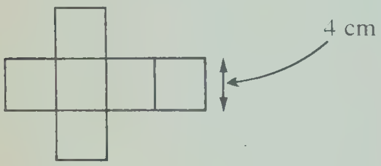
Given a three-dimensional object, the child will be able to recognize flat and curved surfaces, straight and curved edges, and vertices.

Preparation

Materials

cubes (1 per child); sphere and cone, can, box, doughnut, egg (for demonstration)

If you do not have cubes for each child to handle, you might prepare a pattern of a cube on a duplicating master, run it off on heavy paper, and have the children make their own. The following pattern would be suitable:



The sides can be secured with tape, or you can include tab marks on the pattern so that the sides can be fastened together with paste.

If cubes are already available, begin immediately with the investigation.

Investigation

One of the main purposes of this lesson is to help the child distinguish between curved and flat surfaces and curved and straight edges. Since the child's familiar world is three-dimensional, the idea of the two-dimensional figures we see pictured should evolve from examination of three-dimensional objects.

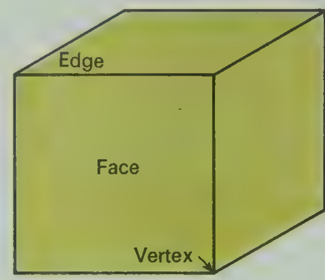
Read the directions with the children and make sure everyone knows what he is to look for in each question. Remind the children to record their results. If you have other rectangular prisms, let the children try to answer the same question about some of these. As you move around the room, ask children if they can think of familiar objects that are cubes.

4 Geometry

Let's count edges, faces, and vertices.

Investigating the Ideas

A cube has "straight" **edges**, "flat" **faces**, and "pointed" **vertices**.



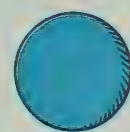
A cube

? Can you find how many of each?

12 edges
6 faces
8 vertices

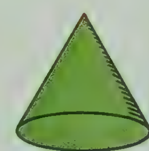
Discussing the Ideas

1. Can you name another object that has edges, faces, and vertices?
   Sample answers: box, tablet, book, domino



A sphere

2. A **sphere** (ball) has no edges and no vertices. It has a "**curved**" **surface**. Can you name another object that has no edges or vertices?
   Sample answers: orange, globe, marble



A cone

3. A **cone** has a **curved edge**.
   - A How many vertices and flat faces does it have? 1 vertex, 1 flat face
   - B Can you name another object with a curved edge?
     Sample answers: drinking glass, can, chalk

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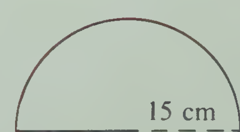
Discussion

When children complete the investigation, let them share their recorded results. If there is any disagreement in their findings, count the edges, faces, or vertices with the class.

As you discuss exercise 1, it would be helpful to display other objects that have edges, faces, and vertices and to count these for a few of the objects. Similarly, have a sphere and a cone to display during discussion of exercises 2 and 3. Use a basketball or tennis ball if you have no other sphere; make a cone from tagboard or construction paper if necessary.



Pattern for a cone (without base)

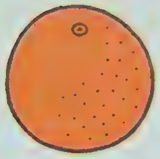
Pass these around the room so that the children have an opportunity to handle them. If the children are slow in responding to exercise 2, give them a start by naming items such as an egg, donut, orange, etc. You might have such objects available to distribute for children's examination.

In exercise 3, point out that a cone has one flat base. Among common objects which have curved

Using the Ideas

A

D

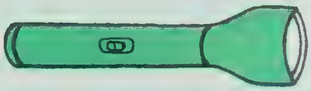


B

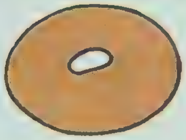
E

C

F



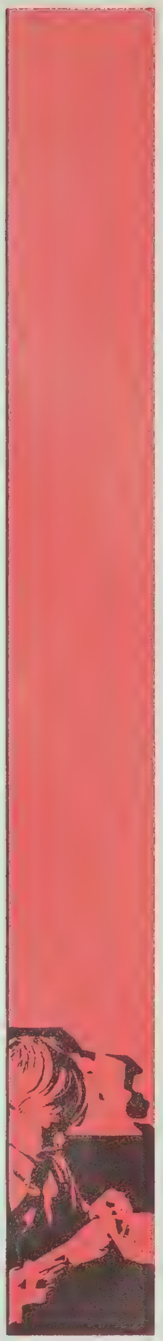
G



1. Name at least one figure above which has

- **A** no edges. D, G
- **B** only straight edges. B
- **C** only curved edges. A, C, F
- **D** both straight and curved edges. E
- **E** no vertices. A, D, F, G
- **F** only one vertex. C
- **G** more than one vertex. B, E
- **H** only flat faces. B
- **I** a curved face or surface. A, C, D, E, F, G
- **J** both flat and curved surfaces. A, C, E, F
- **K** curved edges and both flat and curved surfaces. A, C, E, F

★ **2.** Which figure has **more than one vertex**, both **flat** and **curved surfaces**, and both **straight** and **curved** edges? E



edges are such things as cylindrical cans, sticks of chalk, pointers, circular trash can, etc.

Using the Exercises

Before assigning these exercises, explain that some of them have more than one correct answer. After the children have spent some time working independently, discuss the exercises together. Help the children clarify their understanding of the terms *edges, faces, vertices, straight,* and *curved.* For exercise 2, ask a child to identify the edges, vertices, and surfaces that he recognizes as flat, straight, or curved.

Assignments (page 75)
Minimum: 1. Average: 1.
Maximum: 1–2.

Mathematics

In this lesson the children are introduced to some basic plane geometric figures through the three-dimensional world. This is done by providing the children with a cube and an opportunity to feel and touch and experience physically objects which relate to the abstract notions of point, line, and plane.

The key concepts involved in working with lines and planes are the ideas of straightness and flatness, which children generally understand intuitively. In mathematical geometry, these ideas are taken as undefined, or at least they are defined through certain basic assumptions made in the set of postulates that underly the structure of the mathematical system. For example, the postulate which states that through any two given points there is exactly one line relates to the concept of straightness; and the concept that a line containing two points of a plane lies entirely within the plane relates to the idea that a plane is flat. Such abstract ideas need not be presented to the children, except on an intuitive basis and as they arise naturally.

Follow-up

To follow up this lesson, have the children bring in pictures of three-dimensional objects for display. Suggest that they look for objects with flat surfaces, curved surfaces, and straight edges.

Resources for Active Learning

Developmental Math Cards C²3, C²12, D²6, D²13, Addison-Wesley. [Extends experiences with three-dimensional shapes.]

Franklin Series, *Pencil and Paper Geometry,* "Geometry Around Us," pp. 5–10; "Cubes," pp. 85–88, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Math Activity Cards, "The Cube," A14, Macmillan.

Mathex: Geometry No. 4, "Properties of Solids," pp. 7–8 (pupil pages 13–19), Encyclopaedia Britannica Publications Ltd.

Nuffield Project, *Environmental Geometry,* "Shapes in the Environment," pp. 17–23, Wiley.

PAGES 76–77

Objective

Given representations of points, line segments, rays, lines, and planes, the child will be able to identify them.

Preparation

Materials

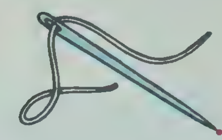
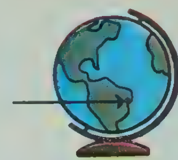
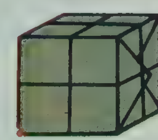
ruler or straightedge (1 per child); unlined paper

Since this lesson begins with the introduction and discussion of new concepts, no special preparatory activity or discussion is necessary.

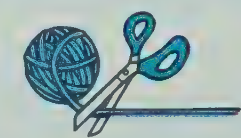
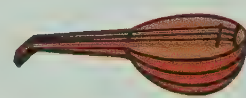
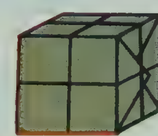
● *Can you name the simplest geometric figures?*

Discussing the Ideas

1. These figures suggest points •
   Can you think of others? Sample answers: pencil tip, point of a compass

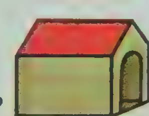
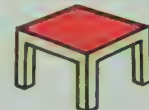
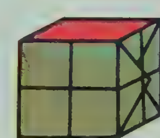
  

2. These figures suggest line segments
   Can you think of others? Sample answers: edge of a ruler, chalkboard frame, edge of a book

3. A beam of light suggests a
   
   ray . A ray has one endpoint and "goes on and on" in one direction.
   Can you think of other examples of rays?
   Sample answers: searchlight beam, beams of a car's headlights

4. The boy looking in opposite directions suggests a
   
   line . A line has no endpoints and "goes on and on" in both directions.
   What other situations suggest lines?
   Sample answers: telephone wires, railroad tracks

5. These pictures suggest
   planes
     
   Can you think of others?
   Sample answers: floor, sheet of paper

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Discussion

As you discuss each of these exercises with the children, note that the pictures are not the points, rays, and so on, but that they represent *ideas* of these figures, just as numerals represent numbers. However, this distinction need not be stressed; the purpose here is to help the children develop correct *intuitive* understandings of these simple figures.

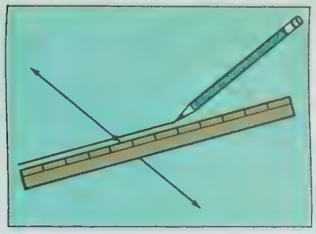
Mention that each of the figures discussed in exercises 2 through 5 may be thought of as a set of points. To demonstrate this, draw two points on the chalkboard and suggest that they can be connected by a series of dots. Show this by adding dots between the points, first spacing the dots widely and then continuing to add dots within the gaps until they are almost closed. Such a demonstration should help the children to think of a line segment as a specified set of points.

In discussing exercise 5, relate the two-dimensional geometric figure to the child's experience of a three-dimensional world; for example, point out the face of a cube and have the children extend it imaginatively to form a plane.

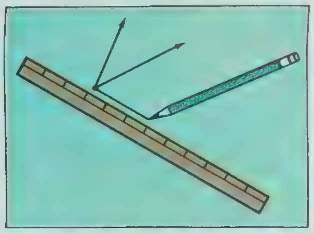
Using the Ideas

See Using the Exercises.

1. Mark a **point** on your paper. Use your ruler to draw 5 different **lines** that pass through that point.

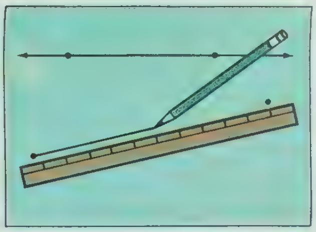


2. Mark a point on your paper. Use your ruler to draw 3 different **rays** from that point.



3. A Mark two points on your paper. Use your ruler to draw **the line** that passes through these two points.

   B Mark two other points on your paper. Draw **the line segment** for these points.



★ 4. Study the chart. Then draw and name a line, a ray, and a segment.

We see the figure	We label some points	We write a name for the figure	We say
	A B	$\overleftrightarrow{AB}$	"line *AB*"
	P Q	$\overrightarrow{PQ}$	"ray *PQ*"
	C D	$\overline{CD}$	"segment *CD*"

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Using the Exercises

The children would benefit from working through at least the first exercise as a class activity under your direction. Urge them to make small, neat dots to represent points. Then, put chalk or pencil on a dot and show how to place a ruler beside it to draw a line. If children have difficulty with their regular rulers, have smaller rulers available as substitutes so they will have less trouble with slipping as they draw the segments.

After the children have completed the exercises, ask volunteers to use chalk and rulers at the chalkboard to draw representations of lines, line segments, and rays.

Call attention to the labelling in starred exercise 4, but you should not expect third graders to master its use.

Assignments (page 77)
Minimum: 1–3. Average: 1–3. Maximum: 1–4.

Mathematics

Children are not given formal definitions of *point, line, line segment,* and *ray;* rather, the ideas are presented intuitively, chiefly through illustrations.

Do not overemphasize the point that the geometric figures we draw are models of ideas that exist only in our minds. Such abstractions are not a vital part of a child's experiences in geometry at this time.

Follow-up

If practical, you might take the class for a walk around the school grounds to have them look for objects which remind them of points, lines, line segments, rays, and planes.

Alternatively, you might play an oral game which relates actual objects to geometric ideas. For example, you might say: "I am a flashlight; I make you think of a ______ (ray)" or "I am your pencil tip, I make you think of a ______ (point)." As the children catch on, let volunteers suggest the "I am . . ." statement and have them call on classmates for a response.

Resources for Active Learning

Franklin Series: *Learn to Fold . . . ,* "Learning About Lines," pp. 51–55, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Franklin Series: *Pencil and Paper Geometry,* "Points," pp. 26-28; "Lines and Line Segments," pp. 30-37; "Rays," pp. 63-65; "Angles," pp. 67-70, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Mathex: Geometry No. 9, "Points, Space and Planes – Activity 2," pp. 19–20; "Rays and Angles – Activity 5," p. 22, Encyclopaedia Britannica Publications Ltd.

Math Workshop: Games and Enrichment Activities, "Nonmetric Geometry with Geo-boards," pp. 80–84, Encyclopaedia Britannica Educational Corp.

Workbook, page 27

Objective

Given from 2 to 6 points, the child will be able to connect the points with line segments and count how many.

Preparation

Materials

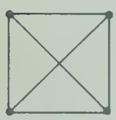
ruler or straightedge (1 per child); unlined paper; geoboard (if available)

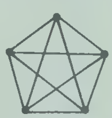
As in the preceding lesson, no formal preparation is necessary.

Investigation

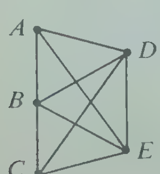
The children should work on this investigation individually. Allow some sharing of ideas, but encourage them not to tell one another how many segments they were able to draw or how they drew them, until each child has had an opportunity to discover what he can on his own. Remind the children not to be overly hasty in concluding that they have drawn all the segments they can; they might be overlooking some less-obvious segments.

To connect 4 points you can draw 6 segments. To connect 5 points you can draw 10 segments.





If some children finish quickly, you might want to have them reposition the same number of points to see how many segments they then can draw. The difficulty of collinear points may arise, but if a child says he can draw only 9 segments for 5 points, ask him to count them for you. Note in the figure below that, even though $\overline{AB}$ and $\overline{BC}$ are on the same line, we can count as segments not only $\overline{AB}$ and $\overline{BC}$ but also $\overline{AC}$, so there are 10 line segments altogether, namely, $\overline{AB}$, $\overline{AC}$, $\overline{AD}$, $\overline{AE}$, $\overline{BD}$, $\overline{BE}$, $\overline{BC}$, $\overline{CD}$, $\overline{CE}$, and $\overline{DE}$.

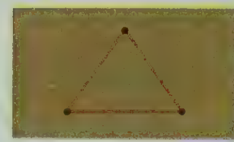


Let's count line segments.

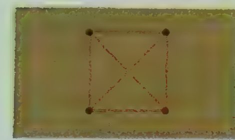
Investigating the Ideas

You can draw only one segment to connect 2 points.

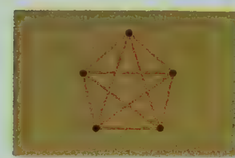
You can draw three segments to connect 3 points.



How many segments can you draw to connect 4 points? Try it. 6

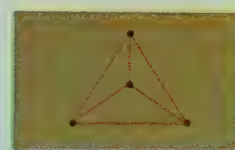


? If 5 points are placed like this, how many segments can you draw to connect them? 10



Discussing the Ideas

1. When 4 points are placed like this, how many segments do you think you can draw to connect them? 6



2. This chart shows the number of segments that connect each set of points shown.

Points	• •	∴	⁘	⁙	• • • • • •
Segments	1	3	6	10	15

A How many segments did you get for 5 points? 10

B Guess how many segments you get for 6 points. Try it. 15 (See Discussion.)

78

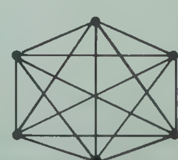
Discussion

Let the children share their investigation results. You might develop a chart on the chalkboard as they agree on how many segments can be drawn for each number of points, letting volunteers draw the figures on the chalkboard and enter the correct numbers on the chart, as shown.

Points	2	3	4	5
Segments	1	3	6	10

During the discussion, the question of three points which lie on the same line may arise, as mentioned previously. Refer to the investigation section for an explanation of this case.

After all the children have tried exercise 2 at their desks, ask a volunteer to show how many segments can be drawn for 6 points.

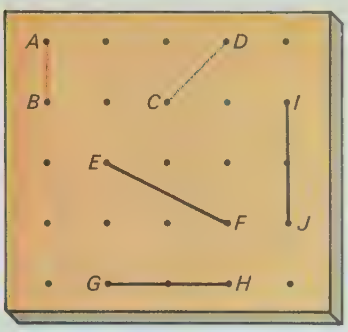


15 segments

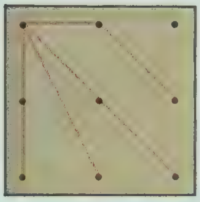
If anyone notices a pattern on the chart, praise him and ask him to give the number of segments for 7 points, but do not overburden the average child with this. (See the discussion in the mathematics section.)

Using the Ideas

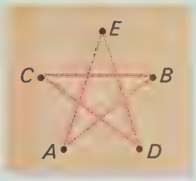
1. The red segment from A to B ($\overline{AB}$) is shorter than the blue segment from C to D ($\overline{CD}$). Use the picture of the nailboard for these questions.
   - A Which segment is the longest? $\overline{EF}$
   - B Which segment is shortest? $\overline{AB}$
   - C Can you name a segment that is longer than $\overline{AB}$ but shorter than $\overline{GH}$? $\overline{CD}$
   - D Name two segments that have the same length. $\overline{GH}$, $\overline{IJ}$



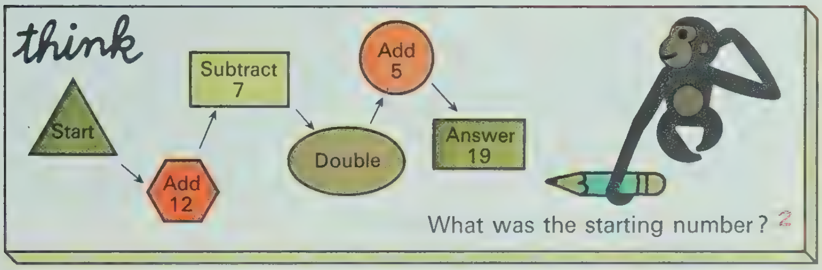
2. How many segments of different lengths can you show using only nine points like these? 5



3. Trace dots A, B, C, D, and E and connect them in this order.
   $A\ B\ C\ D\ E\ A$
   - A What figure did you make? Star
   - B How many segments did you draw? 5



4. A Draw a figure using 4 segments.
   B Draw a figure using 5 segments. Constructions will vary.



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Using the Exercises

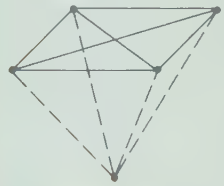
You might have the children work on the exercises on page 79 in small groups, taking turns using geoboards (if available) to work out exercise 2. (If geoboards are not available, children can use dot paper or tracing paper.)

You might wish to use the ideas in the follow-up section to augment these exercises; if so, provide the children an activity period in which to do them.

Assignments (page 79)
Minimum: 1–2. Average: 1–4.
Maximum: 1–4.

Mathematics

The pattern of the number of segments, as it relates to the number of points, is of particular interest. Notice that each time an additional point is added the number of segments increases by the number of points for the previous figure. For example, 4 points determine 6 segments and 5 points determine the same 6 segments plus 4 more segments. That is, the new point can be connected to each of the 4 previous points.



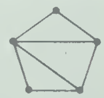
6 segments for 4 points
$6 + 4 = 10$
10 segments for 5 points

In general, then, for n points there are $\frac{n \times (n-1)}{2}$ segments connecting these points.

Follow-up

You might write the following suggestions on the chalkboard:

"Trace dots A, B, C, D, and E in exercise 2, page 79. In how many ways can you connect them so that no line segment crosses another? How many segments did you draw?" Although the children may draw many different figures, the one below includes all the others.



Put the following figures on the chalkboard and ask the children to try to copy them without lifting their pencil from the paper and without tracing back over any lines.

The last one on the right is the only one that cannot be done in this manner.

Resources for Active Learning

Franklin Series: *Learn to Fold . . .*, "Finding End Points," pp. 56–60, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Inquiry in Mathematics via the Geo-Board, "Line Segments," Geo-Cards 4/1, 2, Walker. (Available from Fitzhenry & Whiteside)

PAGES 80–81

Objectives

Given a group of objects, the child will be able to point out those which suggest angles in general and right angles in particular.

Given appropriate materials, the child will be able to construct right angles and other angles.

Preparation

To prepare for this lesson, you might conduct a short oral review of the form suggested in the follow-up section on page 77. To keep it brief, name the objects yourself. Continue the pattern so that it leads into the investigation as suggested below.

Investigation

To introduce this investigation, you might say: "I am the hands of a clock. I make you think of ______" and direct the children to the text.

One of the main objectives of this lesson is to provide children with physical objects, pictured or real, which suggest angles. Let the children study the objects illustrated in the investigation section and identify each. Then ask them how the objects in the first row are alike. Help the children see that each of the objects contains at least two parts that suggest segments which meet or cross. The children should then be able to observe that the scissors and the bird's beak in the second row contain the same elements.

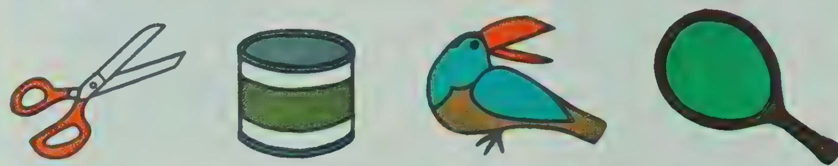
What is an angle?

Investigating the Ideas

All of these objects are alike in some way.

? Which of these objects are like the ones above?

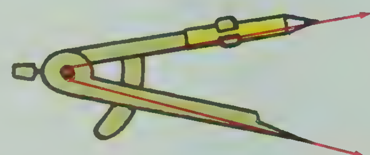
Scissors, bird's beak (See Investigation.)



Discussing the Ideas

See Discussion.

1. The compass suggests the idea of an **angle**. An angle is two rays with the same endpoint. Can you think of other things that suggest angles?



2. Draw an angle using the corner of your tablet. This special angle is called a **right angle**. Can you find some objects that suggest the idea of a right angle?

3. Can you find a way to fold a piece of paper so that some right angles are formed?

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Discussion

In discussing exercise 1, point out that the arrows help us think of rays, which continue on indefinitely. Possible responses to the question may include the corners of desks or books, the angle formed by the flagpole and the wall, the angle formed by a bent arm, etc.

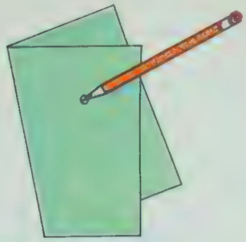
When the children draw the angle for exercise 2, a small ruler or straightedge could be used instead of the tablet if the ruler is easier for the children to manipulate. Children may notice that some objects mentioned in exercise 1 are again mentioned as models of a right angle. You might observe for them that, although every angle is not a right angle, a right angle is a special kind of angle.

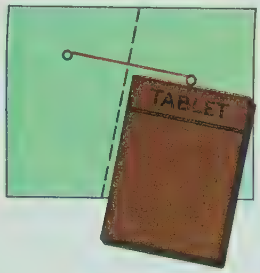
For exercise 3, allow all of the children ample time to work independently at folding sheets of paper to form right angles, and then have volunteers demonstrate and explain the techniques they have discovered.

Using the Ideas

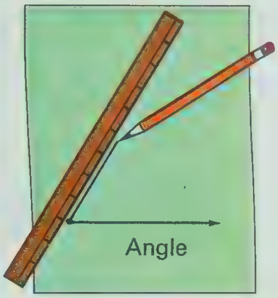
See Using the Exercises.

1. Follow these steps to form a right angle.
   - A Fold your paper any way you wish.
   - B Push your pencil point through both pieces of the folded paper.
   - C Open the paper and draw a line segment connecting the two holes in your paper.
   - D Use a corner of your tablet to see if the fold line and the line you drew form a right angle.





2. A Mark one point on your paper. Draw two rays from this point. The figure you have drawn shows an angle.
   B Draw four more angles. Make each one look different.



★ 3. Study the chart. Then name the angles you drew in exercise 2.

We see the angle	We label some points	We write a name for the figure	We say
	B, A, C	∠ABC or ∠CBA	"angle ABC" or "angle CBA"

81

Using the Exercises

Assign these exercises as independent work, but observe the children carefully as they work and give help freely according to their needs. It is quite likely that some children will need help in understanding and properly executing the directions for exercise 1.

In exercise 2, some children may attempt to make the angles they draw "look different" simply by varying the lengths of the rays. If so, you should remind them that rays extend on and on no matter how long or short we picture them; thus, if we wish to make the angles actually "different," we must vary the amount of space inside the angle.

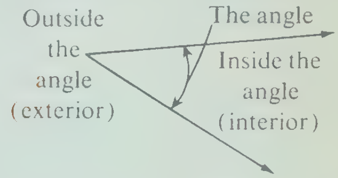
It would be helpful for the entire class to discuss the starred exercise together, even though it is intended primarily for the more capable children.

Assignments (page 81)

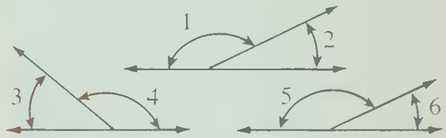
Minimum: 1–2. Average: 1–2. Maximum: 1–3.

Mathematics

A common misunderstanding associated with the concept of an angle is that the angle includes its interior. A precise definition of angle, however, simply defines the angle as the set of points on two rays from a given point. Associated with each angle are an *interior* and an *exterior*, as illustrated below.

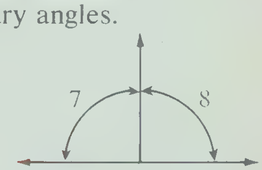


Observe the following illustrations of pairs of supplementary angles. The angles that are supplementary to each other have sides that lie on one line.



Angles 1 and 2 are supplementary to each other, as are angles 3 and 4, and angles 5 and 6.

Now consider this pair of supplementary angles.



Angles 7 and 8 are supplementary to each other, *and* they have the same size (or the same degree measure). Thus, a *right angle* is an angle that is the same size, or degree measure, as its supplement.

Follow-up

As the children study more geometric figures, they might enjoy looking through magazines for pictures which represent the figures, or they might draw the figures on drawing paper and then label and color them.

Resources for Active Learning

Mathex: Geometry No. 4, "Lines and Angles—Activity 3," p. 28, Encyclopaedia Britannica Publications Ltd.

Math Workshop: Games and Enrichment Activities, "Angles," pp. 84–85, Encyclopaedia Britannica Educational Corp.

Mathematics in Modules, SK5, SK6, Addison-Wesley

PAGES 82–83

Objective

Given suitable materials, the child will be able to identify and draw a triangle.

Preparation

Materials

colored strips; 3-by-3 dot paper (*Duplicator Masters*, page 65); unlined paper; ruler or straightedge

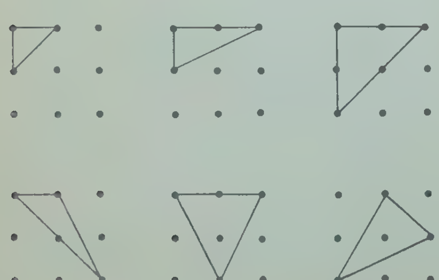
Since you will probably wish to allow considerable time for the investigation, it is recommended that you begin immediately with the text.

Investigation

Guide the children in studying the text and illustration at the top of page 82. Encourage them to observe that the shapes of the figures in the geoboard picture are different because the sides of one figure are not all the same lengths as the sides of the other. Help them to see, too, that a side may contain three pegs (or points) but must include at least two pegs. When you have distributed the 3-by-3 dot paper, be sure the children recognize the correspondence between the dots and the pegs of the geoboard.

Allow the children ample time to find as many different three-sided figures as they can. Some children very likely will repeat figures of the same shape but in different positions, and you should help them see that such figures are not really different in shape.

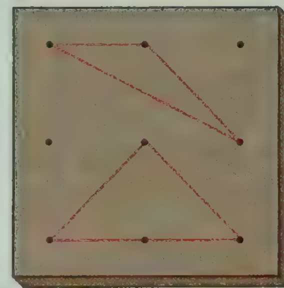
As the children are completing their drawings, you might put on the chalkboard five sets of 3-by-3 dot arrays, and then ask for volunteers, one at a time, to show the different figures they found, until the possibilities are exhausted. Besides the examples in the text, they are:



What is a triangle?

Investigating the Ideas

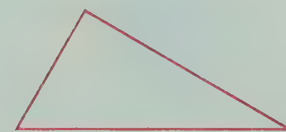
Two three-sided figures of different shapes are shown on the 3-by-3 geoboard.



? How many three-sided figures of different shapes can you draw on 3-by-3 dot paper?

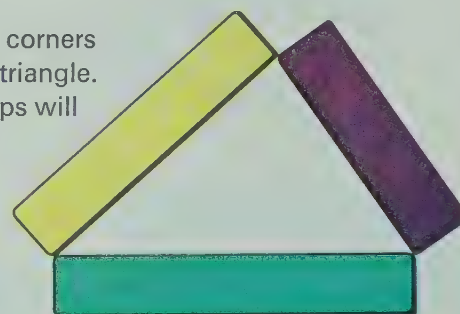
See Investigation.

Discussing the Ideas



1. A closed figure like this one is called a triangle.
   - A A triangle has __?__ 3 line segments.
   - B Can you name some objects that are in the shape of triangles? See Discussion.

2. The three strips have their corners placed together to form a triangle. Which of these sets of strips will not form a triangle? B, D
   - A black, brown, and blue
   - B yellow, orange, and light green
   - C three yellow strips
   - D red, yellow, and brown



3. Suppose you drew a triangle using your light green, purple, and yellow strips and a classmate used the same strips to form a triangle. Do you think the two triangles would have the same size and shape? Yes

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Discussion

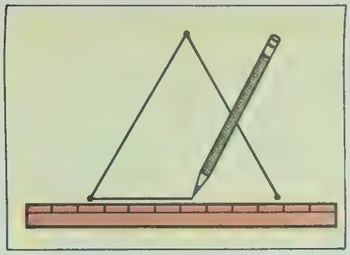
If you have been using the term *triangle* in reference to the children's three-sided figures, they will use the word properly also, without its being given special emphasis.

For exercise 1, the children may have difficulty thinking of objects that suggest triangles. If so, briefly mention such examples as the sails on some sailboats, school pennants, pictures of Indian tepees, church steeples, etc., and move on to the second discussion question. The question there involves the concept of the *triangle inequality:* the sum of the lengths of any two sides of a triangle must be greater than the length of the third side. However, when the children find sets which cannot be used to form a triangle, they will probably reason simply that two of the strips "aren't long enough." The sets of yellow (5), orange (10), and light green (3) and of red (2), yellow (5), and brown (8) are examples.

As you discuss exercise 3, have the children use the strips specified to make triangles and then compare them to see that only one shape can be made with each set of three strips.

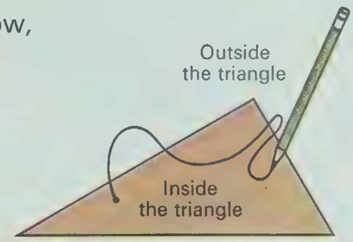
Using the Ideas

1. Mark 3 points as in the figure. Draw line segments to connect those points.
   A How many segments did you draw? 3
   B What is the name of the figure you have drawn? Triangle



2. Draw four triangles. Make each one different.

3. Draw a triangle and mark a point inside the triangle Put your pencil on the point and draw a path that crosses a side of the triangle.
   A Where is your pencil point now, **inside** or **outside**? Outside
   B Cross again. Where is the pencil point now? Inside
   C If you cross 5 times in all, where are you? Outside
   D If you cross 8 times in all, where are you? Inside



★ 4. Study the chart. Then name the triangles you drew in exercise 2.

We see the triangle	We label some points	We write a name for the triangle	We say
		△ ABC	"triangle ABC"

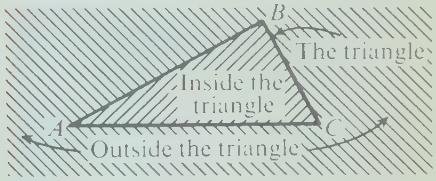
Using the Exercises

The children should use a ruler or straightedge to draw the triangles suggested on page 83. Assign these exercises as independent work and then give help individually as needed. When the children have finished, discuss the exercises together. Call particular attention to exercises 1, 3, and 4. Have the children note that 3 points determine a triangle and the triangle is labelled according to the names of these points. You might have some children try crossing the triangle discussed in exercise 3 a different number of times. Some children may quickly discover the general rule that an even number of crosses will bring you back to the side of your starting point, but let them discover this on their own.

Assignments (page 83)
Minimum: 1–2. Average: 1–3. Maximum: 1–4.

Mathematics

If A, B, and C are three noncollinear points, then we define triangle ABC to be the set of points on the segments AB, BC, and AC. The set of points that are in the interior of each of the angles of the triangle is the *interior* of the triangle, while the set of points that are not in the interior and not on the triangle is the *exterior* of the triangle.



By providing the children experiences in using their strips to form triangles, we introduce on an intuitive level the triangle inequality. That is, the children are exposed to the fact that the sum of the lengths of any two sides of a triangle is greater than the length of the third side. For example, the children would not be able to make a triangle with the 5, 10, and 3 strips, nor with the 2, 5, and 8 strips, because the sum of the two shorter sides is not as a great as the third side. Notice also that once three strips are chosen so that the sum of any two strips is greater than the third, then the shape of the triangle is determined. Every triangle generated from the three strips will be the same size and shape.

Follow-up

Display the "Triangle Cat" (or distribute duplicated copies of it) and ask the children to see how many triangles they can find.

(Answer: 17—head, 6; body and paws, 7; tail, 4)

Resources for Active Learning

Franklin Series: *Pencil and Paper Geometry*, "Triangles," pp. 19–23, Lyons and Carnahan. [This may be used with the next lesson as well.] (Available from McGraw-Hill Ryerson)

PAGES 84–85

Objectives

Given a triangle, the child will be able to recognize whether or not it is a right triangle.

Given a triangular region cut from paper, the child will be able to cut off and arrange the three corners to show the sum of the angles of a triangle.

Preparation

Materials

coloring crayons; paper clips (1 per child); paste; scissors; unlined paper

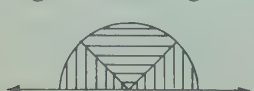
Briefly review the meaning of the terms *angle* and *triangle*. You might point to the hands of the clock and ask what geometric figure is suggested. Or, hold up figures such as a rectangle, square, and triangle for the children to identify.

Investigation

You might find it helpful, and perhaps necessary, to work through the directions for the investigation with the class. Instruct the children to draw the longest side first and then the other two sides. Encourage a variety of sizes and shapes in the triangles drawn by different children.

Assist the children in interpreting the illustration of how to use the paper clip to make an arc at each corner of the triangle. After they have completed the coloring, direct them to cut off the corners where the arcs, or curved lines, are marked. Even if the children were successful in using the paper clip to draw the arcs, they may need help in drawing the circle. If the circle is fairly well drawn and the corners of the triangle are carefully pasted onto the circle side by side, the corners will fit into one half of the circle.

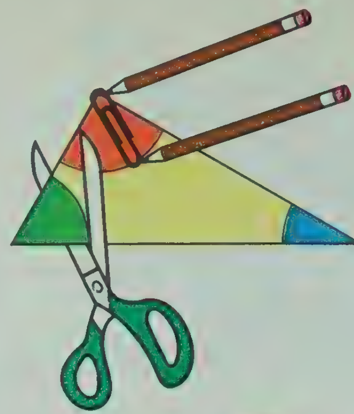
Let children who finish quickly repeat the process starting with a triangle that is different in size and shape. Also, encourage some children to draw line segments and then try to line up the corners of the triangle on the segment.



Let's explore the angles of a triangle.

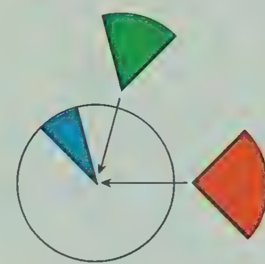
Investigating the Ideas

1. Draw a large triangle. Make the longest side at least 15 centimetres long.



2. Use two pencils and a paper clip to draw part of a circle at each corner. Color each corner a different color. Now cut out the triangle and cut off the colored corners.

? Draw a circle with your paper clip and pencils. How much of the circle do the three corners of the triangle fill if the edges touch but do not overlap? One half (See Investigation.)



Discussing the Ideas

1. How much of a circle can you fill with corners from two triangles? The whole circle

2. A triangle that has one right angle is called a **right triangle**. Do you think the corners of a right triangle will fill half a circle? Yes

RIGHT TRIANGLE

3. Can a triangle have two right angles? Explain. No (See Discussion.)

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When all the children have completed the activity for at least one triangle, give them time to compare their work. They should observe that, no matter how different the size and shape of the triangles they started with, the corners all will fit into one half of the circle they drew.

Discussion

When the children have had ample opportunity to talk about their findings in the investigation, direct their attention to the first discussion question. To provide visual proof of the answer, you might have several pairs of children who drew different-sized triangles for the investigation place together the two half-circles that they formed from the corners of their triangles.

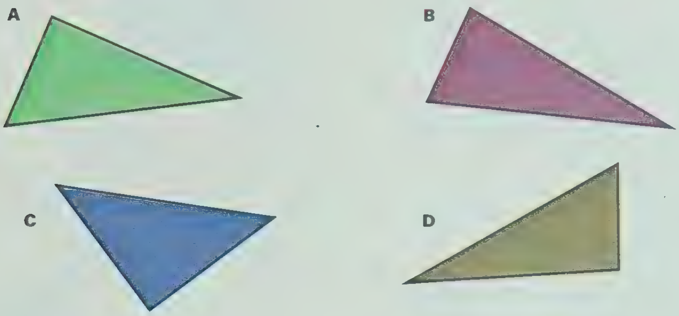
Again, for exercise 2, you might want to have children demonstrate the answer. If so, have the children construct a right angle by folding a plain sheet of paper as illustrated at the right, and when they unfold

Using the Ideas

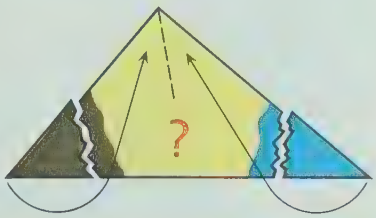
1. Use the corner of your tablet or crayon box and draw a **right triangle** on your paper.
Constructions will vary.

2. Draw a right triangle that has two sides that are the same length.
Constructions will vary.

3. Use the corner of a sheet of paper to help you decide which of these are right triangles. A, C

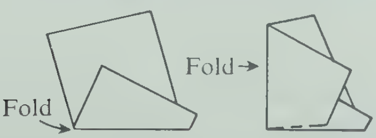


★ 4. Draw a right triangle and color the two angles that are not right angles different colors. Cut out the triangle and then cut off the colored corners. Will they fit exactly into the right angle without overlapping? Yes



85

it, they can trace over one of the four right angles they formed.



The children can use this right angle in making a right triangle and then follow the same procedure as in the investigation to illustrate the answer for exercise 2.

In answering exercise 3, children should reason as follows: Since 2 right angles would fill up half of a circle there would be no room left for a third angle, so a triangle cannot have two right angles.

Using the Exercises

Assign the exercises on page 85 as independent work. You might add to them with some of the items suggested in the follow-up section. Also, encourage all the children at least to try the starred exercise if time permits.

Assignments (page 85)
Minimum: 1, 3. Average: 1–3.
Maximum: 1–4.

Mathematics

The following statement presents the underlying mathematics of this lesson: The sum of the degree measures of the three angles of any triangle is 180 degrees. This geometric theorem can be easily demonstrated by the children in a physical sense. (However, degree measure of angles should not be taught in this lesson.) Notice that an interesting concept arises out of the fact that, if the sum of the angles of any triangle is 180 degrees, then the sum of the two smaller angles of a right triangle is necessarily 90 degrees. (See exercise 4, page 85.)

Follow-up

Any of the following activities would provide appropriate follow-up assignments for this lesson.

1. Use a card or piece of tagboard that has a right angle on the corner to see how many right angles you can find in the classroom.
2. See which of your colored strips you can use to form a right triangle. (3, 4, 5 and 6, 8, 10 strips)
3. On a large piece of paper draw a triangle with sides 9, 12, and 15 units respectively. What kind of triangle did you draw? (a right triangle)
4. Draw two right triangles of the same size. Cut them out and put sides of equal length together. What figures can you form? (Possibilities: square, rectangle, parallelogram, triangle)

Resources for Active Learning

Mathex: Geometry No. 4, "Lines and Angles – Activity 7," pp. 29–30, Encyclopaedia Britannica Publications Ltd.

Mathex: Geometry No. 9, "Angles and Triangles – Activity 2," pp. 26–27, Encyclopaedia Britannica Publications Ltd.

Nuffield Project: *Shape and Size* 2, "Which Shapes Fit Together Best?" pp. 25–35; "Right Angles . . . ," pp. 77–78, Wiley.

Workbook, page 28

PAGES 86–87

Objective

Given a right triangle, the child will be able to identify the legs and the hypotenuse of the triangle. The child will also experience an informal introduction to the Pythagorean Theorem.

Preparation

Materials

tracing paper; scissors; coloring pencils or crayons

To follow the directions of this investigation, the children must be familiar with a few new terms. Before using the text you might display a right triangle. Have children identify the right angle. Then explain that the two sides coming from the right angle in a right triangle are called *legs* and the side opposite the right angle is called the *hypotenuse*. It may be helpful to point out that the hypotenuse of a right triangle is always the longest side of the triangle.

Investigation

Read carefully through the investigation directions with the class. Caution the children to be as accurate and neat as possible in their tracing and cutting. (If you feel your children may have difficulty, you may wish to prepare duplicated sheets containing this pattern drawn to a larger scale, to minimize the demands on the children's motor skills.)

Since the time required to carry out this investigation will probably vary greatly among the children, you may wish to have the faster children try to repeat the activity with another, smaller or larger isosceles right triangle.

Allow children to work individually through the investigation, but do not hesitate to give guidance where necessary.

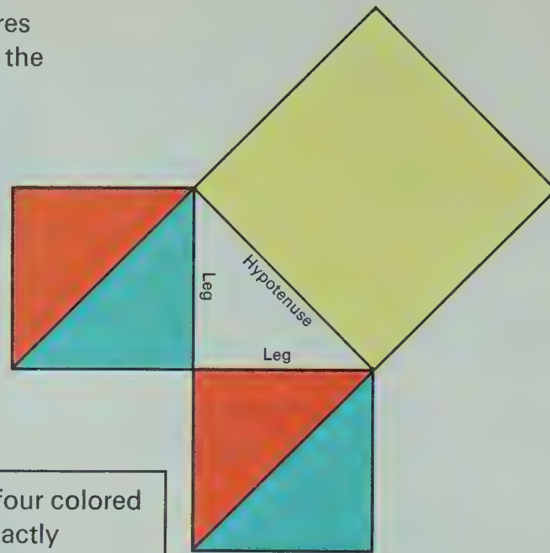
When children have completed the investigation, allow ample time for a comparison and discussion of the children's results. Encourage use of the terms *legs* and *hypotenuse* when appropriate.

Let's explore some right triangles.

Investigating the Ideas

Trace the two small squares that have been drawn on the two legs of the right triangle.

Color and cut out the four small right triangles that form the two squares.

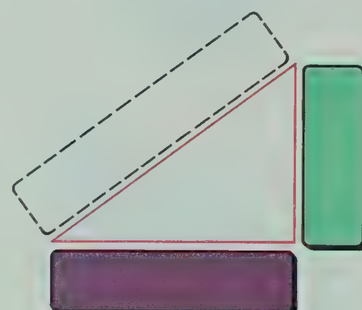


?

Can you make the four colored right triangles fit exactly into the large yellow square on the hypotenuse of the right triangle? Yes (See Investigation.)

Discussing the Ideas

1. In what way are right triangles different from other triangles? See Discussion.
2. Which one of your strips fits in the dotted outline to complete the right triangle? Yellow (5)
3. What strip could you use with your 6-strip and 8-strip to form a right triangle? Orange (10)



Discussion

In discussing exercise 1, the children should note that the distinctive feature of a right triangle is that one of its angles is a right angle. They may also observe that a right triangle has two legs and a hypotenuse. Some children, remembering only the triangles they worked with in the investigation, may suggest that the legs of a right triangle are the same length; others will remember from the previous lesson that such is not always the case.

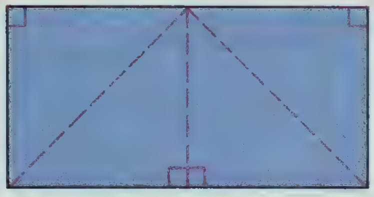
The children should have little difficulty discovering, in answer to exercise 2, that the yellow 5-strip is the only one of the strips that completes the pictured right triangle. Again, encourage the children to identify the longest side (the side opposite the right angle) as the hypotenuse and the two shorter sides as the legs.

Children might work individually or in small groups to find the answer for exercise 3. If they begin by using the 6-strip and the 8-strip as legs, they will quickly find that they can form the hypotenuse with the 10-strip. However, if they try to use the 8-strip as the hypotenuse, they will find that the 5-strip almost *but not quite* fits as the third side of a right triangle. In

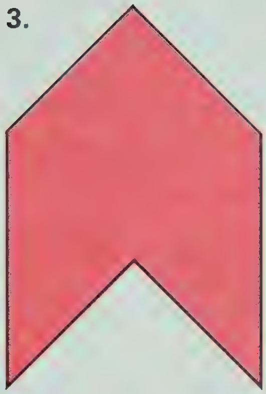
Using the Ideas

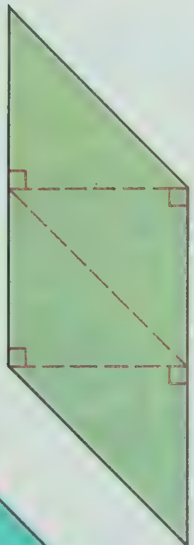
Use the four colored right triangles you cut out for the Investigation. Arrange the four triangles so that they exactly fit each figure. Draw a picture to show how you arranged the triangles for each figure.

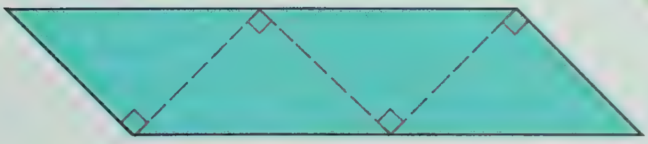
Answers shown are samples; other arrangements are possible.

1. 

2. 

3. 

4. 

5. 

that case, encourage them to keep trying until they find the combination that fits exactly—the 6, 8, 10.

Using the Exercises

Give the children plenty of time to work on the exercises on page 87 independently. You might use suggestions from the follow-up activities on page 85 for any children who finish quickly. When all are finished, allow time for comparing and discussing the answers. You might also allow the children to display samples of their work around the room.

Assignments (page 87)

Minimum: 1–5. Average: 1–5. Maximum: 1–5.

Mathematics

The investigation for this lesson deals with a special case of the Pythagorean Therorem, which is stated as follows:

> For any right triangle, if the lenths of the sides are a, b, and c, where c is the length of the hypotenuse, then $a^2 + b^2 = c^2$.

Since a^2, b^2, and c^2 can be interpreted as the area of squares constructed on three sides of the triangle, the theorem is sometimes stated:

> In any right triangle, the sum of the squares on the legs is equal to the square on the hypotenuse.

Although helping the children discover that this theorem applies to any right triangle would be a desirable mathematical objective for this lesson, the treatment here is restricted to an isosceles right triangle because of the comparative difficulty of demonstrating the theorem with other right triangles. The theorem is easily demonstrated with an isosceles right triangle, because a diagonal of the square drawn on one of the legs of the triangle is the same length as the hypotenuse. The children can think of the hypotenuse as the diagonal of the square and of the two legs as two sides of the square. Hence, by arranging the four small triangles as illustrated in the figure, the children can see that the squares on the two legs have the same area as the square on the hypotenuse.

For right triangles that are not isosceles, the squares on the legs may still be cut so that the pieces fit into the square on the hypotenuse, but the cuts are more complicated.

Resources for Active Learning

Inquiry in Mathematics via the Geo-board, "Pythagorean Theorem." Geo-Cards 21/1–4, Walker. (Available from Fitzhenry & Whiteside)

Duplicator Masters, page 15
Workbook, page 29

PAGES 88–89

Objectives

The child will demonstrate his ability to work with the concepts presented in this chapter.

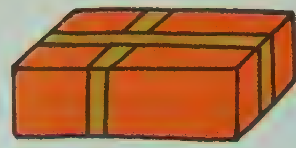
The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

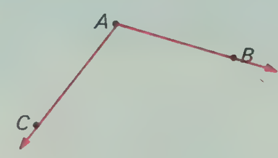
To prepare the children for page 88, review some of the terms developed in this chapter. For example point out certain figures or parts of figures in the classroom and help the children identify them as *edges*, *faces*, or *vertices* or as models of *points*, *rays*, *lines*, *line segments*, *planes*, *angles*, *triangles*, *right triangles*, etc.

Reviewing the Ideas

1. A How many vertices (corners) does the box have? 8
B How many edges does the box have? 12
C How many faces does the box have? 6

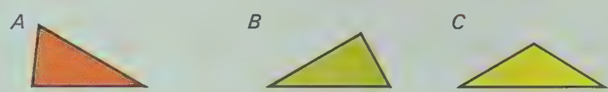


2. A Mark three points like *A*, *B*, and *C* on your paper.
B Draw a ray starting at *A* and going through *B*. Draw a ray starting at *A* and going through *C*.
C The name for the figure is __?__. (ray, angle, triangle)

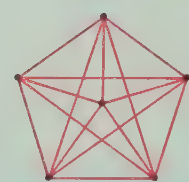


3. Which angle is a **right angle**? B

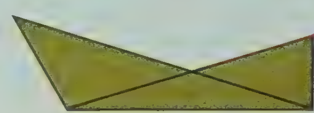
4. Which triangle is a **right triangle**? B



5. Trace these six points on your paper. How many segments can you draw to connect pairs of points? 15



6. How many triangles are in the picture? 5



88

Discussion

You might use page 88 as a review and work through it with the class, or you might use it as an instrument of evaluation. If you choose to use it for evaluation, discuss key topics after the children finish. During your discussion, stress understanding of the terms and identification of the figures introduced in this chapter. You can easily use thin felt strips on the flannelboard to represent simple two-dimensional geometric figures and ask the children to identify them. Then let the children manipulate as many three-dimensional geometric models as are available.

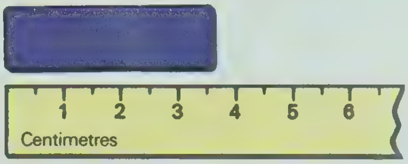
The "Keeping in Touch" exercises on page 89 may be assigned as independent work or used as a group review. You might want to accompany your discussion of children's answers with a game similar to those in Chapter 3, such as "What's My Rule" (page 69).

Keeping in Touch with Measurement, Place value, Addition, Subtraction, Fractions

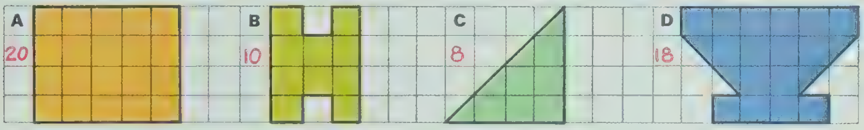
1. A Is the length of the bar closer to 3 centimetres or to 4 centimetres? 4 cm

   B Is the bar nearer to $3\frac{1}{2}$ centimetres or to 4 centimetres? $3\frac{1}{2}$ cm

   C To the nearest half centimetre, the bar is ▮▮▮ centimetres long. $3\frac{1}{2}$



2. Using the unit shown, give the area of each region.



3. Write the numeral for each.

   A 4 tens and 7 ones 47

   B 8 hundreds, 9 tens, and 6 ones 896

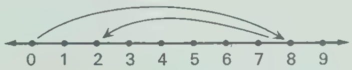
   C seven hundred sixty-three 763

   D six thousand, two hundred eight 6208

4. Write an equation for each number-line picture.

A


4+2=6

B


8-6=2

5. Find the missing addends.

   A $n + 2 = 7$ 5 C $3 + n = 10$ 7 E $n + 1 = 6$ 5

   B $n + 5 = 9$ 4 D $6 + n = 9$ 3 F $3 + n = 8$ 5

6. Find the differences.

   A $9 - 7$ 2 B $8 - 6$ 2 C $10 - 2$ 8 D $6 - 6$ 0 E $9 - 8$ 1 F $10 - 3$ 7

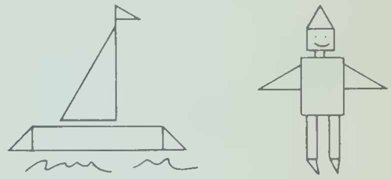
You are invited to explore

ACTIVITY CARD 3
Page 310

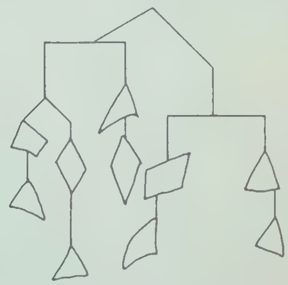
Follow-up

If the children particularly enjoyed any activity suggested earlier, employ it again or modify it to suit your purpose.

Another activity children would enjoy is constructing designs from geometric figures. This may be done with two-dimensional figures using construction paper. Have the children cut simple figures such as triangles, lines, squares, etc., out of one piece, and then design and paste them onto another. (Remind them of the folding method to make a right angle.)



This same idea may be used with cubes, curved surfaces, and flat surfaces used in three-dimensional construction. For example, some might make a mobile using nothing but curved surfaces cut in different shapes, or using curved and flat surfaces together. Others might combine blocks, cones, etc., to make a variety of interesting figures.



Texture Tracing

Show the children how to use a crayon or pencil to shade a thin sheet of paper on top of familiar three-dimensional objects to reproduce distinctive geometric shapes. For example, if you shade over a staple in a piece of paper, a line emerges; if you shade a piece of paper over a paper clip, a figure that has both curved and straight edges emerges. Have the children bring in thick-veined leaves, shade over them, color the result, and label the geometric figures they can see.

Workbook, page 30

CHAPTER 5 | Adding and Subtracting

Pages 90–121

General Objectives

To develop skill in addition (with and without regrouping)
To develop skill in subtraction (with and without regrouping)
To provide word-problem experiences
To provide work with coins
To develop skill in reasoning
To maintain addition and subtraction concepts

The initial pages of the chapter are designed to provide work with collections of coins and to prepare children for addition with numbers of two or more digits. After this introduction, children are asked to work through a carefully organized progression of pages which should lead them to discover a method of column addition for two-digit numbers. Then the text explores regrouping to lead into study of subtracting in columns. The remainder of the chapter develops ideas associated with regrouping in both addition and subtraction. Many pages are included to help develop reasoning abilities through problem sequences.

Although the primary objective in this chapter is a careful development of addition and subtraction processes, a large portion of the material is oriented to thought-provoking activity. Through such activity, we hope to lead children to a better understanding of the mathematical concepts and the relation of these concepts to the algorithms.

Mathematics

Rearranging addends is important in developing the algorithm for addition. Since many steps are needed to show all the order and grouping ideas for rearranging when adding two 2-digit numbers, such a display is impractical here. We can reduce the number of steps, however, by utilizing a generalization of the order and grouping principles.

The following steps for 2-digit addition are similar to those presented to the children.

$$\begin{aligned} 23 + 64 &= (20 + 3) + (60 + 4) \\ &= (20 + 60) + 3 + 4 \\ &= 80 + 7 \\ &= 87 \end{aligned}$$

Now, investigate the mathematics of a simple exercise in regrouping:

$$\begin{array}{r} 28 \\ +64 \\ \hline 92 \end{array}$$

$$\begin{aligned} 28 + 64 &= (20 + 8) + (60 + 4) \\ &= (20 + 60) + (8 + 4) \\ &= 80 + 12 \\ &= 80 + (10 + 2) \\ &= (80 + 10) + 2 \\ &= 90 + 2 \\ &= 92 \end{aligned}$$

Up to and including the expression 80 + 12, we use the same procedures as we do for sums like 23 + 64. But for 80 + 12, we write 12 as 10 + 2 and regroup (carry) to complete the problem.

An intermediate step between this kind of display and a short form for adding two 2-digit numbers can be helpful. The following example shows how the two processes are related and illustrates a way to teach children to understand the algorithm. Hand-lettered numerals show the relationship of the processes.

Intermediate step		Algorithm
$\begin{array}{r} 28 \\ +64 \\ \hline 12 \\ 80 \\ \hline 92 \end{array}$	Use chip chart //	$\begin{array}{r} \overset{1}{2}8 \\ +64 \\ \hline 92 \end{array}$

We rely heavily on the relationship between addition and subtraction in developing subtraction of 2-digit numbers. When we point out that finding a difference is the same as finding a missing addend, we present children with the idea that they can subtract in columns because they can add in columns.

The mathematical concepts associated with this process are quite difficult. To understand why, look at these examples.

$$\begin{aligned} 7 + (3 - 2) &\stackrel{?}{=} (7 + 3) - 2 \\ 7 + 1 &\stackrel{?}{=} 10 - 2 \\ 8 &= 8 \\ 6 + (3 - 3) &\stackrel{?}{=} (6 + 3) - 3 \\ 6 + 0 &\stackrel{?}{=} 9 - 3 \\ 6 &= 6 \end{aligned}$$

Since in both cases, we could regroup, without changing the sum, it is tempting to think that we can use the associative (grouping) principle in either an addition *or* subtraction problem. Some slightly different examples, however, show why the associative (grouping) principle does *not* hold for subtraction. Notice that the subtraction sign precedes the addition sign in this expression and that regrouping changes the result.

$$\begin{aligned} 7 - (3 + 2) &\stackrel{?}{=} (7 - 3) + 2 \\ 7 - 5 &\stackrel{?}{=} 4 + 2 \\ 2 &\neq 6 \end{aligned}$$

Consider the following example of 2-digit subtraction.

$$\begin{aligned} 75 - 23 &= (70 + 5) - (20 + 3) \\ &= [(70 + 5) - 20] - 3 \end{aligned}$$

As you can see, in order to subtract 23, or 20 + 3, we must subtract both the 20 and the 3. (See the second equation.) The equations illustrate why this situation is often confused with regrouping.

Now, read through a subtraction exercise which *does* involve regrouping.

$$\begin{aligned} 34 - 6 &= (30 + 4) - 6 \\ &= (20 + 14) - 6 \\ &= 20 + (14 - 6) \\ &= 20 + 8 \\ &= 28 \end{aligned}$$

Previously in the text, subtraction in the ones' column has been possible without regrouping. It is impossible in this case, since 6 is greater than 4. So 30 + 4 is regrouped as 20 + 14, and then 6 is subtracted from 14.

Teaching the Chapter

Materials

Advertisements, department store catalogues, bulletins, etc.
Calendars
Clothespins, 3 colors
Coins, real or play (at least 10 dimes and 10 pennies per child; nickels, quarters, half dollars for general use)
Colored strips
Counters (paper discs, bottle caps, buttons, etc.
Envelopes (1 per child)
Felt strips (10 each 2-by-20 centimetres marked into 10 units)
Felt units (10 each 2-by-2 centimetres)
Flannelboard
Scale for body weighing, if possible
Thermometer
Yarn

Vocabulary

dime	nickel
dollar	ones
greater than	place value
half dollar	quarter
inequality	regroup
less than	tens

Symbols

. decimal point (to separate dollars and cents)
¢ cent symbol (to show amounts less than 100 cents)
$ dollar sign (to show amounts greater than 100 cents)

The initial work in the use of coins is designed to help children understand the ideas involved in adding 2-digit numbers. For this reason, you should begin by limiting the use of coins to dimes and pennies. When the children combine various collections of coins, focus attention on putting the sets of dimes together and the sets of pennies together.

When the children combine coins such as two dimes and four pennies with three dimes and eight pennies, they will see that they can think of ten pennies as one dime and of the whole collection as six dimes and two pennies, rather than as five dimes and twelve pennies.

Grouping objects by ten is a good way to develop an understanding of the place-value concept in regrouping. The multibase arithmetic blocks are excellent manipulative materials to help a child understand regrouping.

The speed skills, in which shortcuts are developed, will only be meaningful to the child who understands and can use the power skill. Therefore, stress speed skill only after you are sure the child understands the power skill. Then emphasize the algorithmic procedure more than the abstract meaning of the process.

Do not dwell on the decimal point at this time. Stress only that it separates dollars from cents in money notation.

Continue to emphasize that in order to find differences, the children need to find missing addends.

Lesson Schedule

Your time schedule for this chapter should depend on the background and skills of the children. If they have a strong background, plan to cover the material in about four-and-a-half weeks. You should probably devote no more than six weeks to the chapter.

Evaluation of Progress

You can easily prepare a test of the children's proficiency with addition and subtraction algorithms. However, much of this chapter is devoted to the thought processes and reasoning which are necessary to an *understanding* of the algorithms. Therefore, as part of your evaluation of the children's progress in this chapter, observe daily their ability to think and reason about these concepts.

An interviewing technique is very helpful; at convenient times, talk with each child individually and ask him to "think out loud" about a few exercises for you. If you plan these exercises carefully, progressing from a few basic combinations to 2-digit addition and subtraction, the child's understanding of the ideas in this chapter can be observed easily. Then the pages at the end of the chapter may serve to help you evaluate the speed skills of the children.

Resources for Active Learning

General Activities

[Review the listing of Chapter 3 Resources for any activities that might again be useful or interesting to the children.]

Developmental Math Cards, C$^{\underline{4}}$4, C$^{\underline{1}}$8, D$^{\underline{4}}$16, Addison-Wesley
Dienes Multibase Arithmetic Blocks, Tasks and Manual, Cards 6A-12, Herder and Herder (Available from Methuen Publications)
Franklin Series: *Making and Using Graphs and Nomographs*, "Nomographs," pp. 19-24, Lyons and Carnahan (Available from McGraw-Hill Ryerson)
Math Activity Cards, "Addition Slide Rule," B6, Macmillan
Mathex: Operations and Problem Solving No. 8, "Rule Rummy," "Cover Score" (a, b), pp. 1–3 (pupil pages 1, 2), Encyclopaedia Britannica Publications Ltd.
Math Workshop: Games and Enrichment Activities, "Two-Sided Number Cards," pp. 23–28, Encyclopaedia Britannica Publications Ltd. These are good devices for Chapters 6 and 7, too.
Nuffield Project: *Computation and Structure* 2, pp. 66–69; "Addition," pp. 74–77, Wiley
Nuffield Project: *Computation and Structure* 3, pp. 2–13; pp. 21–23, Wiley. Review and problem-solving activities

Manipulative Devices

Abacus or Abacus Board (school supplier)
Dienes Multibase Arithmetic Blocks (Herder and Herder)
Grid Kit (Sigma, Scott Scientific)
Papy Minicomputer (Macmillan)
SEE Calculator (Selective Educational Equipment)
SEE Chips (Selective Educational Equipment)

Commercial Games

Cover-up (Selective Educational Equipment)
TUF (Creative Publications; Cuisenaire Co.; TUF)

PAGES 90–91

Objective

Given a set of dimes and pennies, the child will be able to give its value by using place-value notation.

Preparation

Materials

counters of two different colors to be used as pennies and dimes

To introduce this chapter, play a short oral game to review basic combinations, such as "Names for a Number": put a numeral on the board and ask for many other ways to name the same number. Or play "What's My Rule" (see page 69). Keep this activity brief, using it to stimulate enthusiasm for studying adding and subtracting.

Investigation

The children would benefit most from this investigation by working with partners. For example, you might distribute 9 counters of each color to every child (or to every two children) and challenge them to show the amount of money illustrated in the investigation by using the counters interchangeably as dimes and pennies. Then write several other amounts (less than one dollar) on the chalkboard and direct the children to show their partners a handful of "coins," tell the total value, and ask their partner which counters are the "dimes." Encourage the children to make up other "coin" activities which involve both putting together and taking away.

5 Adding and Subtracting

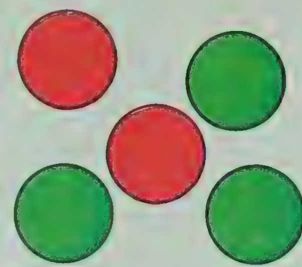
How are dimes and pennies like tens and ones?

Investigating the Ideas

? How many different amounts of money can you show if your "pennies" and "dimes" are these different-colored counters?

23¢ or 32¢

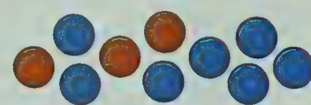
See Investigation.



Discussing the Ideas

1. Why is it important to decide which counters are "dimes" and which are "pennies"? See Discussion.

2. If these counters are worth 37¢, which colored counters are dimes? Brown



3. Which collection is worth more, A or B? B

4. Which collection is worth more, C or D? C

Discussion

When the children have had sufficient practice with the counters, call on a few volunteers to show some amounts, such as those suggested in the text, and explain which counters represent dimes and which represent pennies. Then develop the questions in the text. Relate the importance of deciding which counters are dimes and which are pennies to the importance of knowing which position in a numeral represents tens and which represents ones.

In exercises 3 and 4, have someone write the numeral for each amount on the chalkboard. Again relate the comparison of dimes and pennies to a comparison of numerals. For example, you might say something such as the following (writing on the chalkboard as you speak):

"Just as *2 dimes* and *3 pennies* are less than *3 dimes* and *2 pennies*, so *2 tens* and *3 ones* are less than *3 tens* and *2 ones*. So we write: *23 < 32*."

If necessary, develop other examples that will further review place value.

Using the Ideas

1. Give the value of each of the two coin collections together.
   - A A and B 55¢
   - B A and C 54¢
   - C C and D 47¢
   - D B and D 48¢
   - E B and C 63¢
   - F A and D 39¢

2. Which pair of collections has the greater value?
   - A A and B together or C and D together A and B
   - B A and C together or B and D together A and C
   - C A and D together or B and C together B and C
   - D A and D together or B and D together B and D

3. A Had 1 dime and 7 pennies.
   Spent 5 cents.
   How much left? 12¢

   B Had 2 dimes and 3 pennies.
   Spent 1 dime and 3 cents.
   How much left? 10¢ (or 1 dime)

   C Had 6 dimes and 7 pennies.
   Spent 5 dimes and 5 cents.
   How much left? 12¢
   (or 1 dime and 2 cents)

   D Had 8 dimes and 2 pennies.
   Spent 40 cents.
   How much left? 42¢

   E Had 5 dimes and 9 pennies.
   Spent 50 cents.
   How much left? 9¢

   F Had 4 dimes and 8 pennies.
   Spent 25 cents.
   How much left? 23¢

   G Had 76 cents.
   Spent 56 cents.
   How much left? 20¢

   H Had 79 cents.
   Spent 43 cents.
   How much left? 36¢

★ 4. How much more is in one collection than the other? 16¢

A

B

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Follow-up

Choose one of the following suggestions according to the needs and capabilities of your class.

"Choosing Blindly"

Supply the class with a discarded purse containing 9 dimes and 9 pennies. Ask one child to come up and take out some coins without looking. Let him pick someone to identify the chosen coins and to write the value of the set on the board. The second child should then choose another child to tell how many coins are left in the purse and to write their value on the board. A fourth child, selected by the third, should tell which set is larger, return the coins to the purse, and choose another child to repeat the process.

"Coins to Show Place Value"

Allow the children to use envelopes of play coins or counters, if necessary, to answer questions on this kind of worksheet.

How many dimes (D)? pennies? (P)
How many *cents* altogether?
Mark a ✓ for the set with the *greater* value.

Coins	D	P	Cents	Greater	How many cents greater?
10¢ 1¢ 1¢ 10¢ 1¢	2	3	23		9¢
1¢ 1¢ 10¢ 10¢ 10¢	3	2	32	✓	
1¢ 1¢ 1¢ 10¢ 1¢ 1¢					
1¢ 1¢ 1¢ 1¢ 1¢ 1¢					

Using the Exercises

You may treat these exercises orally or as independent work. Have the children label their answers for exercise 1, such as A, *A and B*, 55¢, so that they can make comparisons in exercise 2 without finding the value of the original sets a second time.

For exercise 3 you might suggest a general approach such as: find out what the 2 sets, or amounts, in each problem are and then ask what you are to do with them, that is, combine them, separate them, or compare them. This page provides an excellent review of place value and simple subtraction, and develops readiness for 2-digit addition and subtraction.

Encourage all the children to try the starred exercise. Even though they may not remember the algorithmic method of subtracting 25 from 41, they may still be able to figure it out by reasoning more or less like this: 25 is 5 less than 30 and 30 is 11 less than 41, so the greater amount must be worth 5 + 11, or 16 cents more than the other.

Assignments (page 91)
Minimum: 1–3, oral. Average: 1–3.
Maximum: 1–4.

Resources for Active Learning

Math Workshop: Games and Enrichment Activities, "The Row and Column Cell Building Game," pp. 60–61, Encyclopaedia Britannica Educational Corp.

Nuffield Project: *Computation and Structure* 2, "Money," pp. 92–102, Wiley.

Workbook, page 31

PAGES 92–93

Objective

Given 2-digit addends that are multiples of ten, the child will be able to find the sums.

Preparation

Materials

sets of colored strips

You may begin immediately with the investigation for this lesson, or if you prefer, conduct a three-minute speed drill on basic addition facts by naming addends for which the children should give the sums.

Investigation

As you discuss the introductory portion of the investigation with the children, help them to see that they should think of the white strip (the basic strip unit) as 1 ten. Thus, the red strip represents 2 tens, the light green strip represents 3 tens, and so on. Have volunteers explain how the equation expresses what is shown by the picture of the strips: the picture shows that 2 tens plus 3 tens equals 5 tens, and this is the same as $20 + 30 = 50$.

Allow children to work individually with their colored strips to find the combinations of strips (addends) that are equivalent to the brown, or "80," strip (sum). If some children use appropriate combinations of more than two strips as addends, this is entirely acceptable, of course, but try to channel the post-investigation discussion so that attention is focussed on equations with two addends.

After all the children have had time to experience some success in finding correct combinations and expressing their results as equations, have volunteers write the possible equations on the chalkboard.

$10 + 70 = 80$ $\quad$ $70 + 10 = 80$
$20 + 60 = 80$ $\quad$ $60 + 20 = 80$
$30 + 50 = 80$ $\quad$ $50 + 30 = 80$
$40 + 40 = 80$

How is 20 + 30 like 2 + 3?

Investigating the Ideas

IF the white strip is 10

THEN these strips

show that $20 + 30 = 50$

? Can you use your strips to help you write some equations that have a sum of 80? See Investigation.

Discussing the Ideas

1. A 4 dimes together with 3 dimes make how many dimes? 7

B 4 tens and 3 tens make how many tens? 7

C $4 + 3 = n$ 7 $\qquad$ D $40 + 30 = n$ 70

2. A 5 bundles together with 4 bundles make how many bundles? 9

B 5 tens and 4 tens make how many tens? 9

C $5 + 4 = n$ 9 $\qquad$ D $50 + 40 = n$ 90

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Discussion

Before discussing the material in the discussion section, help the children realize that their work with the strips for adding multiples of ten was no different from using the strips to add the basic facts. Point out this relation by various examples, such as

$3 + 5 = 8; \quad 30 + 50 = 80.$

Such a discussion should lead into the discussion section in the text. You might want to provide additional demonstration material as you work through exercises 1 and 2. For example, use the felt tenstrips suggested on page 34 or bundles of ten pencils or pipe cleaners and let children combine these groups of tens, always relating the addition of the tens to the basic addition fact. For instance, ask a child to give you the sum of 4 and 5. When he responds "Nine," say, "How much is 4 tens and 5 tens?" When he answers "9 tens," reply, "How much is forty plus fifty?" Show groups of ten items as you do this.

Using the Ideas

1. Find the sums.
   - A Since 7 + 2 = 9, we know that 70 + 20 = *n*. 90
   - B Since 2 + 5 = 7, we know that 20 + 50 = *n*. 70
   - C Since 6 + 4 = 10, we know that 60 + 40 = *n*. 100
   - D Since 7 + 5 = 12, we know that 70 + 50 = *n*. 120
   - E Since 8 + 7 = 15, we know that 80 + 70 = *n*. 150

2. Solve the equations.
   - A 60 + 10 = *n* 70 C 40 + 30 = *n* 70 E 50 + 60 = *n* 110
   - B 50 + 40 = *n* 90 D 30 + 30 = *n* 60 F 70 + 60 = *n* 130

3. Solve the equations.
   - A 30 + 0 = *n* 30 C 30 + 2 = *n* 32 E 40 + 8 = *n* 48
   - B 30 + 1 = *n* 31 D 30 + 7 = *n* 37 F 60 + 4 = *n* 64

4. Find the sums.

   A $\begin{array}{r} 30 \\ +60 \\ \hline 90 \end{array}$ B $\begin{array}{r} 40 \\ +20 \\ \hline 60 \end{array}$ C $\begin{array}{r} 70 \\ +20 \\ \hline 90 \end{array}$ D $\begin{array}{r} 70 \\ +40 \\ \hline 110 \end{array}$ E $\begin{array}{r} 80 \\ +50 \\ \hline 130 \end{array}$

5. Find the sums.

   A $\begin{array}{r} 2 \\ 3 \\ +4 \\ \hline 9 \end{array}$ B $\begin{array}{r} 2 \\ 7 \\ +1 \\ \hline 10 \end{array}$

6. Find the sums.

   A $\begin{array}{r} 70 \\ 20 \\ +50 \\ \hline 140 \end{array}$ B $\begin{array}{r} 60 \\ 40 \\ +80 \\ \hline 180 \end{array}$

   C $\begin{array}{r} 20 \\ 30 \\ 20 \\ +10 \\ \hline 80 \end{array}$ D $\begin{array}{r} 40 \\ 20 \\ 10 \\ +50 \\ \hline 120 \end{array}$

think

With 2 minutes to play, this was the score.

Vanier	48
Beaverbrook	41

At the end of the game, the score was 50 to 47. Vanier

WHO WON THE GAME?

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Using the Exercises

Assign as independent work as many of the exercises on page 93 as seem needed. As you discuss the answers to the exercises, stress the idea that adding columns of multiples of ten is much like adding columns of 1-digit numbers, because the same basic combinations are used.

Encourage everyone to try the *Think* problem. Afterward, let someone explain why the score for Vanier with two minutes left to play makes it possible for us to know that Vanier won the game.

Assignments (page 93)
Minimum: 1–4. Average: 1–6.
Maximum: 1–6.

Mathematics

The following example illustrates the use of the distributive principle in finding a sum such as 30 + 40.

$$\begin{aligned} 30 + 40 &= (3 \times 10) + (4 \times 10) \\ &= (3 + 4) \times 10 \\ &= 7 \times 10 \\ &= 70 \end{aligned}$$

We do not use this approach here, since we have not yet treated multiplication. Rather, we arrive at the correct sum by relating the union of two sets to a simple addition combination.

Follow-up

"Patterns for Multiples of Ten"
A written exercise using patterns like the following may increase the children's efficiency in adding multiples of ten. Encourage them to make up patterns and problems of their own.

Find the pattern. Solve the equations.	
7 + 6 = □ 70 + 60 = □	4 + 8 = □ 40 + 80 = □
5 + 9 = □ 50 + 90 = □	3 + 7 = □ 30 + 70 = □
8 + 7 = □ 80 + 70 = □	9 + 6 = □ 90 + 60 = □
6 + 8 = □ 60 + 80 = □	9 + 3 = □ 90 + 30 = □

Resources for Active Learning

Developmental Math Cards, C-16, F-5, Addison-Wesley.

Duplicator Masters, pages 16, 17
Workbook, page 32
Skill Masters, pages 16, 17

PAGES 94–95

Objective

Given addition and subtraction problems of 2- and 3-digit numbers which do not require regrouping, the child will be able to find the sums and differences.

Preparation

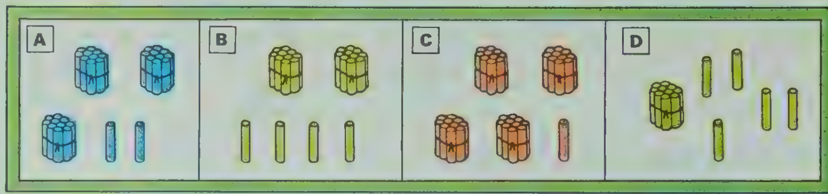
Materials (optional)

felt ten-strips and units; yarn

Warm up with a short oral game such as "Name the Number." For example, give the number 16 and ask for different "names" of 16. After some basic combinations have been reviewed, give a number such as 46 and ask for its place value name, 40 + 6 or 4 tens and 6. Reverse this approach by giving the place-value name and asking for the numeral, such as 2 hundreds, 8 tens, and 4 (to which a child should respond 284).

Let's explore sums and differences.

Discussing the Ideas



1. How many sticks are in these sets? Explain your answers.
   - A A and B together 56
   - C A and C together 73
   - E A and D together 47
   - B B and C together 65
   - D C and D together 56
   - F B and D together 39

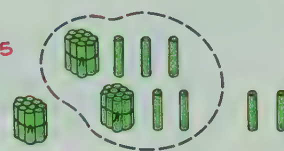
2. Find the sums.

   A $\begin{array}{r} 80 \\ +10 \\ \hline 90 \end{array}$ $\begin{array}{r} 7 \\ +2 \\ \hline 9 \end{array}$ → $\begin{array}{r} 87 \\ +12 \\ \hline 99 \end{array}$

   B $\begin{array}{r} 30 \\ +50 \\ \hline 80 \end{array}$ $\begin{array}{r} 4 \\ +2 \\ \hline 6 \end{array}$ → $\begin{array}{r} 34 \\ +52 \\ \hline 86 \end{array}$

3. Give an easy rule for finding this sum. $\begin{array}{r} 24 \\ +35 \\ \hline 59 \end{array}$
   See Discussion.

4. A How many sticks in all? 37
   B How many sticks in the dotted ring? 25
   C How many sticks not in the dotted ring? 12



5. Find the differences.

   A $\begin{array}{r} 70 \\ -20 \\ \hline 50 \end{array}$ $\begin{array}{r} 6 \\ -3 \\ \hline 3 \end{array}$ → $\begin{array}{r} 76 \\ -23 \\ \hline 53 \end{array}$

   B $\begin{array}{r} 80 \\ -30 \\ \hline 50 \end{array}$ $\begin{array}{r} 5 \\ -2 \\ \hline 3 \end{array}$ → $\begin{array}{r} 85 \\ -32 \\ \hline 53 \end{array}$

6. Give an easy rule for finding this difference. See Discussion. $\begin{array}{r} 67 \\ -42 \\ \hline \end{array}$

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Discussion

As long as the children learn from manipulation of physical objects, having them use the strips, bundles, felt ten-strips, or other appropriate devices, is a benefit. However, the average learner will gradually dispense with concrete materials and be able to imagine them and the processes studied with them. Hence, the presentation of lessons such as this will depend upon the level of your class. Let the children manipulate bundles if necessary, and provide set demonstrations as long as needed. To work and discuss exercise 1, for example, you may wish to have the children use only the pictures. Then, for exercise 2, you might provide a demonstration with bundles, felt ten-strips, or blocks. In both exercises, relate the discussion to the written numeral.

In exercise 3, expect the children to respond with something like: "First add the ones, 4 + 5 = 9; then add the tens, thinking 2 + 3 = 5, so the answer is 59." As you discuss subtraction in exercise 4, write the numerals on the chalkboard:

$\begin{array}{rl} 37 & \text{in all} \\ -25 & \text{in the dotted ring} \\ \hline 12 & \text{not in the dotted ring} \end{array}$

Stress subtraction in the tens' place by helping children understand that they are subtracting tens but need only think "3 − 2 = 1." If necessary, illustrate the problems in exercise 5 with ten-strips, units, and yarn, in a manner similar to the picture in exercise 4. Help the children develop the rule in exercise 6, that is, to subtract first the ones and then the tens. You might have them apply the rule orally:

"7 minus 2 is 5; 6 minus 4 is 2; so the answer is 2 tens and 5, or 25."

Using the Ideas

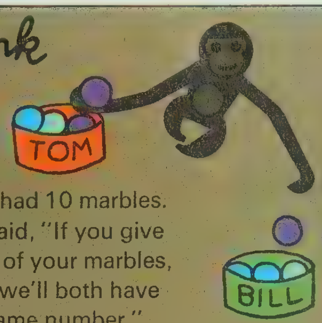
1. Find the sums.

A $15 + 42$ = 57	B $47 + 31$ = 78	C $32 + 65$ = 97	D $41 + 23$ = 64	E $82 + 14$ = 96	F $60 + 70$ = 130	G $12 + 73$ = 85
H $75 + 53$ = 128	I $84 + 12$ = 96	J $24 + 65$ = 89	K $95 + 43$ = 138	L $235 + 162$ = 397	M $740 + 36$ = 776	N $153 + 842$ = 995

2. Find the differences.

A $78 - 32$ = 46	B $93 - 41$ = 52
C $100 - 90$ = 10	D $82 - 62$ = 20
E $648 - 325$ = 323	F $739 - 516$ = 223
G $527 - 524$ = 3	H $607 - 403$ = 204

think



Tom had 10 marbles. Bill said, "If you give me 2 of your marbles, then we'll both have the same number."

How many marbles did Bill have? 6

3. Jane checked out a library book on the thirteenth day of the month. She had to return the book in 14 days. What day was Jane's book due? 27th

4. Tim started reading a science book on page 62. After he read 25 pages, where was he in the book? 87

5. Sue kept a record of the number of days it took her to read a book. How many days in all did it take for Sue to read books **A**, **B**, and **C**? 39

Book **A**	12
Book **B**	14
Book **C**	13

More practice, page A-11, Set 16

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Follow-up

You might prepare a worksheet of short story problems to give the children practice with 2-digit addition and subtraction. Use meaningful situations such as recording temperatures or planning a field trip, but in all cases make sure that regrouping is required only in the place with the greatest value. For example:

1. Garden plants: 20 tomatoes 17 cabbages 12 peppers How many in all?	2. Nuts: 32 pecans 30 almonds 46 walnuts How many nuts?

You might also prepare for addition with regrouping by having the children regroup tens, hundreds, and thousands. The Dienes multibase blocks would provide excellent material for working with this concept. For example, 12 flats (12 hundreds) may be regrouped as 1 block and 2 flats (1 thousand, 2 hundreds), and 17 longs (17 tens) may be regrouped as 1 flat and 7 longs (1 hundred, 7 tens).

Resources for Active Learning

Dienes Multibase Arithmetic Blocks, Tasks and Manual, "Adding and Subtraction," Cards 11, 12, Herder and Herder. (Available from Methuen Publications)

Mathematics in Modules, WN6, WN7, WN11, WN12, Addison-Wesley.

Duplicator Masters, pages 18-20
Workbook, page 33, 34
Skill Masters, pages 18, 19

Using the Exercises

Before assigning the exercises on page 95 as independent work, it would be helpful to have the class think through a few problems together orally. Remind the children to add first the ones and then the tens. Explain that even though we understand that we are adding tens and hundreds we need not think "tens" and "hundreds." Exercise 1N, for instance, would be "3 plus 2 is 5; 5 plus 4 is 9; 1 plus 8 is 9. The answer is 9 hundreds, 9 tens, and 5 ones, or 995." Help the children see that working with 3-digit numerals is similar to working with 2-digit numerals.

When the children finish, check the answers with them and discuss any difficulties. Let children share their methods of solving exercises 3, 4, and 5 but make sure their reasoning is valid.

Encourage all the children to try the *Think* problem and then let a volunteer explain it: Since $10 - 2 = 8$ and $6 + 2 = 8$, Bill must have had 6 marbles.

If more practice is needed, supplementary exercises may be assigned.

Assignments (page 95)

Minimum: 1–2; 3–5, oral. Average: 1–5. Maximum: 1–5.

PAGES 96–97

Objective

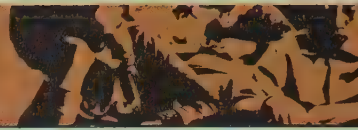
The child will demonstrate his ability to work with the concepts indicated for cumulative review: addition, place value, and inequalities.

Preparation

Review expanded notation by dictating expressions such as, "three tens plus seven" or "five hundreds plus eight tens plus five." As children respond with the proper numerals, 37 or 585, put them on the board or overhead projector. Then reverse the procedure and say the numeral, such as 320, and have the children tell you the expanded form, three hundreds plus two tens plus zero, and write 300 + 20 + 0. Next, write pairs of numerals on the board, and ask several children to draw a ring around the symbol for the greater number in each pair. Review the symbols > and < for *greater than* and *less than*, so that the children may place the proper inequality mark between each pair.

After writing three or four more pairs of numerals on the board, call several children up to write the proper inequality symbols between the numerals *without* ringing the greater first.

Keeping in Touch with Addition, Place value, Inequalities

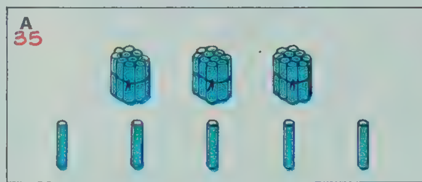


1. Solve the equations.

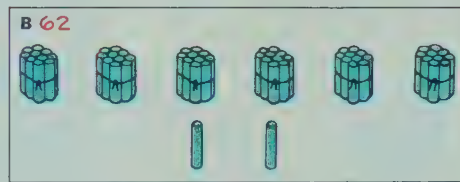
A $38 = 30 + n$ 8
B $56 = 50 + n$ 6
C $70 + 2 = n$ 72
D $80 + 3 = n$ 83
E $358 = 300 + 50 + n$ 8
F $463 = 400 + 60 + n$ 3
G $781 = 700 + 80 + n$ 1
H $640 = 600 + 40 + n$ 0

2. Give the number of each set.

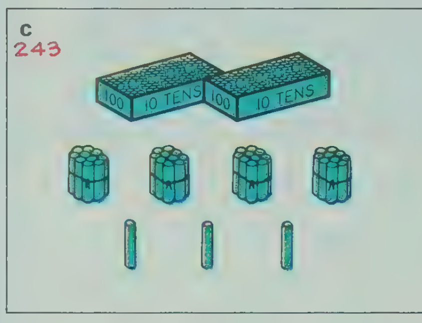
A 35



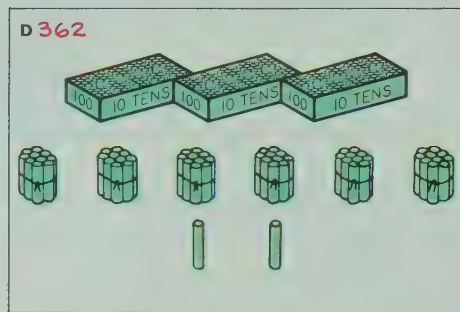
B 62



C 243



D 362



3. Solve the equations.

A $76 = 70 + n$ 6
B $76 = 60 + n$ 16
C $42 = 40 + n$ 2
D $42 = 30 + n$ 12
E $91 = 90 + n$ 1
F $91 = 80 + n$ 11
G $50 = 50 + n$ 0
H $50 = 40 + n$ 10
I $62 = 60 + n$ 2
J $62 = 50 + n$ 12
K $24 = 20 + n$ 4
L $24 = 10 + n$ 14

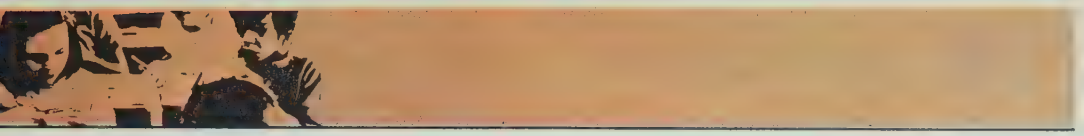
96

Discussion

Both pages may be assigned for independent work. However, you may want to work through exercises 4A and B to make sure the children know what to do. Point out that the sets of exercises in 6 and 7 are ordered in a certain way and the order should help them work the exercises.

When the children finish and you check their work, provide additional help to correct any weaknesses in understanding that you detect. Exercise 3 provides background for regrouping in subtraction. Children who have difficulty with regrouping may need more guidance when they begin to do subtraction requiring this technique.

Similarly, the pattern of exercise 6 will probably point up the concept associated with regrouping. For example, in exercise 6, part A is less than 50, part B is equal to 50, and parts C and D are both greater than 50.



4. Write each number pair on your paper in the order given. Then put the correct mark (< or >) in place of the ◍.

A 65 ◍ 13 (Answer: 65 > 13)
B 27 ◍ 95 <
C 38 ◍ 48 <
D 55 ◍ 56 <
E 615 ◍ 605 >
F 82 ◍ 92 <
G 45 ◍ 54 <
H 69 ◍ 70 <
I 743 ◍ 733 >

think

The sum of two numbers is 20. Their difference is 4. What are the numbers? 8, 12

SUM IS 20
DIFFERENCE IS 4

5. Give the correct mark for each ◍.

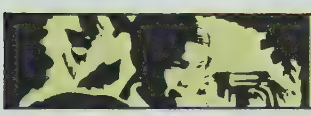
A Since 7 + 5 > 10, we know that 67 + 5 ◍ 70. >
B Since 5 + 4 < 10, we know that 15 + 4 ◍ 20. <
C Since 8 + 6 > 10, we know that 48 + 6 ◍ 50. >
D Since 8 + 4 > 10, we know that 48 + 4 ◍ 50. >

6. Tell whether each sum is less than 50, more than 50, or equal to 50.

A 46 + 3 less than C 46 + 5 more than E 47 + 1 less than G 47 + 3 equal to
B 46 + 4 equal to D 46 + 6 more than F 47 + 2 less than H 47 + 4 more than

7. Tell whether each sum is less than, more than, or equal to 70.

A 68 + 4 more than C 69 + 1 equal to E 67 + 5 more than G 64 + 6 equal to
B 62 + 5 less than D 68 + 1 less than F 63 + 6 less than H 64 + 7 more than



You are invited to explore

ACTIVITY CARD 4
Page 311

Mathematics

The children intuitively understand what it means for one number to be greater than another. Although they may not express it precisely, their concept of this notion is exactly the same as the mathematical definition of the *greater-than* relationship:

To say that $a > b$ means that there is some number c, greater than zero, which adds to b to give a. That is, $a > b$ means $a = b + c$ where c is a number greater than zero.

Follow-up

As an independent activity for capable children, follow up this introduction to regrouping by providing some boardwork problems like the following.

Find the patterns and solve the equations.	
$87 = 80 + \square$ $87 = 70 + \square$	$32 = \square + 2$ $32 = \square + 12$
$62 = \square + 2$ $62 = \square + 12$	$54 = 50 + \square$ $54 = \square + 14$
$210 = \square + 10$ $210 = 190 + \square + 10$	$73 = \square + 3$ $73 = 60 + \square$
$240 = 200 + \square + 0$ $240 = 200 + \square + 10$	★ $302 = \square + 2$ $302 = \square + 12$

Resources for Active Learning

Discovery, Section II, Units 5/2; 6/2, Encyclopaedia Britannica Educational Corp.

[See Chapter 2 Resources relative to place value and inequalities. See Chapter 3 Resources for games and activities to develop competence in basic facts.]

Workbook, page 35

PAGES 98–99

Objective

Given an addition problem of two 2-digit numbers which requires regrouping from the ones' to the tens' place, the child will be able to find the sum.

Preparation

Materials

yearly calendars; felt ten-strips and units (optional)

Since this investigation requires that the child be relatively adept with addition combinations, you might conduct a short oral practice on the basic combinations. You might play "What's My Rule," as described previously (page 69), or you might simply use the pattern: "I'm thinking of the sum 7 + 9. What's my number?" In either case, keep the tempo of the game brisk and spend no more than 4 or 5 minutes for this review.

Investigation

This investigation introduces regrouping by providing a counting activity. To begin, read the questions together with the class and direct the children to work independently for about 5 minutes. Then have the class form small groups, of 3 or 4, and tell the children to work through questions A and B using the birthday of each group member as a separate problem. Thus, any difficulties which occur will become a challenge to a whole group. Have yearly calendars available in case children want to use them for reference. Notice particularly how children handle a birthdate which occurs at the end of a month. Give help when asked or when necessary to correct errors. Generally, though, the children should rely on their own ingenuity and skill to work through these questions. Remind them to record their results.

● *How can you find sums like 36 + 28?*

Investigating the Ideas

Can you use the calendar to help you find sums?

A What is the date 3 days after your birthday?

B Find the date 8 days after your birthday.

S	M	T	W	T	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

? Can you use the calendar to find these sums?

$\begin{array}{r} 23 \\ +7 \\ \hline 30 \end{array}$ $\quad \begin{array}{r} 16 \\ +6 \\ \hline 22 \end{array}$ $\quad \begin{array}{r} 24 \\ +7 \\ \hline 31 \end{array}$ $\quad \begin{array}{r} 17 \\ +9 \\ \hline 26 \end{array}$ $\quad \begin{array}{r} 19 \\ +11 \\ \hline 30 \end{array}$ $\quad \begin{array}{r} 23 \\ +8 \\ \hline 31 \end{array}$

Discussing the Ideas

See Discussion.

1. Study the example below. Explain each step.

Step 1	Step 2	Step 3
$\begin{array}{r} 37 \\ +\ 25 \\ \hline 12 \end{array}$	$\begin{array}{r} 37 \\ +\ 25 \\ \hline 12 \\ 50 \end{array}$	$\begin{array}{r} 37 \\ +\ 25 \\ \hline 12 \\ 50 \\ \hline 62 \end{array}$
7 + 5 = 12	30 + 20 = 50	12 + 50 = 62

2. Now try this one and compare your answer with the answer your teacher puts on the chalkboard. $\begin{array}{r} 65 \\ +17 \\ \hline 82 \end{array}$

98

Discussion

When the children have finished the investigation, encourage them to share their results. Use examples of birthdates to show regrouping. For instance, if a child's birthday is the 18th, the date 3 days after the 18th brings it into the twenties – 19, 20, 21. You might use felt ten-strips and units to illustrate as you count, regrouping 19 + 1 as 20. Then illustrate one of the sums the children found. As an example, on the chalkboard write:

$$\begin{array}{r} 24 \\ +7 \\ \hline \end{array}$$

Show 2 ten-strips and 4 units. Add 7 units. Encourage the children to think: "7 + 4 is 11, but 11 is 1 ten and 1. So we can replace 10 units with a ten-strip and add that new ten-strip to the 2 ten-strips we already have. This gives us 3 tens and 1, or 31."

Direct attention to how such an addition is written in discussion exercise 1. Point out the shaded digits which show that the ones are added first, then the tens, and finally the two *partial sums*, 12 and 50.

Extend exercise 2 with other examples for the children to try.

Using the Ideas

1. Find the sums.

A 46 +28 74	B 37 +44 81	C 29 +65 94	D 67 +15 82	E 48 +6 54
F 34 +27 61	G 68 +19 87	H 17 +35 52	I 54 +9 63	J 37 +27 64
K 46 +35 81	L 27 +67 94	M 58 +14 72	N 42 +49 91	O 36 +27 63

2. Find the sums.

A 76 +40 116	B 76 +45 121	C 67 +60 127	D 67 +66 133	E 56 +56 112
F 83 +30 113	G 83 +39 122	H 56 +75 131	I 88 +33 121	J 95 +28 123
K 76 +72 148	L 66 +84 150	M 97 +14 111	N 59 +79 138	O 88 +88 176

3. Study the example. Then try to find the other sums without pencil and paper.

Example: 58 + 24

First	Then
Think	Think
58 +20 78	78 +4 82

81 A 45 + 36 C 63 + 29 92

62 B 37 + 25 D 59 + 27 86

think

I can be found
Halfway between
Twenty-seven
And seventeen.

WHO AM I?
22

17 27

More practice, page A-11, Set 17

99

Using the Exercises

Depending on the ability of the class, assign this page as independent work or have children work through it at the chalkboard. Let volunteers present some examples step by step to the class. If a child uses the shortcut method correctly (as he might recall from Book 2), encourage him to continue to do so; hopefully, many children will soon be using this method. Also note that exercise 2 differs from exercise 1 because in exercise 2 the sums in the tens' place are greater than or equal to ten.

The *Think* problem is intended for independent work by more capable children. Others may benefit from a discussion of the reasoning of those who solve it, if time allows.

Assignments (page 99)
Minimum: 1. Average: 1–2.
Maximum: 1–3.

Follow-up
Other counting experiences would benefit any child who found this lesson difficult. For this purpose, you might put addition exercises on the chalkboard or on a worksheet, each followed by a suggestion of how to count to solve it. For example:
38 + 27 Count by ones 7 times (39, 40, 41, 42, 43, 44, 45); then count by tens 2 times (55, 65).
56 + 38 Count by ones 8 times; then count by tens 3 times.

If the Dienes blocks are available, you might have the children perform some of the addition exercises with them. This would particularly help any child who had difficulty understanding the regrouping. For example, 67 + 15 would require the child first to take 6 longs and 7 units and combine them with 1 long and 5 units. In so doing, he should realize that the 12 units can be replaced by 1 long and 2 units. Then his total would be 8 longs and 2 units.

An efficient way to test for mastery of the addition combinations is to give a timed test on which children must write only the answers. You might prepare a table such as the following for the children to use.

+	5	2	9	0	3	6	8	1	7	4
4										
7										
1										
8										
6										
3										

Resources for Active Learning
Mathematics in Modules, WN12, Addison-Wesley.
Mathex: Operations No. 3, "Numbers in the Thousands—Activity 1," pp. 23–24, Encyclopaedia Britannica Publications Ltd.
Toward Improving Computation, "Adding with Beans and Sticks," pp. 51–53, Curriculum Development Associates.

Duplicator Masters, page 21
Workbook, page 36
Skill Masters, page 21

PAGES 100–101

Objective

Given 2-digit or 3-digit addends which require regrouping, the child will be able to use the usual algorithm to find the sum.

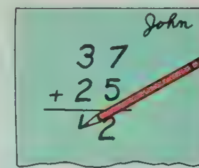
Preparation

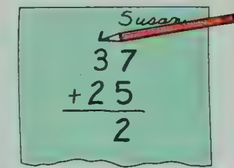
If you did not use the timed test suggested in the follow-up on page 99, you might use it as an introduction to this lesson. Or you might use a short oral review of the basic addition combinations. In either case, keep the preparation time brief. You may prefer to begin immediately with the discussion.

Is there a shortcut for adding with regrouping?

Discussing the Ideas

1. Explain the difference in the way John and Susan started their work.
See Discussion.





2. Explain the two steps in this example.
See Discussion.

Step 1	Step 2
$\begin{array}{r} 1 \\ 37 \\ +25 \\ \hline 2 \end{array}$ (12)	$\begin{array}{r} 1 \\ 37 \\ +25 \\ \hline 62 \end{array}$
7 + 5 = 12	10 + 30 + 20 = 60

3. One of these examples has a mistake. See if you can find it. B

A	B	C
$\begin{array}{r} 1 \\ 45 \\ +38 \\ \hline 83 \end{array}$	$\begin{array}{r} 1 \\ 34 \\ +48 \\ \hline 72 \end{array}$	$\begin{array}{r} 1 \\ 29 \\ +16 \\ \hline 45 \end{array}$

4. Explain each step in this example.

Step 1	Step 2	Step 3
Think 7 + 5 = 12	Think 1 + 5 + 8 = 14	Think 1 + 3 + 2 = 6
$\begin{array}{r} 1 \\ 357 \\ +285 \\ \hline 2 \end{array}$ (12)	$\begin{array}{r} 11 \\ 357 \\ +285 \\ \hline 42 \end{array}$ (14 tens)	$\begin{array}{r} 11 \\ 357 \\ +285 \\ \hline 642 \end{array}$

5. Try these two exercises and check your work with your teacher.

A	B
$\begin{array}{r} 248 \\ +175 \\ \hline 423 \end{array}$	$\begin{array}{r} 657 \\ +195 \\ \hline 852 \end{array}$

Discussion

As you have the children point out the difference in the examples shown for exercise 1, stress the place value of the 1: in John's work, the 1 ten of 12 is being written as a partial sum, while in Susan's work, it becomes part of the tens' column with the addends. Susan's method is further illustrated in exercise 2, which shows that, just as in John's method, the first step in the shortcut method is to find the partial sum and the second step is to find the sum of the tens' column. The only difference lies in the notation. Stress the convenience of writing the 1 in the tens' column with addends; the children should see that this shortcut speeds up their work.

Carefully work through the steps of exercise 4 with the children. The "think clouds" represent the mental process associated with each step. In step 2, help the children understand that 1 + 5 + 8 means 1 + 5 + 8 *tens* but may be treated, for algorithmic purposes, simply as 1 + 5 + 8; similarly, in step 3, 1 + 3 + 2 means 1 + 3 + 2 *hundreds*, but we need only think 1 + 3 + 2 = 6.

The main objective of this lesson is for the child to develop skill in using the algorithm he hopefully now understands. Do not stress speed with a child until he has mastered the process and uses it with understanding. After the power skill is understood, then the child should adopt shortcuts, such as those stressed in this lesson, and develop his speed skill.

Using the Ideas

1. Find the sums.

A 57 + 15 = 72	B 18 + 72 = 90	C 49 + 33 = 82	D 28 + 77 = 105	E 39 + 44 = 83	F 36 + 26 = 62
G 76 + 44 = 120	H 85 + 77 = 162	I 98 + 33 = 131	J 84 + 27 = 111	K 75 + 58 = 133	L 83 + 75 = 158
M 85 + 26 = 111	N 73 + 98 = 171	O 46 + 38 = 84	P 75 + 83 = 158	Q 99 + 24 = 123	R 86 + 67 = 153

2. Find the sums.

A 258 + 137 = 395	B 258 + 167 = 425	C 258 + 187 = 445	D 546 + 125 = 671	E 546 + 175 = 721	F 546 + 675 = 1221
G 546 + 237 = 783	H 546 + 287 = 833	I 123 + 248 = 371	J 193 + 248 = 441	K 648 + 176 = 824	L 435 + 389 = 824

3. Find the sums.

A 804 + 307 = 1111	B 906 + 218 = 1124	C 937 + 105 = 1042	D 893 + 248 = 1141	E 346 + 928 = 1274	F 769 + 452 = 1221

think

My hundreds' place is 9 less than
My tens and ones combined.
Twice my ones will give my tens.
My name you now can find.

WHO AM I?
384

HUNDREDS | TENS | ONES

More practice, page A-12, Set 18

101

Using the Exercises

Let the children do the exercises on page 101 as independent work, and allow time afterward for checking answers and for discussion.

The *Think* problem is intended especially for the more capable children, but all would benefit from an explanation of the solution. In order for the numeral to have a hundreds' digit, the sum of the ones' and the tens' digits must be greater than 9. But also the tens' digit is twice as large as the ones' digit, so the only choice is 4 for the ones' digit and 8 for the tens' digit. This gives 3 for the hundreds' digit, and the number is 384.

Assignments (page 101)
Minimum: 1. Average: 1–2.
Maximum: 1–3.

Follow-up/"Combo"
A game much like bingo can be used to review basic combinations. Cut cardboard shirt forms or tagboard into 10-by-12-cm rectangles. Capable children can rule the cardboard into 30 two-cm squares and label the top row C-O-M-B-O. In the remaining twenty-five squares, write numerals less than 20 in random order, but label the middle square of the third row "Free."

To play the game, hold up an addition combination for all to see, repeating it aloud as you do so. The children should cover the correct sum with buttons, bottle caps, counters, or the like. Remind them that they may cover only one sum on a card for each given combination. To win, children must get five covered sums in a row, column, or diagonal and then call out "Combo." Afterward, the winners should read the sums back to you for quick verification.

(A dual-purpose Combo card can be made by using a different color to rule and label the back of each card with numerals less than 10. With this side of the card, play a game based on subtraction or finding missing addends.)

C	O	M	B	O
10	9	3	16	13
5	2	12	0	18
8	14	FREE	4	11
9	7	9	17	6
12	15	5	10	1

For Addition

Resources for Active Learning
Nuffield Project: *Computation and Structure* 3, pp. 12–13 Wiley. [A "Hundred Square" game]

Duplicator Masters, pages 19, 20
Workbook, page 37
Skill Masters, page 19

PAGES 102–103

Objective

Given word problems pertaining to data on a chart and requiring addition with regrouping and simple subtraction, the child will be able to use the data and solve the problems.

Preparation

To prepare for this investigation you might write *$1.00* on the chalkboard and have children name a few sets of coins they might have that would be worth a dollar. For example, they might have 2 fifty-cent pieces, or 2 quarters and 1 fifty-cent piece, 5 dimes and 2 quarters, and so on. After a few minutes of this discussion, explain to the children that in this lesson they can pretend that they have one dollar (in any set of coins) to use at a lunch counter.

Investigation

The children might gain most from this investigation by first working individually, for approximately ten minutes, and then joining together in small groups to share their solutions. Before they begin, point out the illustration on page 102, noting such items as the most and the least expensive sandwich or the price of milk. When the children join together into small groups, encourage them to share and check each other's lists. As they do so, they will see a variety of answers, and they should correct whatever errors appear. Give the children ample time to check each solution. There are numerous combinations to pick; a few whose sum is exactly one dollar are as follows: cheese sandwich, milk shake, and pie; beef sandwich, bean soup, and ice cream; 2 beef sandwiches and a root beer. (Note that if milk was included the price would never add exactly to one dollar.) When the children have had an adequate opportunity to share their menus, direct them in class discussion.

Can you find the cost of a lunch?

Investigating the Ideas

MENU

Sandwiches		Beverages		Soups	
Cheese	35	Milk	17	Chili	40
Ham	40	Milk shake	35	Vegetable	35
Peanut butter	30	Root beer	10	Bean	30
Hamburger	40	Orange	10	**Desserts**	
Hot dog	30	Coffee	10	Ice cream	25
Tuna	35	Tea	10	Cake	30
Beef	45			Pie	30

Choose a lunch from the menu and find what it costs.

? You have a dollar to spend for lunch. Can you find a way to spend all of it at the lunch counter?
See Investigation.

Discussing the Ideas

How is your "mental" arithmetic? Try to find the cost of these lunches in your head.

1. Ham sandwich
   Bean soup
   Milk
   87¢
2. Hamburger
   Chili
   Root beer
   90¢
3. Peanut butter sandwich
   Vegetable soup
   Orange
   Pie $1.05
4. 2 Hot dogs
   Milk
   Cake
   $1.07
5. Beef sandwich
   Chili
   Ice cream
   Milk
   $1.27
6. 2 Hamburgers
   2 Milks
   Cake
   $1.44

102

Discussion

To lead from the investigation, you might have a few volunteers put their price lists on the chalkboard and use them to review regrouping in column addition. For example:

cheese sandwich	35
milk shake	35
pie	30
	100

(Monetary notation need not be introduced here; it will be treated in a subsequent lesson. If you wish, however, you may use the ¢ (cents) symbol).

After the brief review of column addition, direct attention to the discussion section of the text. Here the children are given the opportunity to do some "mental arithmetic." Help the children to use reasoning to get the correct sum. For example, in exercise 1 the ham sandwich and bean soup are easy to add since they are multiples of ten: 40 + 30. To add the price of milk to 70, the children may think "70 + 7 is 77 and 10 more is 87."

Using the Ideas

1. What costs the most on the menu? Beef sandwich
2. Which soup costs the least? Bean
3. How much more is cake than ice cream? 5¢
4. For lunch Sue had a hot dog, vegetable soup, and a root beer. How much did her lunch cost? 75¢
5. Cindy bought a hamburger, a milk shake, and a piece of pie. How much did her lunch cost? $1.05 or 105¢
6. How much more is a beef sandwich than a tuna sandwich? 10¢
7. Andy had a sandwich, a soup, and a beverage. He spent the least amount that he could. How much did he spend? 70¢
8. Bill spent 77 cents. What did Bill have to drink? Milk

★ 9. Ann had 95 cents to spend for lunch. She spent more than 85 cents. What might Ann have ordered?
Sample answer: hamburger, milk, bean soup (87¢)

★ 10. Tom did not like soup or dessert. He spent 95 cents for lunch. What might Tom have ordered if he had only one beverage?
Sample answer: 2 hot dogs and a milk shake

★ 11. Jean had one soup, one beverage, and one dessert. She spent 95 cents. What did she have to drink? Milk shake

103

Follow-up/"Pick a Lunch"

If your school has a cafeteria, the children may wish to make charts showing typical food choices and prices. They may also make up problems concerning their own lunches. Learning to make change efficiently should be an incidental outcome of such an activity.

If you do not have a school cafeteria, ask the children to study the basic food groups and to design charts showing good lunch menus. Some may enjoy finding illustrations for such charts, while others could find up-to-date prices for lunch selections in local restaurants. With this information, the children could devise realistic problems about choosing the best lunch for a given amount of money and then determine and count out the proper change.

Resources for Active Learning

Nuffield Project: *Computation and Structure* 2, "Money," pp. 91–102, Wiley. [Handling money in the classroom]

Duplicator Masters, page 24
Workbook, page 38

Using the Exercises

You may choose to work through these problems with the class. However, if you choose to assign them as independent work, remind the children to study the menu on page 102 carefully. Also, if they have difficulty finding the answer by using the usual algorithm, encourage them to try to use the type of reasoning employed in solving the "mental" arithmetic problems on the preceding page. (This suggestion is particularly applicable for exercise 5.)

Note that for starred exercises 9 and 10 several different combinations are possible.

Assignments (page 103)

Minimum: 1–8, oral. Average: 1–8.
Maximum: 1–11.

PAGES 104–105

Objective

Given a 2-digit numeral, the child will be able to regroup tens into the ones' place.

Preparation

To prepare for this lesson, use a short oral game to review place value. For example, say, "I'm thinking of 5 tens and 9 ones. What's my number?" Gradually lead into expressions such as 5 tens and 10 ones, 5 tens and 11 ones, or 5 tens and 12 ones and expect the child to respond with answers such as 60, 61, 62. This activity will lead into the discussion exercises in the text.

Let's explore regrouping.

Discussing the Ideas

1. Solve the equations.

A	$35 = 30 + n$ 5
B	$35 = 20 + n$ 15
C	$27 = 20 + n$ 7
D	$27 = 10 + n$ 17
E	$42 = 40 + n$ 2
F	$42 = 30 + n$ 12

2. Find the missing numbers.

A $48 = 40 + n$ 8
$48 = 30 + n$ 18

B $72 = 70 + n$ 2
$72 = 60 + n$ 12

C $65 = 60 + n$ 5
$65 = 50 + n$ 15

D $45 = 40 + n$ 5
$45 = 30 + n$ 15

E $54 = 50 + n$ 4
$54 = 40 + n$ 14

F $61 = 60 + n$ 1
$61 = 50 + n$ 11

G $42 = 40 + n$ 2
$42 = n + 12$ 30

H $99 = 90 + n$ 9
$99 = n + 19$ 80

I $68 = n + 8$ 60
$68 = 50 + n$ 18

104

Discussion

Study the sets of tens and ones in exercise 1 with the children and discuss with them the different ways to think of 35, 27, and 42. You might include $35 = 10 + 25$, $42 = 20 + 22$, and $42 = 10 + 32$. Use these examples to point out that the different expressions for each number all have the same value. Then write pairs of equations which demonstrate the kind of regrouping which is used in the subtraction algorithm:

$82 = 80 + 2 \qquad 44 = 40 + 4$
$82 = 70 + 12 \qquad 44 = 30 + 14$

Include pairs to emphasize regrouping with missing addends, as in the following four examples.

$51 = \square + 1 \qquad 37 = 30 + \square$
$51 = \square + 11 \qquad 37 = 20 + \square$

Work through exercise 2 with the children and then, to prepare them for the independent study exercises, demonstrate the short way of showing this regrouping. For example, write 34 on the chalkboard and say that, in order to help us think of 34 as $20 + 14$, we write:

$$\overset{2}{\cancel{3}}\ \overset{14}{\cancel{4}}$$

Using the Ideas

1. Study examples A and B.

A To think of 52 as 40 + 12, we can write $\overset{4}{\cancel{5}}\overset{12}{\cancel{2}}$.

B To think of 37 as 20 + 17, we can write $\overset{2}{\cancel{3}}\overset{17}{\cancel{7}}$.

Find the matchings. Part C is matched with 4.

C 40 + 13	1 $\overset{2}{\cancel{3}}\overset{14}{\cancel{4}}$
1 D 20 + 14	2 $\overset{7}{\cancel{8}}\overset{11}{\cancel{1}}$
6 E 60 + 12	3 $\overset{5}{\cancel{6}}\overset{16}{\cancel{6}}$
2 F 70 + 11	4 $\overset{4}{\cancel{5}}\overset{13}{\cancel{3}}$
5 G 30 + 18	5 $\overset{3}{\cancel{4}}\overset{18}{\cancel{8}}$
3 H 50 + 16	6 $\overset{6}{\cancel{7}}\overset{12}{\cancel{2}}$

2. Complete each of the following as in the examples above.

A To think of 48 as 30 + 18, we can write ▥. $\overset{3}{\cancel{4}}\overset{18}{\cancel{8}}$

B To think of 65 as 50 + 15, we can write ▥. $\overset{5}{\cancel{6}}\overset{15}{\cancel{5}}$

C To think of 72 as 60 + 12, we can write ▥. $\overset{6}{\cancel{7}}\overset{12}{\cancel{2}}$

D To think of 49 as 30 + 19, we can write ▥. $\overset{3}{\cancel{4}}\overset{19}{\cancel{9}}$

E To think of 83 as 70 + 13, we can write ▥. $\overset{7}{\cancel{8}}\overset{13}{\cancel{3}}$

F To think of 54 as 40 + 14, we can write ▥. $\overset{4}{\cancel{5}}\overset{14}{\cancel{4}}$

G To think of 21 as 10 + 11, we can write ▥. $\overset{1}{\cancel{2}}\overset{11}{\cancel{1}}$

H To think of 96 as 80 + 16, we can write ▥. $\overset{8}{\cancel{9}}\overset{16}{\cancel{6}}$

105

Follow-up
Worksheets similar to the following would help the children recognize correct regrouping.

Put in the circle every correct name for the number in the box.

70 + 15 100 − 25 25 + 25 + 25 75 70 + 5 15 80 − 5 60 + 15 40 + 35 30 + 45 50 + 45

Use one of these symbols
<, >, or =
to make each sentence true.

40 + 24 ○ 65	52 ○ 50 + 2
40 + 14 ○ 54	77 ○ 60 + 17
35 + 10 ○ 50	81 ○ 70 + 1
27 ○ 20 + 17	65 ○ 60 − 15

Resources for Active Learning
Discovery, Section 1, Activity 4, pp. 4–6, Encyclopaedia Britannica Educational Corp. [A "fewest moves" activity with special rules for regrouping]

Duplicator Masters, page 25
Workbook, page 39
Skill Masters, page 25

Using the Exercises
Assign the exercises as independent work. You also might write several other 2-digit numerals on the chalkboard and have the children rewrite these to show regrouping. To check the answers, have several children write the new notation on the chalkboard for each exercise.

To challenge the more capable children, you may wish to distribute a worksheet as suggested in the follow-up.

Assignments (page 105)
Minimum: 1–2. Average: 1–2.
Maximum: 1–2.

PAGES 106–107

Objective

Given a 2-digit subtraction problem which requires regrouping, the child will be able to find the difference by using the usual algorithm.

Preparation

This lesson calls for a short review of subtraction combinations. To review finding missing addends, play a short game of "What's My Rule," as suggested on page 69. If you use the written method, your chart might look something like this:

a	17	12	15	14	11	16	18
b	8	3	6	?	?	?	?
c	9	4	7	?	?	?	?

and your rules would be:

$a - \square = b$ or $a = \square + b$ $(\square = 9)$

$a - \square = c$ or $a = \square + c$ $(\square = 8)$

Investigation

This investigation is short, so you might have the children study the patterns independently after you have read the directions with the class. Remind them to record their answers. You might put a few more patterns on the chalkboard, particularly for those who finish quickly. You may use patterns such as the following:

52	52	52	52	52	52
−1	−2	−3	−4	−5	−6
51	50				

36	36	36	36	36	36
−14	−15	−16	−17	−18	−19
22	21				

Encourage the children to describe these and the other patterns in their own words.

● *How is regrouping used to find differences?*

Investigating the Ideas

Look for a pattern.

43	43	43	43	43	43	43
−1	−2	−3	−4	−5	−6	−7
42	41	40	39	38	37	36

? Can you find the rest of these differences without subtracting?

75	75	75	75	75	75	75
−32	−33	−34	−35	−36	−37	−38
43	42	41				

Discussing the Ideas

1. Explain when the number of tens changes in the sets of answers above. See Discussion.

2. Explain each of the steps below.

Step 1	Step 2	Step 3	Step 4
64 − 26	$\overset{5\,14}{64}$ − 26	$\overset{5\,14}{\cancel{6}\cancel{4}}$ − 26 = 8	$\overset{5\,14}{\cancel{6}\cancel{4}}$ − 26 = 38
4 − 6 = !!!	64 = 50 + 14	14 − 6 = 8	50 − 20 = 30

3. Find this difference by following the steps above. The answer is 48. $\overset{6\,15}{\cancel{7}\cancel{5}} - 27 = 48$

4. Explain the steps for the first example. Then try the second one on your own.

$\overset{6\,11\,13}{\cancel{7}\cancel{2}\cancel{3}} - 146 = 577$ $\qquad$ $\overset{5\,13\,12}{6\cancel{4}\cancel{2}} - 256 = 386$

106

Discussion

As you discuss exercise 1 with the class, help the children to realize that the number of tens in the answer changes when the digit in the ones' place of the number being subtracted is greater than the digit in the ones' place of the other number. This situation is shown in step 1 of exercise 2, and should be stressed so that the children will regroup only when necessary. Use the equations below each step to point out the thinking process.

Work through the example in discussion exercise 3 with the class, discussing each step as you proceed.

$75 - 27 \rightarrow \overset{6\,15}{\cancel{7}5} - 27 \rightarrow \overset{6\,15}{\cancel{7}5} - 27 = 8 \rightarrow \overset{6\,15}{\cancel{7}5} - 27 = 48$

Once children understand the regrouping from tens to ones, they will readily extend the same process from hundreds to tens. However, you should work carefully through exercise 4 to be sure the children understand each step. It would be helpful to demonstrate such a problem in steps as follows:

$723 - 146 \rightarrow 7\overset{1\,13}{\cancel{2}\cancel{3}} - 146 = 7$ $\qquad$ $723 = 710 + 13$ $\qquad$ $13 - 6 = 7$

Using the Ideas

1. Cover the answers and work the problems.

A	B	C	D	E
74	52	92	83	61
−26	−35	−64	−27	−25
48	17	28	56	36

2. Check each answer in exercise 1 by addition.

3. Find the differences.

A	B	C	D	E	F
43	33	72	54	81	68
−16	−15	−44	−17	−23	−25
27	18	28	37	58	43

G	H	I	J	K	L
42	95	76	69	20	27
−19	−77	−38	−62	−17	−18
23	18	38	7	3	9

M	N	O	P	Q	R
42	50	68	156	143	143
−38	−18	−29	−82	−71	−74
4	32	39	74	72	69

S	T	U	V	W	X
120	165	142	122	130	115
−68	−76	−65	−47	−56	−88
52	89	77	75	74	27

★ 4. Find the differences.

A	B
124	224
−56	−56
68	168

C	D
132	432
−95	−95
37	337

E	F
143	543
−68	−168
75	375

G	H
724	631
−157	−256
567	375

think

It's true our sum is 60,
And we differ by just 2.
If you want to find our names,
Some thinking you must do.

WHO ARE WE? 29,31

More practice, page A-13, Set 19

$$\begin{array}{r} 6\,11\,13 \\ \cancel{7}\cancel{2}3 \\ -146 \\ \hline 7 \end{array} \rightarrow \begin{array}{r} 6\,11\,13 \\ \cancel{7}\cancel{2}3 \\ -146 \\ \hline 577 \end{array} \qquad \begin{array}{l} 723 = 600 + 110 \\ \qquad\qquad + 13 \end{array}$$

$$110 - 40 = 70$$
$$600 - 100 = 500$$

Using the Exercises

Before assigning several of the exercises on page 107 as independent work, remind the children of the missing-addend approach and show them how to check their work by addition. The ability of your class should determine your assignment of the starred exercise.

The *Think* problem will be solved when the children realize that since $30 + 30 = 60$, then $29 + 31 = 60$.

Assignments (page 107)

Minimum: 1–3. Average: 1–3.
Maximum: 1–4.

Follow-up

After a lesson of this type, the children would probably enjoy a game. If you made the Combo cards (page 101) in the dual-purpose manner with numerals less than 10 on one side, you might play Combo, giving subtraction equations instead of addition.

Or you might use the "Arithmetic Baseball" game: Divide the class into two teams. Pick a number to see which team bats first. If team A is up, a member from team B "pitches" an addition or subtraction phrase such as "7 + 9," "15 = 8 + ?" or "2 plus what is 11," and a "batter" from team A responds. (The phrases may be written on cards which the "pitcher" then reads.) The "umpire" (teacher) marks progress on a baseball diamond drawn on the chalkboard. Each batter gets one chance to answer. One mistake is an out, and one out retires the side and brings the other team to bat. A correct answer serves as a base hit. Every time a team is out the diamond is cleared. The umpire writes the team member's symbol to indicate base hits. Before a team can score, three consecutive answers must be given to get the bases loaded and the fourth correct answer will bring in a run.

Resources for Active Learning

Mathex: Operations No. 3, "Numbers in the Thousands—Activity 2," p. 24, Encyclopaedia Britannica Publications Ltd.

Nuffield Project: *Computation and Structure* 2, p. 76, Wiley.

Toward Improving Computation, "Sequences with Pattern and Purpose," pp. 39–44; "Subtraction with Beans and Sticks," pp. 53–54, Curriculum Development Associates.

Duplicator Masters, pages 26–28
Workbook, pages 40, 41
Skill Masters, pages 26-28

PAGES 108–109

Objective

Given word problems requiring addition and subtraction of 2-digit numerals and data illustrated in chart form, the child will demonstrate his ability in problem solving by performing the suitable operation.

Preparation

Material

scale for weighing children (if available)

Little preparation is needed for this investigation. It would be helpful to remind the children that to estimate means to make a careful guess. You might review this meaning by referring to the man at the circus or the fair who tries to guess people's ages and weights, and explain that in this lesson they will have a chance to do some of that kind of estimating. But since everyone in the third grade is just about the same age, they will be guessing only weights.

Investigation

This investigation necessitates separating the class into groups of, say, four or five. The composition of the groups should reflect a consideration for the possible sensitivity of any children with weight problems. You can minimize the likelihood of embarrassment for such children by putting them in groups with their friends and by trying to avoid having extreme weight ranges within a single group.

Read the directions for the investigation with the class and, if necessary, guide them in making the charts. If you prefer, prepare duplicated copies of the chart for the children to use. Suggest that they write down their guesses before sharing them, so that they will not be influenced by each other's answers. Also remind them to be honest when telling the others their actual weight. If possible, have at least one scale in the classroom so that children who do not know how much they weigh can find out.

Can you estimate someone's weight?

Investigating the Ideas

How close can you come to guessing someone's weight?

Name	Guess	Weight	Difference
Ann	23	26	?
Fred	30	24	?

See Investigation.

?

Can you guess the weight of each classmate in your group?	Make a chart like the one above to record your findings.

Discussing the Ideas See Discussion.

1. A Did you guess anyone's weight exactly?
   B Whose weight did you come closest to guessing?
   C How many times did you guess "over"?
   D How many times did you guess "under"?

2. A Did you use what you know about **your** size and weight to help you make your guesses? How?
   B How could you make better guesses?

3. A man 173 centimetres tall might weigh about 70 kilograms.
   A Can you find two students who together weigh this much?
   B Can you find three students who together weigh this much?

4. A What is your guess for the total weight of your class?
   B Did you miss the total weight by more than 500 kilograms?

108

Discussion

The first discussion exercise provides children with an opportunity to interpret data on a chart – the chart which they themselves made in the investigation. Help them understand the phrases "guess 'over,'" and "guess 'under.'"

Throughout the discussion of exercises 1 and 2, stress the importance of using known information in making estimates. Use discretion in treating exercise 3; it would be difficult to find two average third graders who together weigh 70 kilograms. In exercise 4, it is not necessary to calculate exactly the total weight of the class: if you have 30 third graders with an average weight of 25 kilograms, a child would have to guess less than 250 or more than 1250 in order to miss the total weight by more than 500.

Throughout the discussion, stress the importance of understanding given information and the helpfulness of estimates in finding solutions to problems.

Using the Ideas

Name	Guess	Weight	Difference
Bobby	30	24	? 6
Susan	28	27	? 1
Tom	25	28	? 3
Sara	21	19	? 2
Alan	22	25	? 3
Bill	29	29	? –
Jane	24	21	? 3
Tony	23	22	? 1
Joan	27	26	? 1
Ann	26	22	? 4
Rick	31	30	? 1

1. Which child weighs most? Rick
2. Who weighs least? Sara
3. How many children weigh less than 25 kilograms? 5
4. How many children weigh more than 30 kilograms? 0
5. The table shows the guess and the measured weight for each student. Find how close the guess was for each student.
6. The boys' names are in red boxes. How much heavier is the heaviest boy than the lightest boy? 8 kg
7. How much do these children weigh together?
   A Bobby and Alan 49
   B Susan and Sara 46
   C Rick and Tony 52
   D Ann and Joan 48
   E Sara and Ann 41
   F Bill and Alan 54
8. How much heavier is the heaviest girl than the lightest girl? 8 kg

★ 9. Two children got on the scales together. The scales showed 58. Who were the children? Tom and Rick

★ 10. How much greater is the total weight of the boys than the total weight of the girls? 43 kg

think

Number thirty-one
Is slightly more than me.
I'm that much larger
Than number twenty-three.

WHO AM I? 27

109

Using the Exercises

If you prefer, you may have the children use the chart they made in the investigation as the data source for exercises 1–4. However, if a child in the class has an obvious weight problem, it would be better to use the chart in the text.

You might assign these exercises to be worked independently, but working through some of the questions together would highlight helpful guidelines in problem solving. For example, you might discuss the importance of understanding what information is given in a problem. (Here the information is given in chart form.) Stress, too, the importance of clearly understanding what the question is and how the given information can be used to answer it. Finally, suggest that the answer be examined for its reasonableness: "Does the answer make sense for the question asked?"

If necessary, review subtraction with regrouping to help children in their computation.

Assignments (page 109)

Minimum: 1–8, oral. Average: 1–8.
Maximum: 1–10.

Follow-up

Depending on class ability and need, for review you may wish to choose a game such as the one in the follow-up section of the previous lesson or have a class activity such as the following.

Instruct the children to separate into the groups in which they worked during the investigation. This time, instead of marking weight, they are to guess and then chart the *height* of each group member *in centimetres*. They should then make up questions similar to those on page 109 comparing the heights of persons in their group. They might find it interesting to compare the difference between weight in kilograms and height in centimetres for each child in the group. These questions should not only give practice in using subtraction with 2-digit numerals but also help children develop insight in forming relationships, charting data, and making comparisons.

Resources for Active Learning

Mathex: Matching and Graphing No. 1, "First Steps in Graphing," pp. 16–25, Encyclopaedia Britannica Publications Ltd.

[The following references provide graphing activities as readiness for reading tables and charts, like the "multiplication table" in the next chapter.]

Developmental Math Cards, $E^2 15$, Addison-Wesley.

Mathex: Measurement and Estimation No. 5, "Weight-Activity 1," p. 37, Encyclopaedia Britannica Publications Ltd.

Nuffield Project: *Computation and Structure* 2, "Weight," pp. 23–32, Wiley.

Nuffield Project: *Pictorial Representation* 1, "Graphs," pp. 4–6, 9, Wiley.

PAGES 110–111

Objective

Given word problems which require addition and subtraction of 2-digit numerals, and data presented in chart form, the child will demonstrate his ability to solve problems by using the given data and answering the questions.

Preparation

Material

thermometer (optional)

You may treat these problem-solving pages in a variety of ways. You might use both pages for discussion and assign the other as independent study.

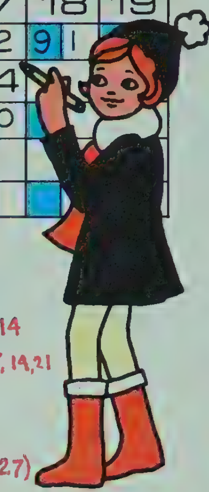
If you use page 110 as a basis for discussion, you might introduce the lesson with a review of the calendar. It would also be helpful to display a thermometer. Also mention common temperatures such as comfortable room temperature (about 21° C); the freezing point of water (water to ice — 0° C); and the boiling point of water (water to steam — 100° C).

High and Low Temperatures

Miss Smith's class decided to keep a record of high and low temperatures for days in November.

For each day, the students wrote the two temperatures on the school calendar.

NOVEMBER						
S	M	T	W	T	F	S
		1	2	3	4	5
		17 8	16 8	11 4	9 3	9 2
6	7	8	9	10	11	12
8 1	9 0	11 4	10 1	9 1	8 1	7 1
13	14	15	16	17	18	19
3 0	10 0	18 3	17 3	15 2	9 1	
20	21	22	23	24		
7 5	2 0	3 1	3 1	4 0		
27	28	29	30			
1 0	8 1	9 8	11 3			



Use the calendar above for these problems.

1. A Which Monday had the highest temperature? November 14
   B Which Monday had the lowest temperature? November 7, 14, 21
2. Water freezes at temperatures of 0 or lower. How many days are shown with freezing temperatures? 6 (November 7, 13, 14, 21, 24, 27)
3. Which Sundays had freezing temperatures? November 13 and 27
4. How many days are shown with temperatures above 16? 3 (November 1, 15, 16)
5. What was the difference between the high and the low temperature on each of these days?
   A November 2 8°
   B November 27 1°
   C November 1 9°
   D November 8 7°
6. How much more was the high temperature on November 14 than the high temperature on November 6? 2°
7. What day was the warmest day of the month? What was the difference between the highest and the lowest temperatures in November? November 15, 18°

110

Discussion

Read the introductory section for page 110 and discuss what is meant by high and low temperatures. You might read off a few dates and ask children to give you the high and low temperatures recorded for those days. As you work through the questions, the children would benefit from a few simple guidelines. For instance, as you discuss a problem, suggest using these questions to approach it:

What do I know? (What information is given in the problem or on an accompanying chart?)

What must I find? (sum, difference, how much more, etc.)

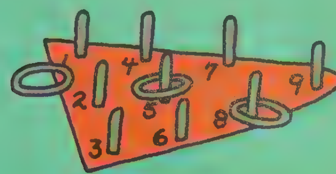
What must I do? (What operation, addition or subtraction, should be used?)

Does my answer make sense? (Does my "answer" answer the question that was asked?)

These are not rules to be memorized by children, but using them will give the average and slower child a method of attack in problem solving, and will help him develop logical thinking.

Page 111 may be treated in a similar manner or assigned as independent work. Stress the importance of reading the chart correctly. Call particular attention to Dan's score in Game 3. Show the chil-

FIGURING SCORES



The numbers given in the chart are the children's scores for each game. For example, Dan scored 47 points in game 3.

	Ann	Bill	Carol	Dan	Ed	Fay
Game 1	28	27	62	81	60	47
Game 2	34	26	58	27	54	23
Game 3	39	43	72	47	34	52
Game 4	50	76	40	56	67	58
Game 5	63	18	43	64	29	67
Game 6	31	71	42	54	34	36

1. How many points did Fay score in games 2 and 3 together? 75

2. How many more points did Bill score in game 4 than in game 3? 33

3. In game 6, how much more was the highest score than the lowest score? 40

4. In game 2, how many points did Carol and Dan score together? 85

5. In game 5, how much less was Ed's score than Dan's? 35

6. Fay found the sum of her two best scores. Was this more or less than the sum of Ed's two best scores? Less

7. In game 1, the sum of the two highest scores was 143. In what other game was the sum of the two highest scores 143? Game 4

8. What is the difference of the highest score on the chart and the lowest score? 63

★ 9. The children decided that the winner should be the one who had the highest sum for his best 3 games. Who won? Dan

More practice, page A 14, Set 20

dren how they can arrive at 47 by tracing across the Game 3 row to the column labelled *Dan*. Explain that reading this table is much like reading a multiplication or addition table. Encourage children to discuss any problems which cause particular difficulty.

Follow-up

More capable children might enjoy making a chart to show scores and players of games played at recess or during activity time.

Others may make a chart to record temperatures taken at different times of the day for a certain number of weeks. This could be a class activity recorded for all to see over a period of one month. If any children have a thermometer at home, they may make a similar chart and keep a record for a week or two. The charts may be like the one on page 110 or like this:

Date	Temperature		Diff.
	9:00 A.M.	2:30 P.M.	

Resources for Active Learning

Mathematics in Modules, M9, S1, S2, Addison-Wesley.

Nuffield Project, *Pictorial Representation* 1, "Graphs," pp. 4–6, 9, Wiley.

Duplicator Masters, page 29
Workbook, page 42

Assignments (page 111)
Minimum: 1–8, oral. Average: 1–8.
Maximum: 1–9.

Objective

Given collections of coins less than one dollar, the child will be able to solve problems of comparing and combining.

Preparation

Materials

advertisements, department store catalogues, price lists, etc.

You might prefer to omit any specific preparation and begin immediately with the investigation. Otherwise, conduct a short, brisk review of basic combinations.

Investigation

Distribute as many advertisements and price lists as possible. If necessary, list some items and their prices on the chalkboard; for example:

5 pencils............................32¢
1 eraser.............................15¢
4 candy bars ....43¢
6 cupcakes..........................88¢
3 tablets ....77¢
1 binder ....96¢
2 packages of binder fillers.....66¢

Read the directions at the top of page 112 with the class, explaining the meaning of *fewest*. This investigation does not require group work, but as the children work individually, allow them to share their results. You might want to distribute previously duplicated copies of the chart for children to use in recording their choices of coins. They can check one another's work, trying to find items for which fewer coins than are listed might be used.

In doing this investigation, the children are constantly working both with the basic combinations to find missing addends and with combinations that involve 2-digit numerals.

● *Let's count money.*

Investigating the Ideas

Find the prices of some things you would like to buy for less than one dollar.

Item and Cost	quarters	dimes	nickels	pennies
Pen 69¢ 	2	1	1	4
Game 98¢ 	3	2	0	3

? Can you fill in a table like this by giving the **fewest** coins you could use to buy each one?

See Investigation.

Discussing the Ideas

1. Give some other ways you could pay for the pen.
Sample answers: 1 quarter, 4 dimes, 4 pennies, 6 dimes, 9 pennies

2. What are some other ways you could pay for the game?
Sample answers: 2 quarters, 4 dimes, 8 pennies; 9 dimes, 1 nickel, 3 pennies

3. What change would you get back from one dollar for each item above? Pen, 31¢; game, 2¢

4. Give some different ways you could give someone change for a dollar.
Sample answer: 3 quarters, 2 dimes, 1 nickel

5. How much money would you have if you had two of each coin listed in the chart? 82¢

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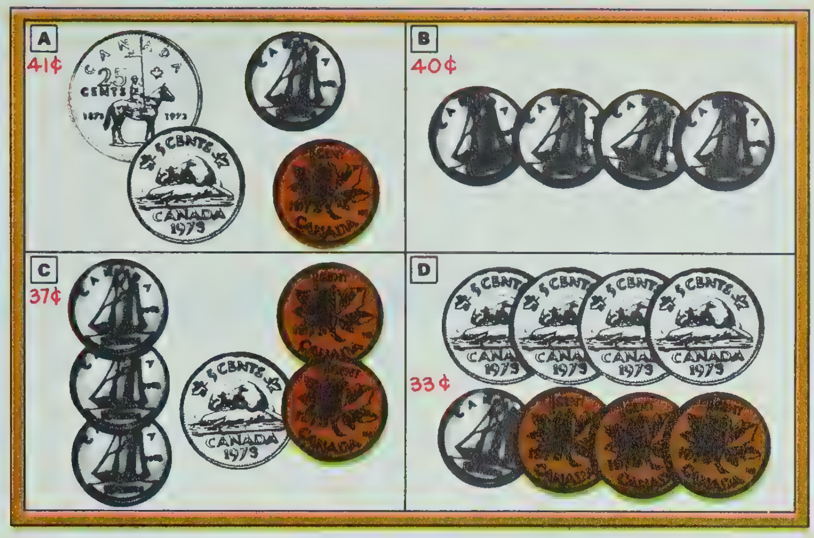
Discussion

Encourage children to suggest a variety of coin combinations as answers for exercises 1, 2, and 4, but do not have children attempt to give all possible combinations. It is sufficient that they realize that many combinations are possible and that they have the experience of using their knowledge of place value to see that 10 pennies can be substituted for 1 dime or for 2 nickels, that 2 dimes and 1 nickel can be substituted for 1 quarter, and so on.

For exercises 3 and 5, you might wish to have the children work the subtraction problems independently before discussing the answers or work through the problems at the chalkboard.

Using the Ideas

1. Give the value of each coin collection.



2. For each part, tell which coin collection has the greater value.

A B or C B A or B C A or D D C or D

3. Give the value of each of the two coin collections together.

A A and B 81¢ C A and C 78¢ E A and D 74¢
B B and D 73¢ D B and C 77¢ F C and D 70¢

4. 1¢ A How much more is A than B? D How much more is B than C? 3¢
4¢ B How much more is A than C? E How much more is A than D? 8¢
4¢ C How much more is C than D? F How much more is B than D? 7¢

5. Which pair of collections has the greater value?
A A and B or C and D
B A and C or B and D
C A and B or A and C

★ **6.** A Give the total value of all the coin collections together. $1.51 or 151¢
B How much more would you need to have 5 dollars? $3.49

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Using the Exercises
You might wish to use parts of this section as a basis for further discussion. For example, after the children carefully record the values of the collections in exercise 1, you might treat exercise 2 orally.

Exercises 3, 4, and 5 provide practice in 2-digit addition and subtraction, so they should be assigned as written work. If the children include the set labels in their answers for exercise 3 (e.g., A and B: 30 + 40 = 70, 70¢, or 70 cents), the information will help them in answering exercises 5 and 6.

Give everyone an opportunity to think about exercise 6. Although it is starred, all the children would benefit from having the problem displayed on the chalkboard and discussed, after they have had an opportunity to try it on their own.

If necessary, allow slower children to manipulate "coins" to clarify problems causing difficulty.

Assignments (page 113)
Minimum: 1–4. Average: 1–5.
Maximum: 1–6.

Follow-up
You may choose from a variety of follow-up activities according to the needs of the class. If the children need more practice with basic combinations choose a game like "Combo" (page 101) or "Arithmetic Baseball" (page 107).

Some children may benefit from practice with cardboard coins, giving change for purchases suggested by their classmates. Or you might distribute a worksheet similar to the chart in the investigation section of the text; for example:

Give the *fewest* coins you could use to purchase an item for each amount.

	Half Dollars	Quarters	Dimes	Nickels	Pennies
72¢	1		2		2
85¢					
75¢					
48¢					
53¢					
64¢					
98¢					

Resources for Active Learning
Franklin Series: *Learning About Measurement*, "Making Change", pp. 99-101, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)
Mathex: Operations No. 3, "Shopping," pp. 15–16, Encyclopaedia Britannica Publications Ltd.
Nuffield Project: *Computation and Structure* 2, "Money," pp. 91–102, Wiley.

Duplicator Masters, page 30
Skill Masters, page 30

PAGES 114–115

Objective

Given a variety of problems involving money, the child will be able to use the appropriate algorithms and proper dollar-and-cent notation in solving such problems.

Preparation

Materials

advertisements, department store catalogues, price lists, etc.

To introduce this lesson, you might use a short oral warm-up such as this: "If I have one dollar, and buy a 47¢ item, what change will I get back?" The children may respond either by giving the total amount (53¢) or by specifying the coins you would receive in change (3 pennies and a half dollar). Continue with other amounts similarly, but only for a few minutes.

Investigation

This investigation provides opportunity not only for the child to work with amounts of money in purchases and figuring change, but also for the child to observe and discover for himself the use of dollar-and-cent notation.

Read the directions with the class and guide the children in making a chart like the one in the text, or distribute duplicated copies to record their amounts of money. (Do not specifically point out the dollar-and-cent notation; this will be developed in the discussion.)

As in the previous lesson, distribute as many advertisements as are available and on the chalkboard provide supplementary information such as:

plastic ball	$ .99	shirt	$2.79
harmonica	$1.00	sandals	$1.58
doll clothes	$1.29	books	$2.98
water gun	$1.25	puzzle	$1.36

Encourage the children to work on their charts individually, but be sure to allow for free exchange of ideas. If some children finish very quickly, suggest that they choose three things to buy at a total cost of less than 5 dollars.

Can you add and subtract amounts of money?

Investigating the Ideas

Choose two things you can buy at a total cost of less than 3 dollars.

? Can you fill in a chart like the one below?
See Investigation.

Had	Bought	Bought	Spent in all	Had left
$3.00	Record $.95	Game $1.50	$ 2.45	$.55

Discussing the Ideas

1. Give the missing numbers.

Example: $5.28 means 5 dollars and 28 cents.

A $7.16 means ▒ dollars and 16 cents. 7

B $4.75 means 4 dollars and ▒ cents. 75

C $2.09 means ▒ dollars and 9 cents. 2

D $17.34 means ▒ dollars and ▒ cents. 17, 34

2. Give the missing numbers.

Example: $4.38 is 438 cents (438¢).

A $6.52 is ▒ ¢. 652¢ C $1.00 is ▒ ¢. 100¢ E $.50 is ▒ ¢. 50¢

B $1.27 is ▒ ¢. 127¢ D $.76 is ▒ ¢. 76¢ F $2.86 is ▒ ¢. 286¢

3. Give the number of dollars and cents for each exercise.

Example: 364¢ is $3.64

A 125¢ $1.25 C 506¢ $5.06 E 650¢ $6.50 G 1000¢ $10.00 I 1250¢ $12.50

B 326¢ $3.26 D 400¢ $4.00 F 100¢ $1.00 H 1100¢ $11.00 J 1795¢ $17.95

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Discussion

Call on some children to write their purchases, total expenditure, and change on the chalkboard. Encourage the class to check their figures and to notice any error. The dollar-and-cent notation is likely to be noticed. If it is used in the figures shown on the board, point it out and proceed to the discussion in the text; if it is not used, ask questions that will lead the children to see how the figures on the board are different from those in the book.

As you discuss these exercises, do not elaborate on the decimal point specifically. Indicate only that this dot (or period, or point) separates the number of dollars from the number of cents. Children should recognize this readily, and it is not likely to cause any difficulty.

You may wish to have volunteers write the answers for exercise 3 on the chalkboard, while the others write at their desks.

Using the Ideas

1. Find the total amounts.

Example: $5.68 + 2.71 = $8.39

A $3.27 + 4.61 = $7.88	B $5.38 + 1.25 = $6.63	C $7.64 + 1.75 = $9.39
D $2.76 + 1.85 = $4.61	E $3.79 + 4.48 = $8.27	F $9.72 + 8.99 = $18.71

2. Find the total amounts.

Examples:
$1.23 and $2.45 is $3.68.
$2.45 and $3.72 is $6.17.

$3.67 A $2.14 and $1.53 is ▒.
$7.58 B $5.21 and $2.37 is ▒.
$7.00 C $3.50 and $3.50 is ▒.

3. Find the difference in the amounts.

Example: $5.27 − 1.43 = $3.84

A $6.34 − 1.52 = $4.82	B $7.65 − 2.24 = $5.41	C $8.32 − 4.18 = $4.14
D $9.21 − 1.65 = $7.56	E $12.72 − 5.28 = $7.44	F $10.64 − 3.95 = $6.69

Solving Story Problems

1. Debra had this much money.
She bought a book for 99 cents
and paid 5 cents tax.

A How much did she have at the start? $2.40
B How much did she spend? $1.04
C How much did she have left? $1.36

2. Craig had this much money.
He bought some goldfish for $1.39.
His tax was 7 cents.

A How much did he have at the start? $1.81
B How much did he spend? $1.46
C How much did he have left? $.35

More practice, page A-15, Set 21

Using the Exercises

Even though exercise 1 provides an example, children might benefit from a developmental activity built around finding the total of two amounts. For example:

$2.45 is ? cents. 245
3.72 is ? cents. 372
617 cents

617 cents is ? dollars and ? cents.

Several examples of this kind will help the children see that they should think of adding and subtracting *numbers*, not money, and then interpret their answer in terms of money. When they understand this, using the dollar-and-cent notation directly, as shown in exercise 1, should be encouraged.

It would be helpful to point out the regrouping for exercise 3F:

$$\begin{array}{r} {\scriptstyle 9\ 15\ 14} \\ \$\cancel{10.64} \\ \underline{3.95} \end{array} \qquad 1064 = 900 + 150 + 14$$

You may assign the story problems as part of the independent study. Remind the children to use dollar-and-cent notation.

Assignments (page 115)*
Minimum: 1, 3; story problems 1–2, oral. Average: 1–3; story problems 1–2. Maximum: 1–3; story problems 1–2.

Follow-up

The children would probably enjoy further development of the charts they made in the investigation. Under the same headings they could use different amounts of money and list purchases of items you have suggested on the chalkboard or items they have found in advertisements. For example:

Had	Bought	Bought	Spent in all	Had left
$5.00	Game $1.25	2 Books $2.98	$4.23	$.77

These charts might then be displayed around the room. Pictures of some of the items purchased might be included.

Resources for Active Learning

Franklin Series, *Learning About Measurement*, "Money," pp. 94–98, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)
Mathex: Measurement and Estimation No. 5, "Adding Money . . .," pp. 42–44, Encyclopaedia Britannica Publications Ltd.
Nuffield Project: *Computation and Structure* 2, "Money," pp. 91–102, Wiley.

Duplicator Masters, page 31
Workbook, page 43
Skill Masters, page 31

PAGES 116–117

Objective

The child will demonstrate his understanding of regrouping in addition and subtraction of 2- and 3-digit numerals by writing sums and differences in exercises and word problems.

Preparation

Since these two pages provide addition and subtraction practice, you might use a short oral warm-up of basic combinations such as "What's My Rule" (page 69) or "Names for a Number" (page 90). In either case, keep the activity brisk and brief.

How sharp are your adding and subtracting skills?

1. Find the sums and differences.

A	B	C	D	E	F
24 + 15 = 39	35 − 12 = 23	56 + 22 = 78	78 − 53 = 25	92 − 42 = 50	64 + 23 = 87

2. Find the sums.

A	B	C	D	E	F
75 + 16 = 81	38 + 25 = 63	67 + 17 = 84	67 + 57 = 124	84 + 92 = 176	84 + 98 = 182

3. Find the differences.

A	B	C	D	E	F
52 − 16 = 36	52 − 36 = 16	73 − 18 = 55	73 − 38 = 35	126 − 53 = 73	126 − 59 = 67

4. Find the sums and differences.

A	B	C	D	E	F
75 + 83 = 158	67 + 54 = 121	56 − 9 = 47	70 − 26 = 44	78 + 65 = 143	156 − 88 = 68

G	H
132 − 54 = 78	324 + 247 = 571

I	J
384 + 247 = 631	432 − 154 = 278

5. Find the sums.

A	B
21 + 25 + 23 = 69	34 + 23 + 18 = 75

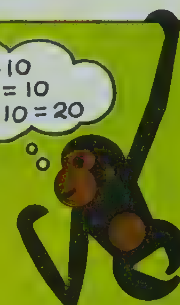
★ **6.** Find the differences.

A	B
602 − 24 = 578	500 − 34 = 466

116

think

2 + 8 = 10
1 + 9 = 10
10 + 10 = 20



Find each sum quickly without pencil and paper.

1. 1 + 5 + 9 15
2. 1 + 2 + 5 + 8 + 9 25
3. 1 + 2 + 3 + 5 + 7 + 8 + 9 35
4. 1 + 50 + 99 150
5. 1 + 2 + 50 + 98 + 99 250
6. 1 + 2 + 3 + 50 + 97 + 98 + 99 350

Discussion

How you treat these pages will depend on the needs of your class. If your class as a whole has understood regrouping and can work successfully on 2-digit addition and subtraction, you might use page 116 to time their skill, or you might assign only the exercises which involve 3-digit numerals and the word problems on page 117. If your class would benefit from further development of regrouping problems, choose several of the exercises and have children present their solutions step by step for the rest of the class.

The children would also benefit from an explanation of the patterns developed in the *Think* problem:

1 + 5 + 9

1 + 2 + 5 + 8 + 9

1 + 2 + 3 + 5 + 7 + 8 + 9

When discussing the story problems on page 117, use the approach mentioned in the discussion section on page 110; that is, in exercise 1 you might ask, "What information has been given in the problem?" (the prices of two items, 45¢ and 15¢); "What am I being asked to find?" (the total of both prices; you might point out here that another

Solving Short Story Problems

1 45 cents for a sandwich.
15 cents for milk.
How much for both? 60¢

2 Had a dollar.
Spent 49 cents.
How much left? 51¢

3 Weighed 58 kilograms.
Gained 14 kilograms.
Weigh how much now? 72 kg

4 52 marbles. 35
17 are taken away.
How many are left?

5 43 cookies.
28 ice cream bars.
How many more cookies than ice cream bars? 15

6 84 cents for a game. 56 cents for a puzzle. 28¢
How much less for the puzzle than for the game?

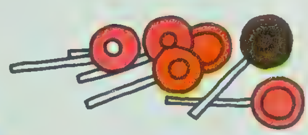
7 14 apples in a sack.
More are put in.
32 apples in the sack now.
How many were put in? 18

8 Lunch period: 45 minutes.
Recess: 25 minutes.
How much longer for lunch? 20 min

9 Room A: 32 children.
Room B: 27 children.
How many more in Room A than in B? How many less in Room B than in A? 5, 5

10 65 cars. 19 are black. How many are not black? 46

11 60 eggs.
One dozen are broken.
How many are not broken? 48



12 45 doughnuts.
27 of them are eaten.
How many are left? 18

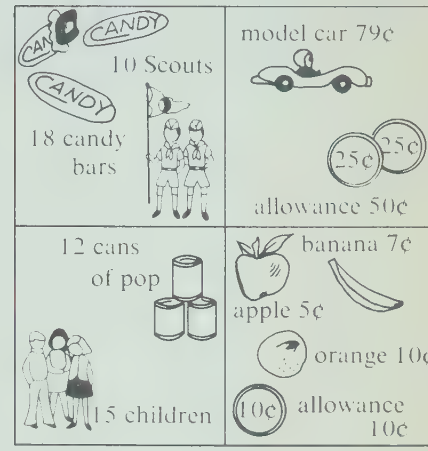
★ **13** 25 children.
34 lollipops.
Each child gets at least one lollipop. How many children can have two? 9

More practice, page A-16, Set 22

question *could* be given, such as "What is the difference in the cost of the two items?); "What must I do—add or subtract?" (since we are asked to find the price of both, we add: $45 + 15 = ?$); and finally, "Does my answer make sense?" (reread the problem and see if 60¢ would be reasonable; if someone subtracted, he would notice here that his cost for both items was less than for the sandwich by itself and so his answer could not be correct). You need not belabor this process, for it is a very natural way of approaching a problem and should be stressed only with children who have difficulty with word problems. Note that in exercise 11 the child must recall that one dozen equals twelve.

Assignments (page 117)
Minimum: Even-numbered problems. Average: 1–12. Maximum: 1–13.

Follow-up
To give children more practice with problem solving, you might develop a worksheet like the one pictured below. Encourage the children to write a short story for each picture and to solve their own problems. Then suggest that they make up problems on their own and, if they want to, draw pictures for them.



Duplicator Masters, pages 32–34
Skill Masters, pages 32-33

PAGES 118–119

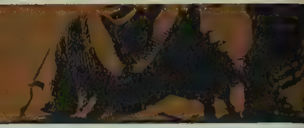
Objective

The child will demonstrate his ability to work with the concepts presented in this chapter.

Preparation

The preparation for this review lesson should vary according to the needs of your class. If the children have had difficulty with a particular concept, spend a few minutes reviewing it.

Reviewing the Ideas



1. Find the sums and the differences.

A	B	C	D	E	F
23	78	56	60	57	82
+51	−24	+13	+16	−36	−32
74	54	69	76	21	50

G	H	I	J	K	L
27	62	48	75	43	51
+54	−24	+36	+15	−15	−27
81	38	84	90	28	24

M	N	O	P	Q	R
67	86	70	38	39	81
+27	−67	−26	+57	+48	−44
94	19	44	95	87	37

2. Find the sums.

A	B	C	D	E	F
23	31	42	30	32	38
16	26	14	50	57	24
+50	+12	+25	+20	+26	+10
89	69	81	100	115	72

3. Find the sums and differences.

A	B	C	D	E	F
78	120	95	137	87	153
+49	−57	+36	−68	+76	−75
127	63	131	69	163	78

4. Find the differences.

A	B
326	9782
−114	−1542
212	8240

C	D
3641	4372
−1216	−144
2425	4228

★ **5.** Find the differences.

A	B
1682	8002
−427	−643
1255	7359

think

Diane has 48 cents.

1. What coins does she have if she has 9 coins? 3 dimes, 3 nickels, 3 pennies
2. What is the fewest number of coins she could have? 6
3. What is the fewest number of coins she could have and still have at least one of each coin pictured? 7

118

Discussion

Pages 118 and 119 may both be used as evaluation or as review material. If you have used the method of evaluation discussed in the introductory remarks for this chapter, you may use page 118 to evaluate the children's speed skills. Or you may use the pages as review and work through several exercises with the children. If starred exercise 5B gives those who try it difficulty, suggest that they think of 8002 as 799 tens and 12. (7990 + 12) and write

$$\begin{array}{r} 799\,12 \\ \cancel{8002} \\ -643 \\ \hline \end{array}$$

This direct regrouping of tens, hundreds, and thousands in one step will be developed more fully in a subsequent grade level.

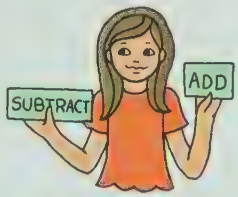
Exercise 7 gives children an opportunity to think about the operation involved without having to demonstrate their skill with an algorithm. Again, you might use these problems as a review, working through some together, or you might assign them as independent work and evaluate each child's ability to think through word problems.

After the children have completed these pages, allow time for discussion.

6. Find the amounts.

A	B	C	D	E	F
$1.25	$3.84	$1.84	$6.40	$7.04	$5.00
+2.41	−1.22	+2.09	−1.25	+1.88	−4.98
$3.66	$2.62	$3.93	$5.15	$8.92	$ .02

7. In these exercises, no numbers are given. You should decide whether you would add or subtract if numbers were given. Answer **A** if you would **add** to answer the question. Answer **S** if you would **subtract**. Think carefully before giving your answer.



A Joe had ▯ marbles. He gave ▯ to Tom. How many did he have left? S

B Jane counted ▯ cars on one train and ▯ cars on another. How many did she count in all? A

C Sue had ▯ cents. She spent ▯ cents for an apple. How much did she have left? S

D Tom is ▯ years old, and Jim is ▯. How much younger is Jim than Tom? S

E Susan spent ▯ cents for a sandwich and ▯ cents for milk. How much did she spend? A

F In Jane's class there are ▯ children. There are ▯ boys in the class. How many girls are in Jane's class? S

G Tim checked ▯ books out of the library. Later he returned ▯ of the books. How many did he still have? S

H Beth and Sara have ▯ dolls in all. ▯ of the dolls are Sara's. How many of the dolls are Beth's? S

I Jack read ▯ pages in his book. He has ▯ pages yet to read. How many pages are in the book? A

J Ann said, "▯ years ago, I was ▯ years old." How old is Ann now? A

Follow-up

You might suggest to the more capable children that they extend exercise 7 by substituting numbers for the placeholders. Then they may solve their own problems, or they may exchange papers with a classmate and solve the problems using the numbers he has substituted.

Resources for Active Learning

Nuffield Project; *Computation and Structure* 3, p. 8, Wiley. [A Cross Number Puzzle assignment card]

Workbook, page 44

PAGES 120–121

Objective

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

You might ask the children to suggest a review game or activity of their choice. However, help them choose one which does not require a great length of time. Alternatively, you may begin immediately with the exercises in the text.

Keeping in Touch with Addition, Subtraction, Place value, Inequalities

1. Four of these statements are false. Which ones are they? A, C, F, H

A $15 - 5 = 5$
B $26 + 7 > 30$
C $34 - 8 > 30$
D $39\,427 > 38\,998$
E $48 - 9 < 40$
F $48 - 19 > 30$
G $67 + 8 = 70 + 5$
H $48\,265 = 48\,165 + 1000$
I $528 = 500 + 20 + 8$

2. Write the correct numeral for each ▮.

A For 6 tens and 2, we write ▮. 62
B For 8 tens and 7, we write ▮. 87
C For 2 tens and 5, we write ▮. 25
D For 5 tens and 2, we write ▮. 52
E For 7 tens and 0, we write ▮. 70
F For 1 ten and 0, we write ▮. 10

3. Solve the equations.

A $6 + 5 = n$ 11
B $4 + 3 = n$ 7
C $8 + 7 = n$ 15
D $9 + 2 = n$ 11
E $6 + 4 = n$ 10
F $8 + 8 = n$ 16
G $n + 6 = 8$ 2
H $n + 4 = 11$ 7
I $n + 9 = 10$ 1
J $n + 8 = 12$ 4
K $n + 6 = 6$ 0
L $n + 5 = 10$ 5
M $14 - 5 = n$ 9
N $9 - 6 = n$ 3
O $12 - 5 = n$ 7
P $11 - 4 = n$ 7
Q $15 - 7 = n$ 8
R $18 - 9 = n$ 9
S $n + 6 = 8$ 2
T $4 + n = 9$ 5
U $6 + 8 = n$ 14
V $n - 2 = 4$ 6
W $9 - n = 4$ 5
X $15 - 5 = n$ 10

4. Copy each column and complete the counting.

A	B	C	D	E
6	16	86	96	436
7	17	87	97	437
8	18	88	98	438
▮ 9	▮ 19	89	▮ 99	439
▮ 10	▮ 20	▮ 90	▮ 100	▮ 440
▮ 11	▮ 21	▮ 91	▮ 101	▮ 441
▮ 12	22	▮ 92	▮ 102	▮ 442

Discussion

The exercises provided on these pages are concerned more with understanding than with speed. Review the inequality symbols before assigning exercise 1. Similarly, before assigning exercise 7, demonstrate several reconstruction problems, such as:

$$\begin{array}{r} 2▮ \\ -\,19 \\ \hline 4 \end{array}$$

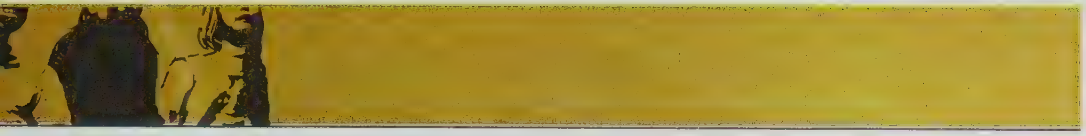
If children recall the check for subtraction or if they simply understand the role of each numeral in subtraction, they will realize that they can find the answer by adding: $4 + 19 = 23$.

In an exercise such as

$$\begin{array}{r} 58▮ \\ +\,▮55 \\ \hline 9▮4 \end{array}$$

the children might begin by examining the ones' place. Here they should realize that 9 is the only digit which when added to 5 will result in a numeral ending in 4. This immediately determines the middle digit in the sum. Continuing to regroup leaves only a missing addend to find, 3.

The *Think* problem is intended primarily as enrichment. Since this exercise is quite difficult to visualize, you might have some of the children paint shells (or drawings



5. Solve each equation.

A $9 + 1 = n$ 10 B $99 + 1 = n$ 100 C $999 + 1 = n$ 1000 D $9999 + 1 = n$ 10 000

6. Give the correct sign < or > for each ▒.

A 26 ▒ 28 <	I 238 ▒ 228 >	Q 672 ▒ 658 >
B 94 ▒ 90 >	J 654 ▒ 664 <	R 180 ▒ 159 >
C 87 ▒ 77 >	K 275 ▒ 272 >	S 338 ▒ 242 >
D 59 ▒ 99 <	L 542 ▒ 546 <	T 665 ▒ 571 >
E 127 ▒ 147 <	M 556 ▒ 561 <	U 862 ▒ 955 <
F 152 ▒ 122 >	N 474 ▒ 439 >	V 347 ▒ 274 >
G 219 ▒ 209 >	O 621 ▒ 612 >	W 631 ▒ 713 <
H 257 ▒ 277 <	P 389 ▒ 398 <	X 865 ▒ 568 >

7. Copy the problems and give the missing digits.

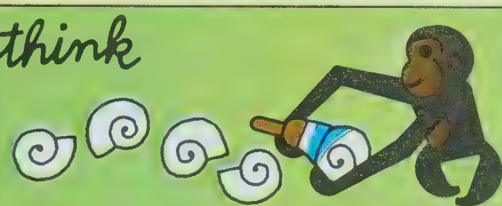
A 1▒7 − 6 = 11 B 26 + ▒8 = 34

C 2▒6 + 19 = 4▒5 D 6▒4 + 2▒7 = 91 (2, 4)

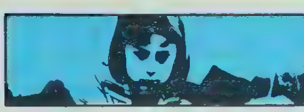
E 72 − 3▒4 = 3▒8 F 6▒▒3 − 47 = 16

G 43▒5 + 2▒36 = 6▒98 H 48▒9 + 4▒54 = 9▒34

think



Nan picked out 5 white shells. She put some blue paint on 3 of them. Then she could see white on only 3 of the 5 shells. On how many could she see both blue and white? 1



You are invited to explore

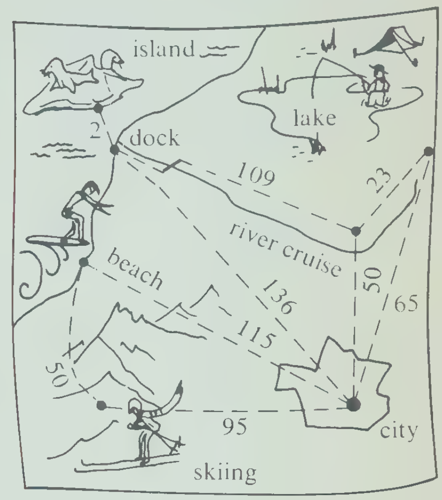
ACTIVITY CARD 5
Page 311

of shells) in the described manner. An actual demonstration will certainly be necessary for general understanding if those who solve the problem present it to the class.

You may wish to give the children two days to do these pages somewhat thoroughly, so you can spot areas of weakness that should be retaught. Do any necessary reteaching of place value, inequalities or 2-digit addition and subtraction after these pages have been completed.

Follow-up

The children might be stimulated to make up their own word problems about where to go on a vacation, what side-trips to take, and so on, if you put this or a similar map on the chalkboard or bulletin board.



Also try to incorporate the use of addition and subtraction of 2- and 3-digit numerals into any project on which the class is working. For instance, if the class is studying bridges, or aviation, or trees, encourage them to include numbers in their research findings; then help them formulate questions or make up problems including these numbers.

CHAPTER 6 | Multiplication

Pages 122–169

General Objectives

To introduce the concept of multiplication
To associate products with groups of equivalent sets
To associate products with product sets
To associate multiplication with repeated addition
To associate multiplication with the number line and skip counting
To provide word problems in multiplication
To work toward mastery of multiplication facts
To introduce some basic multiplication principles
To introduce an element of logic in finding products

Although the emphasis in Chapter 6 is on the concept of multiplication, it is likely that by the time they complete the chapter many of the children will have attained a fair mastery of multiplication facts. Several interpretations of multiplication are presented to provide a broad perspective that will help strengthen overall understanding of multiplication concepts, as well as give more tools for finding products.

The early part of the chapter develops the following topics: multiplication and equivalent sets, multiplication and the number line, multiplication and rectangular arrays, multiplication and repeated addition.

Later, basic principles for multiplication—the order (commutative), grouping (associative), zero, and one principles—are explored, followed by the multiplication-addition (distributive) principle.

The lessons which introduce multiplication facts by means of the multiplication table illustrate how we use previously developed concepts to help build mastery of the basic multiplication facts. We recognize the necessity for a certain amount of memorization; children must *eventually* learn the facts. However, by combining facts already learned with the basic principles, we have minimized the total amount of memorization that is required for the children's work at this level.

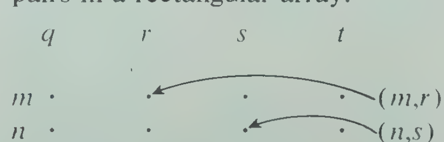
The remainder of the chapter develops the relationship between product sets and rectangular arrays, introduces missing factors, and provides a variety of story problems involving multiplication processes.

Mathematics

We begin the study of multiplication by defining an operation on sets. Consider two sets, $C = \{q, r, s, t\}$ and $D = \{m, n\}$. From these sets, we form pairs of elements such that the first member of the pair is from set C and the second from set D. That is, each of the four elements of set C is paired in turn with each of the two elements of set D. This results in a new set of eight pairs, known as the *Cartesian product* of sets C and D (written $C \times D$ and read "C cross D"). The elements of $C \times D$ are the following pairs:

(m, q) (m, r) (m, s) (m, t)
(n, q) (n, r) (n, s) (n, t)

The next figure describes these pairs in a rectangular array.



Careful examination of this array will reveal how it is related to the multiplication concept.

The Cartesian product $C \times D$ has 8 elements (pairs); 8 is the product of 4 and 2. The numbers 4 and 2 are called the *factors* of 8, and 8 is a *multiple* of each of these factors.

The products $C \times D$ and $D \times C$ have the same number of pairs. By changing the order of the members in each pair of $C \times D$, we obtain the product $D \times C$. This illustrates the *order* principle for multiplication. We demonstrate this principle for children by having them think about rectangular arrays, first by rows and then by columns.

Teaching the Chapter

Materials

Apron
Boxes
Centimetre rulers
Centimetre strips
Counters (buttons, chips, beans, paper discs, etc.)
Crayons
Duplicated multiplication tables, blank (1 per child)
Felt objects
Flannelboard
Gift-wrapped boxes (3)
Gloves
Graph paper
Index cards, large size (2 per child)
Index cards, small size (3 per child). (Slips of paper may be substituted for these.)
Objects to demonstrate product sets (different colored cups and saucers, colored paper cutouts, and so on)

Vocabulary

combination
factor
function machine
function rule
grouping principle for multiplication
input
multiplication
multiplication-addition principle
order principle for multiplication
output
parentheses
product
skip counting
times

To study equivalent sets and rectangular arrays, the children manipulate counters and use strips to

form rows and columns. To demonstrate these concepts in discussion, you can use diagrams on the chalkboard or overhead projector as well as felt materials available for demonstrating product sets; for example, two cups and three saucers, all of different colors. (To illustrate the product 2×3, you could demonstrate the color combinations that can be obtained by putting each cup with the three saucers one at a time, recording the color combinations on the chalkboard to show that there are six different combinations in all; thus, $2 \times 3 = 6$.)

In developing the vocabulary for this chapter, pay particular attention to the word *factor*. Emphasize the idea of a missing factor, since this idea is used very often in teaching division concepts.

It may not be reasonable to expect every child in an average class to master all the multiplication facts by the end of this chapter. Those who do not will be given additional opportunities throughout the book to increase their skill. Most children will have an adequate command of multiplication facts when they reach Chapter 8, in which the multiplication algorithm is presented. It is essential that they be familiar with these facts when they study the division algorithm in Chapter 9. Again, we recognize that some of the slower children will not attain these goals, but they will gain much from an exposure to the ideas which rely on the multiplication facts.

You may wish to provide additional drill for children who do not master the multiplication facts. *Later in the book, it may also be necessary to provide these students with a multiplication table for reference while they work the more difficult exercises. Mastery of the multiplication facts through* 9×9 *should not be expected of many third graders.*

Lesson Schedule

Plan to cover this material in about four-and-a-half weeks. You may need to adjust this time schedule slightly, depending on the background of the children.

Evaluation of Progress

To determine whether each child understands how multiplication relates to the various concepts developed in the chapter, you should carefully watch his participation in and contribution to the lessons.

For a thorough evaluation of the children's progress toward mastery of the multiplication combinations use the interview technique: throughout the chapter, ask each child individually to think through scaled multiplication equations aloud for you. Carefully observing the speed with which the child arrives at products will enable you to determine whether he really knows the combinations. Some children may be able to find a product, such as 6×7, by using repeated addition, and this is fine, but it does not prove mastery of the combination. Children who can do the repeated addition but have not mastered the combination should not be criticized; it is important that they have some method for finding products. However, they should be encouraged to work toward mastery of the combinations so that they need not continue to rely on slower methods.

After we introduce product sets, we associate them with rectangular arrays. For the children who do not see this relationship, emphasize that they can think about product sets in much the same way as they think about rectangular arrays. When children become more knowledgeable, the relationship between product sets and rectangular arrays will become apparent.

Pages 166 and 167 contain a chapter review which will help you evaluate the children's understanding of concepts and principles of multiplication. Pages 168 and 169 provide a cumulative review of material covered so far in Book 3.

Resources for Active Learning

General Activities

[You will be able to use many of the activities listed here again in Chapter 7 (Division), so it would be a good idea to keep them handy.]

Mathex: Operations No. 3, "Multiplication and Division," pp. 28–38 (pupil pages 39–45), Encyclopaedia Britannica Publications Ltd.

Mathex: Operations and Problem Solving No. 8, "Understanding Multiplication," pp. 8–12, Encyclopaedia Britannica Publications Ltd.

Math Workshop: Games and Enrichment Activities, "Multiplication and Division," pp. 62–69, 74–79, Encyclopaedia Britannica Educational Corp.

Nuffield Project: *Computation and Structure* 3, "Multiplication," pp. 24–55, Wiley

Nuffield Project: *Mathematics Begins* 1, "Toward Multiplication," pp. 55–57, Wiley

Manipulative Devices

Cuisenaire Cubes, Squares, and Rods (Cuisenaire Co.)

Dienes Multibase Arithmetic Blocks (Herder and Herder)

"Invicta" Math Balance (Math Media; Selective Educational Equipment)

Pegboards (school supplier)

SEE Calculator (Selective Educational Equipment)

Sigma Chips (Sigma, Scott Scientific)

Unifix Math Lab Kit (Educational Teaching Aids; Math Media; Responsive Environments Corp.)

Commercial Games

Kalah (Creative Publications; World Wide Games)

Quinto (Hammett; Selective Educational Equipment)

PAGES 122–123

Objective

Given groups of equivalent sets, such as four sets of 5 elements each, the child will be able to state a multiplication fact for the sets.

Preparation

Materials

paper cups (at least 5 per child); counters (about 40 per child, plus some extras) (Beans may be substituted if there are not enough counters.); boxes, if available (1 per child)

To introduce this lesson and the new concept of the chapter, you might write an addition and a subtraction symbol on the chalkboard (+, −) and then ask a volunteer to write an equation with each and read each with the class. Next, put the multiplication symbol (×) on the chalkboard and ask if anyone knows what it means. After writing the symbol in a multiplication equation and reading the equation with the class, direct the children's attention to the text.

Investigation

In this investigation, the child studies a group of equivalent sets and writes a multiplication equation to describe it. After the children have worked a few minutes with the cups and counters, explain that they must put the *same number* of counters in each cup. Some, of course, may realize this on their own. It would be helpful and conserve time if you provide the children with duplicated copies of the chart they are to fill in. The children could do this investigation in groups of two or three, but, if so, be sure that every child in each group actually does the activity and writes his own equations. You might challenge the more capable children to use a larger number of cups and counters and invent more "interesting" multiplication equations. Encourage children who finish quickly to work out more than just the suggested 4 equations.

6 Multiplication

- *What is multiplication?*

Investigating the Ideas

Get some paper cups, counters, and a collection box.

Number of cups	Number in each cup	Total number in product box	Multiplication equation
5	4	20	5×4=20

? Can you use different numbers of cups and counters and write 4 more multiplication equations ? See Investigation.

Discussing the Ideas

1. Give the missing numbers and explain how to write a multiplication equation.

We see — We think — We write

A

3 boxes

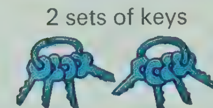
5 crayons in each box

3 fives are 15 ▯.

? 3 x 5 = 15

B

2 sets of keys



4 keys in each set

2 fours are 8 ▯.

? 2 x 4 = 8

2. Make up an example like those above. See Discussion.

122

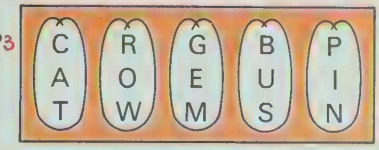
Discussion

Have a few volunteers display on the chalkboard one of the equations they wrote on their chart and then use the cups and counters to illustrate the equation. After this, display a few demonstration sets on the overhead projector or with sets of felt objects on the flannelboard. Group the objects into equivalent sets, ringing them for easy identification (as in the pictures on page 123). Accompany each set demonstration with the related multiplication equation.

During your demonstration and while discussing exercises 1 and 2, use phrases such as "three fives are fifteen" to help children associate multiplication with groups of equivalent sets. Later, begin saying "three *times* five" and explain to the children that this is how they read the expression "3 × 5." Emphasize that *times* helps them think about multiplication.

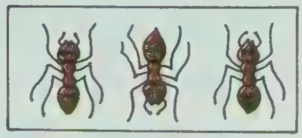
Using the Ideas

1. A How many words? 5
   B How many letters in each word? 3
   C How many letters in all? 15
   D Solve: $5 \times 3 = n$ 15

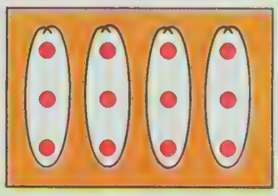


2. A How many pairs of shoes? 4
   B How many in each pair? 2
   C How many shoes in all? 8
   D Solve: $4 \times 2 = n$ 8

3. A How many ants? 3
   B How many legs on each ant? 6
   C How many legs in all? 18
   D Solve: $3 \times 6 = n$ 18

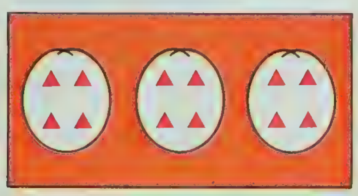


4. A How many sets? 4
   B How many dots in each set? 3
   C How many dots in all? 12
   D Solve: $4 \times 3 = n$ 12



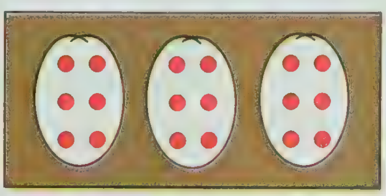
5. A How many nickels? 5
   B How many cents for each nickel? 5
   C How many cents in all? 25
   D Solve: $5 \times 5 = n$ 25

6. Solve the equation.



$3 \times 4 = n$ 12

7. Write a multiplication equation.



3 × 6 = 18

Using the Exercises
Ask the children to work the exercises on page 123 by themselves. Allow time for questions and discussion when everyone has finished. If the children did not encounter multiplication last year, however, you may prefer to use the exercises on both pages as the basis for class discussion, providing demonstration sets where needed.

Point out to the children that, when they are asked to "write a multiplication equation," as in exercise 7, they should show both the numbers they multiply and their answer (in this instance, $3 \times 6 = 18$).

Assignments (page 123)
Minimum: 1–6. Average: 1–7.
Maximum: 1–7.

Mathematics
We relate multiplication to groups of equivalent sets and use this relationship as a definition of multiplication, since it is easy for children to grasp. Then they can readily relate multiplication to repeated addition, skip counting, and number-line activities. Later in the chapter, we associate multiplication with product sets, and product sets with rectangular arrays.

Follow-up
Throughout this chapter, involve the children in as many projects using multiplication as possible. For example, spend some time having the class divide themselves into equivalent groups and write an equation on the chalkboard to illustrate each grouping. If you have 30 children in the class, tell them to divide into 2 groups, count how many are in each group, and write

$$2 \times 15 = 30.$$

Or have them form 3 groups, count each group, and write

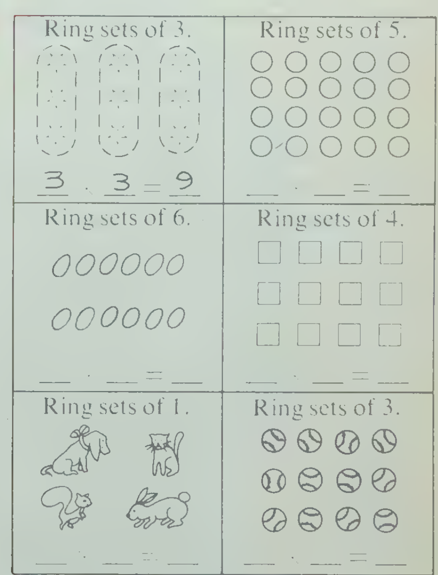
$$3 \times 10 = 30.$$

Or have them form 5 groups, count each group, and write

$$5 \times 6 = 30.$$

The point in such a project would be to stress writing a multiplication equation to express the total number in all the sets.

If you prefer, duplicate a worksheet with a different set in each section, similar to the one shown.



Workbook, page 45

PAGES 124–125

Objective

Given the colored strips and a number line, the child will be able to find products and write related multiplication equations.

Preparation

Materials

colored strips (1 set per child); centimetre rulers (1 per child)

Although the children have used the colored centimetre strips previously, it would be helpful to review the number-name of each strip. You might also review using the edge of a ruler as a number line.

Investigation

In this investigation, children use the strips and the number line (ruler edge) to write and solve multiplication equations. Briefly discuss the pictured strips and ruler edge and the corresponding equation as a model of what to do. After the children have had sufficient time to perform the illustrated activities and to discuss how their answers show on the number line after their movement of the strips, have them work independently to find other equations.

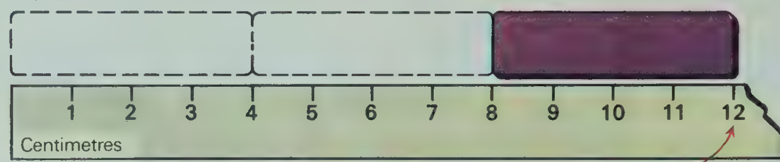
You may find that it is easier for the children to place several strips of the same color end to end above the ruler, rather than moving a single strip along the ruler. Remind the children to record their equations including the answer. If any children seem to need help in getting started, you might ask them to show you what strips they would use and what the solution would be for an equation such as $4 \times 5 = ?$. If the child is capable, help him understand that the number of times the strip is to be laid down (or the number of like-colored strips laid end to end) is expressed by the first number in the equation and that the length of the strip to be used is expressed by the second number.

After the children have had enough time to complete the activity and discuss their equations, direct attention to the discussion questions in their book.

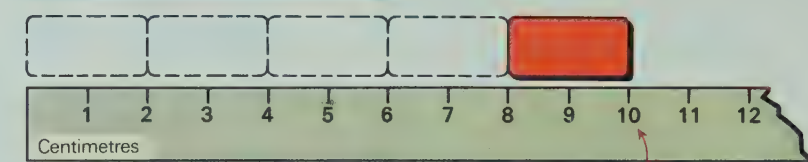
Can the number line help you think about multiplication?

Investigating the Ideas

You can use your strips and a centimetre ruler to help you think about multiplication.



Put down the 4-strip 3 times. $3 \times 4 =$? 12

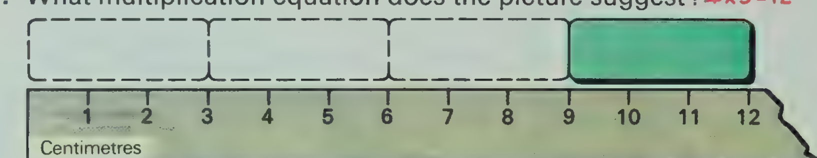


Put down the 2-strip 5 times. $5 \times 2 =$? 10

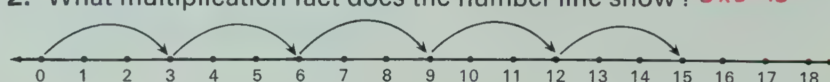
? Can you use your strips and a ruler and write at least six more multiplication equations? See Investigation.

Discussing the Ideas

1. What multiplication equation does the picture suggest? $4 \times 3 = 12$



2. What multiplication fact does the number line show? $5 \times 3 = 15$



Discussion

Read the questions and discuss the suggested multiplication facts with the class. Then it would be helpful for you to show a number line on the overhead projector, or on the chalkboard, and discuss jumps shown first with strips and then with arrows. This would also be an appropriate time to show children how to use a centimetre ruler to draw a number line, as they will be asked to do in the next section of the lesson.

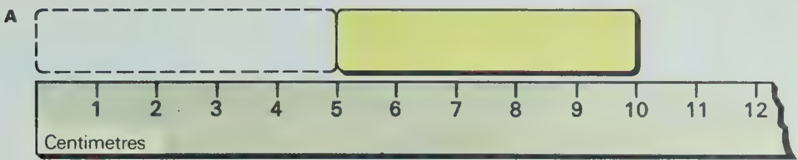
Before assigning the exercises on page 126, you might wish to ask for volunteers to put on the chalkboard some of the equations they found in the investigation and then show how those equations could be illustrated by jumps on the number line.

Throughout the entire discussion stress the association of the number-line picture (using strips or arrows) to the related multiplication equation.

Using the Ideas

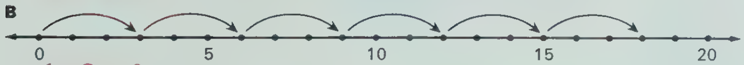
1. Write a multiplication equation for each picture.

A



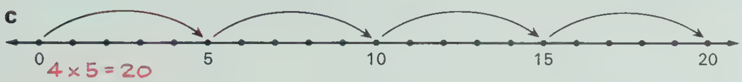
2 × 5 = 10

B



6 × 3 = 18

C

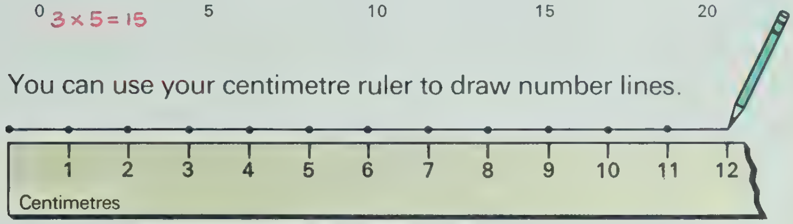


4 × 5 = 20

D

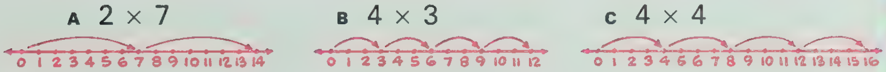
3 × 5 = 15

2. You can use your centimetre ruler to draw number lines.



Draw 3 number lines. Use them to show these multiplications.

A 2 × 7 **B** 4 × 3 **C** 4 × 4



★ **3.** Find the missing numbers.

A 2, 4, 6, ▥, ▥, ▥, ▥, 16 8, 10, 12, 14

B 3, 6, 9, ▥, ▥, ▥, ▥, 24 12, 15, 18, 21

C 4, 8, 12, ▥, ▥, ▥, ▥, 32 16, 20, 24, 28

D 5, 10, 15, ▥, ▥, ▥, ▥, 40 20, 25, 30, 35

E 6, 12, 18, ▥, ▥, ▥, ▥, 48

F 7, 14, 21, ▥, ▥, ▥, ▥, 56

G 8, 16, 24, ▥, ▥, ▥, ▥, 64

H 9, 18, 27, ▥, ▥, ▥, ▥, 72

E 24, 30, 36, 42
F 28, 35, 42, 49
G 32, 40, 48, 56
H 36, 45, 54, 63

125

Using the Exercises

After you have helped the class draw the number lines to be used in exercise 2, assign all of the exercises as independent work.

Exercise 3 is designed primarily for the faster children. When they have completed it, call attention to the related multiplication facts they can find by skip counting. For instance, in exercise 3A, they can think of 8 × 2 = 16. Ask for volunteers to suggest some of the many other products that they can find by counting the number of skips. (In exercise 3B, for example; they can think of 8 × 3 = 24 or 7 × 3 = 21.)

Assignments (page 125)
Minimum: 1–2. Average: 1–2.
Maximum: 1–3.

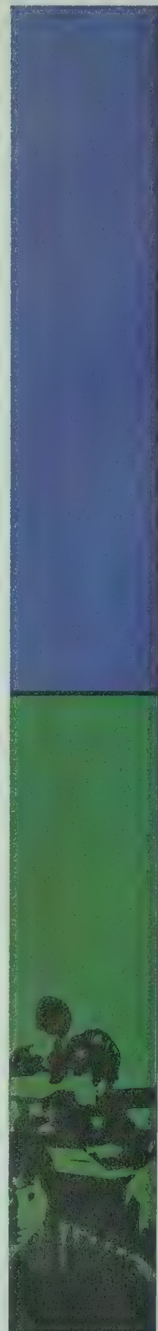
Follow-up

The children would benefit from oral games based on skip counting. For instance, you could give two numbers in a sequence and have them respond with the next two, ("Five, ten . . ." – ". . . fifteen, twenty").

A similar game the children would enjoy is the "Buzz" Game. In this game, children count whole numbers consecutively but for every third (or fifth, or second) number the child counting substitutes the word "Buzz." For example, if the pattern is every fourth number, the counting would be 1, 2, 3, Buzz, 5, 6, 7, Buzz, 9, 10, 11, Buzz, and so on. You may have someone write on the chalkboard every number for which Buzz was used, and the result will be a list of the multiples of four. Also, you might distribute worksheets on which children are asked to complete patterns or skip counting, as in the sample below. The children might like to try making up skip countings of their own and picture them with arrows on a number line.

Find the patterns. Fill in the blanks.
2, 4, ____, 8, 10, ____, 14, 16, ____, 20, 22.
5, ____, 15, 20, ____, ____, 35, ____, 45.
3, ____, ____, 12, 15, 18, ____, ____, 27, 30.
20, 18, 16, ____, ____, ____, 8, 6, ____, ____.
____, 12, 18, 24, ____, 36, ____, 48, ____, 60.
7, ____, 21, ____, 35, ____, 49, ____, 63.

Duplicator Masters, page 35
Workbook, page 46
Skill Masters, page 35

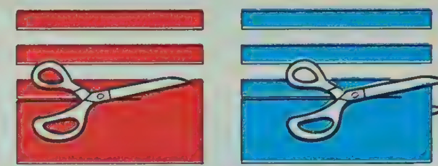


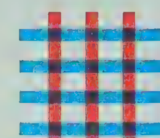
● *Let's think about multiplication in another way.*

Investigating the Ideas

Cut five thin red strips and five thin blue strips.

Now make a table like the one below.



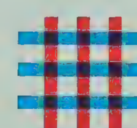


Number of red strips	Number of blue strips	Number of crosses +	Multiplication equation
3	4	12	$3 \times 4 = 12$
5	3	? 15	$5 \times 3 = \ ?$ 15

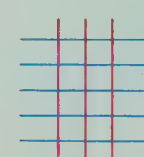
? Can you use your strips to help you write other multiplication equations? See Investigation.

Discussing the Ideas

1. Show how you would use your strips to solve this equation. See Discussion. $4 \times 2 = n$ 8



2. What equation can you solve by laying the strips down like this? Show how to do this. $3 \times 3 = 9$



3. You could draw lines instead of using your strips. What equation can you solve by counting crosses on these lines? $3 \times 5 = 15$

4. What are the largest numbers you could multiply using your strips? 5×5

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PAGES 126–127

Objective

Given a rectangular array, the child will be able to associate it with multiplication by writing an appropriate multiplication equation.

Preparation

Materials

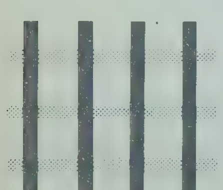
crayons; 15-by-20-cm cards or 15-by-20-cm pieces of red and blue construction paper; flannelboard; red and blue demonstration strips

Since the investigation requires specific materials which the children can help make, you might begin immediately with the investigation.

Investigation

If you are using 15-by-20-cm cards, have each child color one blue and another red, and then use the lines on the card as guides for cutting five strips from each card. If you prefer, you might have the strips cut before class and distribute five of each color to every child. Also, you might wish to duplicate copies of the chart rather than have the children try to copy from the book.

It would be helpful to read the directions with the class and make sure that they understand what to do. (You may need to point out what is meant by "number of crosses.") The colored numerals are to be associated with the color of the strips; however, if a child writes 3×4 for the following arrangement, he is still correct:



Although the emphasis should be on the number of crosses related to the number of red and blue strips rather than on the relative positions of the red strips and the blue strips, the text consistently relates the red vertical strips to the first number in the equation. Also, the child should be careful that all red strips are laid in the same direction and the blue strips in the other direction. Otherwise, the number of crosses suggested by the table is not valid. If a child is not well co-ordinated and has difficulty in using the strips, you might have the child use a notched box and straws.

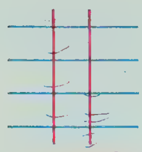
Discussion

After the children have had time to write a half dozen or so multiplication equations, ask for volunteers to display on the flannelboard an arrangement of strips and then to write a corresponding equation on the chalkboard. In discussing exercise 1, associate the red 4 with 4 red strips and the blue 2 with 2 blue strips, but allow the children

Using the Ideas

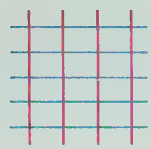
1. Write and solve a multiplication equation for each picture.

A



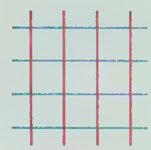
2×4=8

B



4×5=20

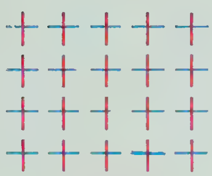
C



4×4=16

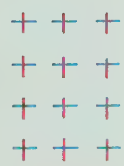
Parts of the lines have been erased in exercises D, E, and F, but you can still see the crosses.

D



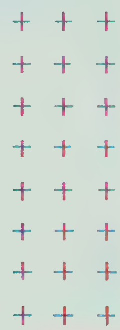
5×4=20

E



3×4=12

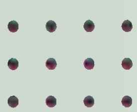
F



3×8=24

Only the dots where the lines cross are left in exercises G, H, and I, but you can still tell how many lines and how many crosses.

G

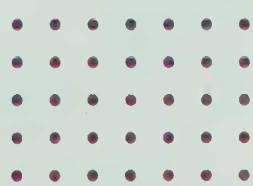


4×3=12

H

6×4=24

I



7×5=35

2. Use strips, lines, or dots and solve these equations.

A $2 \times 6 = ?$ 12 B $4 \times 5 = ?$ 20 C $3 \times 7 = ?$ 21 D $8 \times 3 = ?$ 24

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to follow their own preference in deciding whether to position the red strips vertically and the blue strips horizontally or vice versa.

As the children respond to exercise 2, you might expect them to respond "3 down, 3 across" or "3 × 3 is 9." To check their thinking, they could count the number of crosses. It would be helpful to have students draw lines for other equations that you write on the chalkboard. If the children have 5 strips of each color, they should realize that a 5-by-5 arrangement is the largest they could use (exercise 4).

Using the Exercises

Since each child might be ready to work independently of concrete materials at a different time, allow any who still need the physical objects to use the strips as long as necessary. However, since this lesson progresses from work with physical strips to written lines, to arrangements of crosses, and finally to arrays of dots, encourage all who can to do the exercises on page 127 without using the strips.

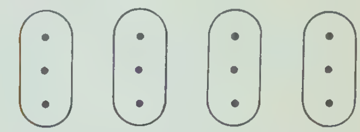
Work through parts of exercise 1 with the class so that you can point out the progression from lines to dots. You might extend exercise 2 by writing other equations for the children to solve.

Assignments (page 127)

Minimum: 1. Average: 1–2. Maximum: 1–2.

Mathematics

Using an array of dots enables us to present multiplication to children in terms of something already familiar to them, namely addition. For example, we can introduce 4×3 as 4 threes, or $3 + 3 + 3 + 3$. By showing them a dot array like that in the following figure, we can help them to think of 4×3 in terms of 4 sets of 3.



In this way the children "see" multiplication in terms of addition. At the same time, they become aware of product sets, although their working definition for multiplication centres on the idea of dot arrays and repeated addition. Notice that the number of rows and the number of columns of a dot array can be thought of as factors of the *product*, which is the total number of dots in the array.

Follow-up

Each child might like to make a poster illustrating a multiplication equation by gluing colored strips in place or by drawing red and blue lines.

More capable children would benefit by extending the investigation to include 9 blue and 9 red strips and recording the multiplication equations they investigate.

Resources for Active Learning

Mathex: Numeration No. 2, "Arrays," pp. 41–42, Encyclopaedia Britannica Publications Ltd.

Mathex: Operations and Problem Solving No. 8, "Arrays," pp. 13–15, Encyclopaedia Britannica Publications Ltd.

PAGES 128–129

Objective

Given groups of equivalent sets, the child will be able to write both an addition equation and a multiplication equation to describe them, thus showing multiplication as repeated addition.

Preparation

It would be appropriate to begin immediately with the investigation. However, if you prefer, use a few rounds of the Buzz game as a short warm-up activity (see page 125).

Investigation

In the preceding lessons of this chapter, the children learned to think about multiplication in terms of equivalent sets, rectangular arrays, and the number line. This investigation utilizes those now familiar devices to demonstrate that multiplication may be thought of as repeated addition.

Read the investigation directions with the children. Allow the children to work in groups of two or three, but make sure each child writes the multiplication equations himself, after he discusses them with members of his group. It would be helpful to put other pictures on the chalkboard for those who finish quickly. For example:

xx xx xx
xx xx xx

$4 + 4 + 4 = ?$ $(3 \times 4 = 12)$

$2 + 2 + 2 + 2 = ?$ $(4 \times 2 = 8)$

$3 + 3 + 3 + 3 + 3 = ?$ $(5 \times 3 = 15)$

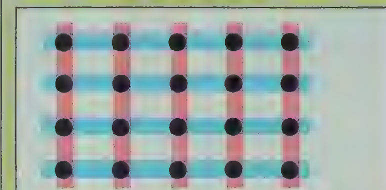
How are addition and multiplication related?

Investigating the Ideas

Find the sum for each picture.

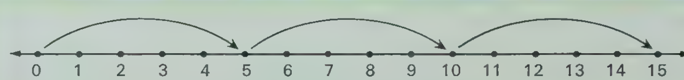
$3 + 3 + 3 + 3 = ?$ 12

4 × 3 = 12



$4 + 4 + 4 + 4 + 4 = ?$ 20

5 × 4 = 20



$5 + 5 + 5 = ?$ 15

3 × 5 = 15

? Can you write a multiplication equation for each picture? See above.

Discussing the Ideas

1. Carol said, "Multiplication is just a short way to do addition." What do you think she meant? See Discussion.

2. A Can you make up an example using sets to show $6 + 6 + 6$? What multiplication equation goes with this? 3 × 6 = 18

   B Can you make up an example using dots to show $4 + 4$? What multiplication equation goes with this? 2 × 4 = 8

   C Can you make up an example using the number line to show $3 + 3 + 3 + 3 + 3$? What multiplication equation goes with this? 5 × 3 = 15

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Discussion

As you discuss exercise 1, it would be helpful to exhibit rectangular arrays on the chalkboard or flannelboard and show the repeated addition equations associated with these arrays. For example:

★ ★ ★ ★ ★
★ ★ ★ ★ ★
★ ★ ★ ★ ★

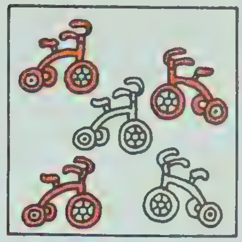
$3 + 3 + 3 + 3 + 3$

The children should see that adding 5 threes is the same as multiplying 5 times 3. Have them count the items in a few arrays and then help them conclude that the multiplication approach to repeated addition is quicker than counting. Also emphasize that they can use repeated addition to figure out multiplication facts that they do not know. For example, if a child does not know 4×5, he may simply add 4 fives.

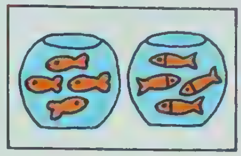
The children may draw upon the ideas suggested by the illustrations in the investigation to make up the examples called for in each part of exercise 2. If feasible, allow them to use physical objects to illustrate the examples they invent.

Exhibit other examples with the number line and groups of equivalent sets and ask the children to write the related addition and multiplication equations for each.

Using the Ideas

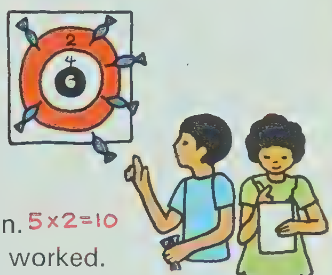


1. A How many tricycles? 5
   B How many wheels on each tricycle? 3
   C How many wheels in all? 15
   D Write an addition equation about the sets. 3+3+3+3+3 = 15
   E Write a multiplication equation about the sets. 5 × 3 = 15



2. A How many bowls? 2
   B How many fish in each bowl? 4
   C How many fish in all? 8
   D Write one addition and one multiplication equation about the sets.
   4 + 4 = 8 ; 2 × 4 = 8

3. Solve the equations.
   A $4 + 4 + 4 = n$ 12
   $3 \times 4 = n$ 12
   B $6 + 6 + 6 = n$ 18
   $3 \times 6 = n$ 18
   C $3 + 3 + 3 + 3 + 3 = n$ 15
   $5 \times 3 = n$ 15
   D $2 + 2 + 2 + 2 = n$ 8
   $4 \times 2 = n$ 8



4. A Ken made a dart board.
   The first time he threw 6 darts, the board looked like this.
   Nancy was scorekeeper. She wrote
   $2 + 2 + 2 + 2 + 2 = 10$.
   Ken found his score by multiplication. 5 × 2 = 10
   Give the multiplication problem Ken worked.
   B In one game Bob threw all 6 darts in the 4 ring.
   What was his score? 24
   ★ C Jane threw 3 darts. Her score was 14. Where did the darts land? 2 in 6 ring and 1 in 2 ring (6+6+2=14) or 2 in 4 ring and 1 in 6 ring (4+4+6=14)
   ★ D John threw 3 darts. His score was 12. Where could his darts have landed? 3 in 4 ring (4+4+4=12) or one in each ring (6+4+2=12) or 2 in 6 ring and one outside the scoring area (6+6+0=12)

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Using the Exercises

Assign the exercises on page 129 as written work. If some children need extra help, present it in the form of a guided discussion. The starred exercises are optional, but all the children might enjoy discussing them. Exercise 4C may be expressed by the addition sentence $2 + 4 + 8 = 14$, but since this is not repeated addition, multiplication does not apply.

Assignments (page 129)
Minimum: 1–3. Average: 1–4B.
Maximum: 1–4.

Mathematics
We can prove $5 + 5 + 5 = 3 \times 5$ by the use of an extension of the distributive principle:

If $a, b, c, d, \ldots$ are whole numbers, then
$a \times (b + c + d + \cdots) =$
$(a \times b) + (a \times c) + (a \times d) + \cdots$.

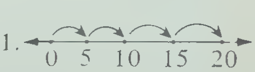
The development of $5 + 5 + 5 = 3 \times 5$ is as follows:
$5 + 5 + 5 = (5 \times 1) + (5 \times 1) + (5 \times 1)$
$5 \times (1 + 1 + 1) = 5 \times 3 = 3 \times 5$

Other principles are involved. However, the children are not ready for such ideas, so for our working definition we relate multiplication to groups of equivalent sets. We consider the total number of objects, or form the union of sets, which automatically leads to the interpretation of multiplication as repeated addition.

A problem arises when we attempt to work with products involving zero and one. At this stage, however, children are working primarily with the ideas of multiplication that do not involve such factors. Repeated addition will not be considered when there is a first factor of zero or one, since addition is an operation performed on two numbers, and since it is confusing to speak of adding 0 threes.

Follow-up
To help children better relate multiplication to repeated addition, prepare a worksheet similar to this:

Write an addition equation and a multiplication equation for each exercise.

1.


(5 + 5 + 5 + 5 = 20
4 × 5 = 20)

2. 3.


Resources for Active Learning
Developmental Math Cards, E-1, Addison-Wesley. [Showing repeated addition by peg counting]
Mathematics in Modules, WN13, Addison-Wesley.

Duplicator Masters, page 36
Workbook, page 47

PAGES 130–131

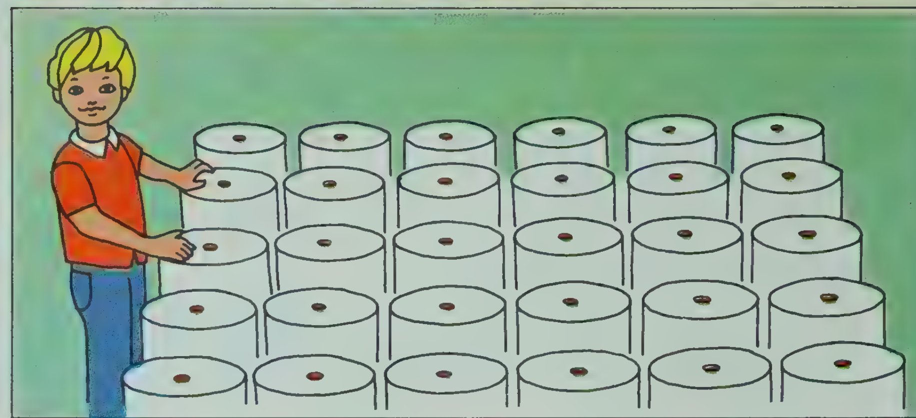
Objective

Given word problems using multiplication, the child will be able to solve the problems by using rectangular arrays and repeated addition, or by applying known multiplication facts.

Preparation

To remind the children that repeated addition can be used when a multiplication fact is not known, spend a few minutes with an appropriate oral activity. For instance, give the children a multiplication equation, such as 3×5, and ask them to respond with a related repeated addition equation, such as $5 + 5 + 5$; then ask a volunteer to give the sum of the addition equation.

NEWFOUNDLAND



Newfoundland is well known for its fishing. It also has a large pulp and paper industry.

1. Bowater's mills at Cornerbrook make newsprint. It is stored and shipped in large rolls. In the diagram above, how many rolls are in each row ? 6 How many rows are there ? 5 How many rolls of newsprint in all ? 30
2. In the diagram above, how many rolls are in each column ? 5 How many columns are there ? 6 How many rolls of newsprint in all ? 30
3. Newfoundland became Canada's tenth province in 1949. Before then it was a British colony and had its own coins. Jim has 4 rolls of George VI Newfoundland pennies. There are 5 pennies in each roll. How many of these pennies does Jim have ? 20
4. Paul, John and Eddie went fishing. They each caught 5 fish. How many fish did the boys catch ? 15
5. Jim had 3 rows of Edward VII nickels with 6 in each row. How many Edward VII nickels did he have ? 18

★6. Suppose a certain penny is worth 5 cents and an older penny is worth 10 cents. Which is worth more, five of the pennies worth 5 cents or three of the pennies worth 10 cents ?

three pennies worth 10 cents.

Discussion

Although these problems incorporate multiplication facts which all children may not yet have memorized, the problems may be solved by association with rectangular arrays and/or repeated addition. Work through some of the problems together so that the children realize this. For example, in exercise 3, page 130, the problem may be thought of as $5 + 5 + 5 + 5$. You may find it helpful to ask the children not only to answer the question but also to write the appropriate multiplication equation for each problem.

Similarly in exercise 5, page 130, suggest that the children draw the array referred to in the problem:

○ ○ ○ ○ ○ ○

○ ○ ○ ○ ○ ○

○ ○ ○ ○ ○ ○

He may then write $6 + 6 + 6 = 18$ or $3 \times 6 = 18$.

Centennial Coins

To mark Canada's one hundredth birthday in 1967, the Canadian mint issued a special series of coins.

1. A set of 6 Centennial coins includes a penny, a nickel, a dime, a quarter, a fifty-cent piece and a dollar. Bob has 4 sets. How many coins does Bob have ? 24
2. Gerry has a page that will hold pennies in slots. The page has 5 rows and 5 columns. How many pennies does Gerry have if the page is full ? 25
3. Bill put 10 dimes in a roll. How many cents is the roll worth ? 100
4. Bill put 4 rolls in a box. How many dimes did Bill put in the box ? 40
5. Ted had 6 rows of Centennial nickels with 3 nickels in each row. How many Centennial nickels did he have ? 18

★6. 58 884 849 Centennial nickels were made in 1967. 9 028 507 Sudbury nickels were made in 1951 to mark the 200th anniversary of the discovery of nickel in Canada. Which of these nickels would you be most likely to find ? Centennial nickels

★7. Nancy has a page with slots to hold Centennial dimes. It has spaces for 7 rows with 4 dimes in each row. There are only 7 empty spaces. How many dimes are already on the page ? 21

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Follow-up
If possible, have the children bring collections of insects, rocks, buttons, shells, stamps, or any other objects displayed in boxes or on pages in arrays. These will readily suggest interesting word problems.

Using the Exercises
After you have used a few problems as a basis for discussion, and when you think the children are capable, assign the rest of the exercises on pages 130 and 131 as independent work. If necessary, form a small group of those having difficulty and work through the problems together. Have the children note that in order to solve problem 4 on page 131 they must use information given in problem 3.

The starred exercises are for the more capable children. Also, for these children you might draw some sets or arrays on the chalkboard and have them make up problems to exchange with one another.

When the children have finished working the problems, discuss and correct their work.

Assignments (page 131)
Minimum: 1–5, oral. Average: 1–5.
Maximum: 1–7.

Objective

Given a multiplication equation, the child will be able to identify the factors and the product.

Preparation

Materials

small blank tagboard cards (12 or 15 per child)

To prepare for this lesson, write an addition equation on the chalkboard and review the terms *addend* and *sum*. For example, write

$$3 + 7 = 10$$

and ask for volunteers to recall the special names for these numbers. (3 and 7 are addends; 10 is called the sum.) Then explain that the numbers in a multiplication equation also have special names.

Investigation

This investigation introduces the terms *factor* and *product* while providing further experience in expressing some basic multiplication facts as equations.

You may wish to divide the children into small groups and have them work together to label the cards with the same numbers as illustrated in the text. Then they might be allowed to continue working in groups to make the multiplication equations.

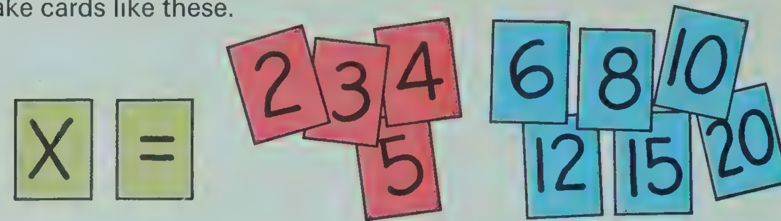
Note that the use or understanding of the terms *factor* and *product* is not essential to the investigation. However, it is hoped that by relating the terms in the text to the appropriate cards children will discover the meaning of them. As you circulate through the room, you might ask children to notice which cards are called *factor* cards and which are called *product* cards, and ask them to try to figure out why they are so grouped.

Remind the children, if necessary, that they can use repeated addition to find or check products that they do not know.

Let's explore factors and products.

Investigating the Ideas

Make cards like these.



Symbol cards **Factor cards** **Product cards**

Here is one multiplication equation you can make with your cards.

2 X 3 = 6

2×4=8 4×3=12
2×5=10 4×5=20
3×2=6 5×2=10
3×4=12 5×3=15
3×5=15 5×4=20
4×2=8

? How many more multiplication equations can you make?	Record each one.

Discussing the Ideas

The numbers we multiply are called factors of the product.
The answer in multiplication is called the product of the factors.

A $\underset{\text{factor}}{4} \times \underset{\text{factor}}{3} = \underset{\text{product}}{12}$

B
$$\begin{array}{rl} 7 & \leftarrow \text{factor} \\ \times\ 4 & \leftarrow \text{factor} \\ \hline 28 & \leftarrow \text{product} \end{array}$$

1. Suppose the product is 24. One factor is 6. The other factor is 4. How would you write this as in A? as in B? $\begin{array}{r} 4 \\ \times 6 \\ \hline 24 \end{array}$
6×4=24

2. How are factors and products like addends and sums?
See Discussion.

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Discussion

Before taking up the discussion questions, ask for volunteers to explain why some cards in the investigation were called factor cards, and others product cards. Then work through the text material.

Help children to read the vertical multiplication example properly. For instance, point out that example B is read as, "Four times seven equals 28."

In exercise 2, children may be able to relate factors with addends by thinking of them as the numbers which are brought together, and to relate sums with products by thinking of them as the answers.

It would be helpful to provide children with other examples of factors and products and then ask them to use the numbers to write multiplication equations, as was done in exercise 1. For example, "If 5 and 7 are the factors and 35 is the product, write this as a multiplication fact in two ways."

$5 \times 7 = 35$ or $7 \times 5 = 35$ $\qquad \begin{array}{r} 7 \\ \times 5 \\ \hline 35 \end{array}$ or $\begin{array}{r} 5 \\ \times 7 \\ \hline 35 \end{array}$

Using the Ideas

1. A In $3 \times 5 = 15$, the number 15 is the product of what factors? 3,5
   B In $4 \times 3 = 12$, what is the number 12 called? Product
   C In $6 \times 7 = 42$, the number 42 is the product of what factors? 6,7
   D In $8 \times 9 = 72$, what is the number 72 called? Product
   E In $3 \times 5 = 15$, the numbers 3 and 5 are factors of what product? 15
   F In $4 \times 3 = 12$, what are the numbers 4 and 3 called? Factors

2. Write a multiplication equation for each exercise. Write an f over each factor and a p over the product as in the example: $\overset{f}{3} \times \overset{f}{4} = \overset{p}{12}$

A

$\overset{f}{4} \times \overset{f}{2} = \overset{p}{8}$

B $4 + 4 + 4 + 4 + 4 = 20$

$\overset{f}{5} \times \overset{f}{4} = \overset{p}{20}$

C

$\overset{f}{4} \times \overset{f}{3} = \overset{p}{12}$

3. What is the product when
   A 2 and 3 are the factors? 6
   B 3 and 4 are the factors? 12
   C 2 and 7 are the factors? 14
   D 9 and 2 are the factors? 18
   E 3 and 7 are the factors? 21
   F 6 and 4 are the factors? 24

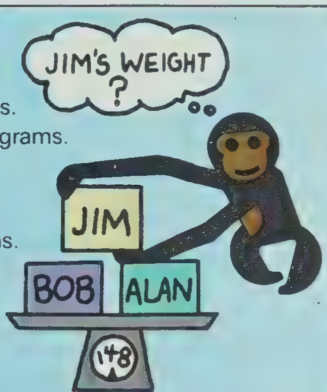
think NB

Alan and Bob together weigh 41 kilograms.
Alan, Bob and Jim together weigh 67 kilograms.

1. How much does Jim weigh? 26 kg

Alan and Jim together weigh 48 kilograms.

2. How much does Alan weigh? 22 kg
3. How much does Bob weigh? 19 kg



133

Using the Exercises

Treat exercise 1 on page 133 as an oral exercise. You may choose to add similar questions of your own. Then, as the children work through exercises 2 and 3 independently, you might draw other sets on the chalkboard and use them to extend exercise 2.

Encourage those children who solve the *Think* problem to explain their solution to the class.

Follow-up

To give children more experience in determining the factors and products of multiplication equations, you might display on the chalkboard or overhead projector an exercise like the one illustrated below. Direct the children to copy each equation and to determine what is missing. Use more difficult products for more capable children.

What is missing? Write *f* for a missing factor. Write *p* for a missing product. Then find the number for each *n*.

1. $4 \times n = 16$	10. $n \times 7 = 14$
2. $n \times 5 = 15$	11. $3 \times 3 = n$
3. $7 \times 3 = n$	12. $5 \times 2 = n$
4. $6 \times n = 12$	13. $7 \times n = 21$
5. $n \times 3 = 12$	14. $n \times 3 = 9$
6. $8 \times 2 = n$	15. $4 \times 3 = n$
7. $2 \times n = 6$	16. $3 \times 6 = n$
8. $n \times 2 = 16$	17. $5 \times n = 10$
9. $3 \times n = 18$	18. $n \times 7 = 21$

Resources for Active Learning

Developmental Math Cards, E[1]14, Addison-Wesley.

Math Workshop: Games and Enrichment Activities, "Three Boxes," pp. 10–11, Encyclopaedia Britannica Educational Corp. [An "If-Then" activity]

Assignments (page 133)
Minimum: 1–2. Average: 1–3.
Maximum: 1–3.

PAGES 134–135

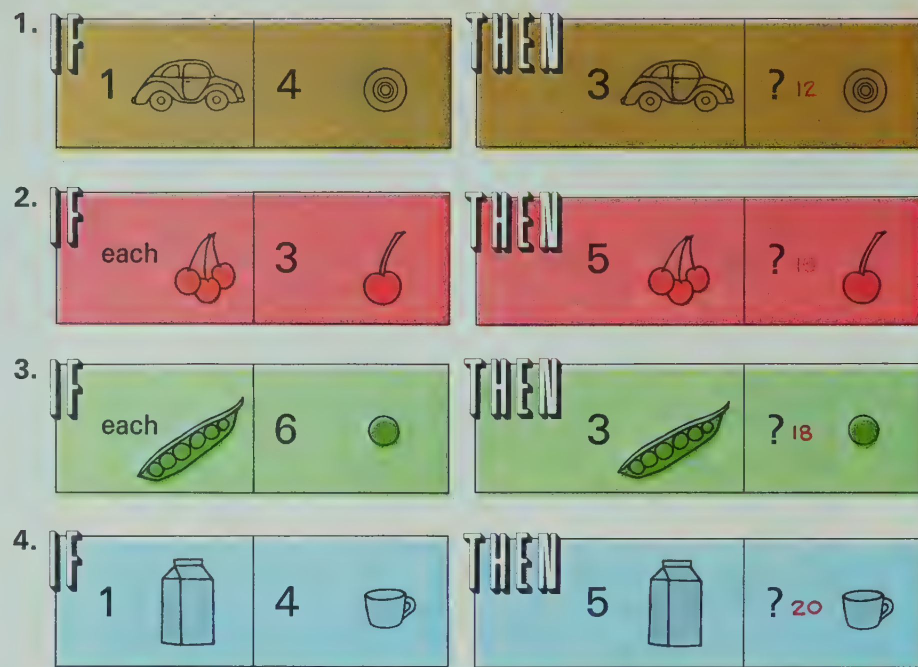
Objective

The child will demonstrate his ability to apply his understanding of multiplication in short story problems and to work with the concepts indicated for cumulative review.

Preparation

If you choose to use these pages as a single review lesson, use a short oral activity as warm-up. For example, start with a simple game such as: "I'm thinking of the difference 12 − 9. What's my number?" Work up to more involved chain games such as: "Start with 2 . . . Multiply by 5 . . . Subtract 7 . . . Add 3. What's my number?" (6)

Short Picture Problems



Short Stories

1 3 engines on each jet.
5 jets.
How many engines? 15

2 4 chairs in each row.
3 rows.
How many chairs? 12

3 5 puppies.
4 legs per puppy.
How many legs? 20

4 6 legs on an insect.
3 insects.
How many legs? 18

5 8 legs on a spider.
2 spiders.
How many legs? 16

134

Discussion

The short picture problems at the top of page 134 can best be treated as an oral activity. The If-Then problems (using exercise 1 as an example) should be read: "If 1 car has 4 tires, then 3 cars have how many tires?" (If children evidence difficulty in reading the picture problems properly, you might build up to it by having the children tell short stories about the pictures.) After the children supply the answer for each problem, have them give the corresponding multiplication fact. If you choose to have a volunteer put the fact in equation form on the chalkboard, ask the children to identify the factors and the product.

The short story problems at the bottom of the page may be assigned as independent work.

After children have completed the exercises on pages 134 and 135, allow ample time to discuss any exercises which caused difficulty. Stress the topics which children have found troublesome and include additional exercises on a specific topic if necessary.

Assignments (page 134)

Minimum: 1–4. Average: 1–4; 1–3

Maximum: 1–4; 1–5

Keeping in Touch with

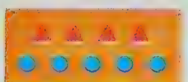
Addition | Inequalities
Subtraction | Basic principles
Fractions | Measurement

1. For each pair, write the larger number on your paper.
   A 3764; 4764 B 67 289; 67 290

2. A What is the area of the rectangle? 8
   B What is the area of $\frac{1}{2}$ of the rectangle? 4
   C What is the area of $\frac{1}{4}$ of it? 2

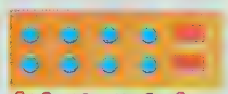
3. Solve the equations.
   A $295 = 200 + n + 5$ 90 B $6285 = 6000 + n + 80 + 5$ 200

4. Write 2 addition and 2 subtraction equations for each exercise.

A

4+5=9 9−5=4
5+4=9 9−4=5

B

6+4=10 10−4=6
4+6=10 10−6=4

C

8+2=10 10−2=8
2+8=10 10−8=2

5. Solve the equations.
   A $6 + 7 = n$ 13 B $7 + 6 = n$ 13 C $8 + 4 = n$ 12 D $4 + 8 = n$ 12

6. Solve the equations.
   A $(4 + 3) + 6 = n$ 13 C $(5 + 2) + 4 = n$ 11
   B $4 + (3 + 6) = n$ 13 D $5 + (2 + 4) = n$ 11

7. Find the sums and differences.

A	B	C	D	E	F
34 + 27 = 61	45 + 38 = 83	74 − 26 = 48	93 − 47 = 46	26 + 49 = 75	32 − 18 = 14

G	H	I	J	K	L
85 + 69 = 154	123 − 64 = 59	48 + 75 = 123	97 − 69 = 28	84 + 66 = 150	147 − 75 = 72


You are invited to explore

ACTIVITY CARD 6
Page 312

Follow-up
More capable children might like to make up some If-Then problems of their own. You might list a few items such as those below to help them get started.

1 cat 4 paws	1 box 3 tennis balls
1 team 9 players	1 plane 2 pilots

To review place value, the children might enjoy the place-value Card Game suggested on page 41.

Also, to provide some variety in reviewing addition and subtraction combinations, you could duplicate several written chain games like the ones below. Remind the children that each sum or difference becomes part of the next problem.

Start

4	+5		+8		−7		→ End
	−5		+8		−9	4	

Start

10	+8		−9		+5		→ End
	−4		−7		+10	13	

Start

7	−7		+10		+8		→ End
	−5		−5		+6	14	

Resources for Active Learning

Franklin Series: *Making and Using Graphs and Nomographs*, pp. 19–24, Lyons and Carnahan. [Using "Nomographs" to "Keep in Touch"] (Available from McGraw-Hill Ryerson)

Math Workshop: Games and Enrichment Activities, "Game of 8's, 4's, 2's and 1's," pp. 20–21, Encyclopaedia Britannica Educational Corp.

[See also games and activities from previous chapter for addition and subtraction review.]

PAGES 136–137

Objective

Given a simple multiplication equation in which one factor is zero or one, the child will be able to recognize the special multiplicative properties of zero and one and will be able to find the product.

Preparation

Materials

paper cups; counters; colored strips

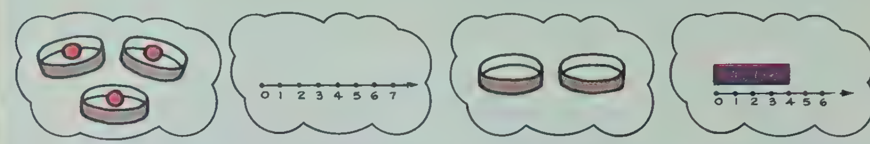
Although the children are directed only to *think* about the materials pictured, you might want some children actually to work with them. Begin immediately with the investigation unless you prefer to use a short oral warm-up such as "What's My Rule."

Investigation

Read the directions together with the children. Suggest that each child record his answers and then check them with a classmate. This investigation is very short and need not involve much activity. Its purpose is to lead into the discussion section.

Let's explore multiplying by 0 and 1.

Investigating the Ideas



A $3 \times 1 = ?$ 3 B $0 \times 5 = ?$ 0 C $2 \times 0 = ?$ 0 D $1 \times 4 = ?$ 4

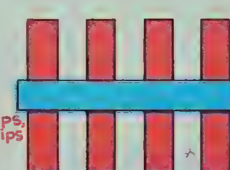
? How many of these products can you give correctly? Check your answers with a classmate.

Discussing the Ideas See Discussion.

1. Think of cups and counters.
   A How does this show 4×1?
   B How would you show 1×4? 1 cup with 4 counters
   C How would you show 3×0? 3 cups with no counters

2. Think of strips and a number line.
   A How does this show 1×5?
   B How would you show 5×1?
   5 1-unit strips

3. Think of the red and blue strips that cross.
   A How does this show 4×1?
   B How would you show 4×0? 4 red strips, no blue strips
   C How would you show 0×4?
   No red strips, 4 blue strips



4. Can you give a rule for multiplying by 0? by 1?
   The product of any number and 0 is 0.
   The product of any number and 1 is that number.

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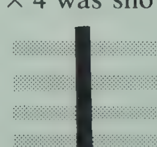
Discussion

Decide with the children which answers in the investigation problems are correct. Then work through exercises 1, 2, and 3.

If children have trouble responding to exercise 3B or 3C, ask them to consider how they would show 1×4. If 1×4 was shown as



then 0×4 would be shown as

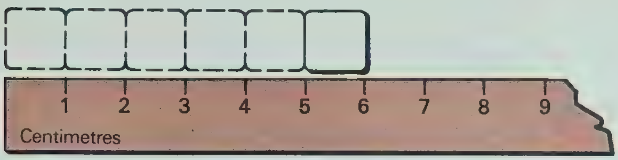
During this discussion make sure the children realize that the numbers of the multiplication equation correspond to the *number of crosses* formed by the strips; thus, 0×4 shows four strips but zero number of crosses.

For exercise 4, help the children generalize the rules in their own words, and then summarize them by writing the following on the chalkboard:

$$n \times 1 = n \qquad 1 \times n = n$$
$$n \times 0 = 0 \qquad 0 \times n = 0$$

Using the Ideas

1. Write a multiplication equation for the figure. $6 \times 1 = 6$



2. Complete the statements.

A The product of any number and **1** is __?__. That number

B The product of any number and **0** is __?__. 0

3. Find the products.

A $9 \times 0 = n$ 0	D $1 \times 7 = n$ 7	G $35 \times 1 = n$ 35
B $0 \times 9 = n$ 0	E $12 \times 0 = n$ 0	H $0 \times 35 = n$ 0
C $7 \times 1 = n$ 7	F $1 \times 12 = n$ 12	I $97 \times 0 = n$ 0

4. Solve the equations.

A $8 \times n = 0$ 0	D $19 \times n = 0$ 0	G $n \times 62 = 62$ 1
B $n \times 7 = 7$ 1	E $1 \times 56 = n$ 56	H $58 \times n = 0$ 0
C $0 \times 6 = n$ 0	F $74 \times n = 74$ 1	I $0 \times 254 = n$ 0

★ **5.** Some of these equations have just **one** solution, some have **many** solutions, and some have **no** solution. Answer: **one, many,** or **none.**

A $n \times 6 = 0$ One

B $0 \times n = 6$ None

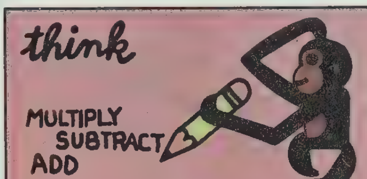
C $0 \times n = 0$ Many

D $6 \times 0 = n$ One

E $n \times 0 = 0$ Many

F $n \times 0 = 8$ None

think



1. What two numbers have a product of 20 and a difference of 19?
2. Find two numbers so that their product is less than their sum.

Any number greater than 0 and 0 or any number and 1

137

Mathematics

In this lesson, we stress the special multiplicative properties of zero and one.

For all whole numbers a,

$$a \times 1 = a.$$

For all whole numbers b,

$$b \times 0 = 0.$$

Follow-up

For practice with multiplication facts, modify the Combo game suggested on page 101 to employ factors and products.

To make cards for multiplication, cut tagboard or cardboard shirt forms into 10-by-12-centimetre pieces and have the children rule each rectangle into 2-cm squares. Show them how to label the top row C-O-M-B-O, leaving five rows of five squares each. Write products for factors less than 5 on the chalkboard, and let the children choose numerals from this list to write at random in 24 of the squares. (The centre square should be marked FREE.) To play, have the leader hold up a flash card and call out the fact at the same time. The other children should cover the appropriate product, if they have it, with buttons, discs, paper counters, or whatever is available. A winner must cover all the spaces in a row, column, or diagonal.

For more capable children, display an exercise like the following one on the chalkboard or overhead projector and assign it as an independent written activity.

Solve the equations.
$(2 \times 3) + (8 \times 3) = n$
$(4 \times 1) + (6 \times 1) = n$
$(5 \times 5) + (5 \times 5) = n$
$(9 \times 2) + (1 \times 2) = n$
$(7 \times 4) + (3 \times 4) = n$
$(4 \times 3) + (6 \times 3) = n$
$(8 \times 2) + (2 \times 2) = n$
$(3 \times 3) + (7 \times 3) = n$
$(0 \times 7) + (10 \times 7) = n$
$(4 \times 0) + (6 \times 0) = n$

Workbook, page 48

Using the Exercises

You may wish to work through exercises 1 and 2 together before assigning the remaining exercises as independent work. Also, you may find that the children require guidance for exercise 5. Help them realize that $0 \times n = 0$ (exercise 5C) means that zero times any number is zero, and $n \times 0 = 0$ (exercise 5E) means that any number times zero is zero. Both of these equations exemplify the statement that the children completed in exercise 2B.

The *Think* problem is intended especially for more capable children, but the whole class would benefit from a discussion of ways to solve it. Trial and error with reasoning should help them solve this problem.

Assignments (page 137)

Minimum: 1–3. Average: 1–4.
Maximum: 1–5.

PAGES 138–139

Objective

Given two factors, the child will understand that he can multiply them in any order.

Preparation

Materials

colored strips

To prepare for this lesson, use a short oral activity such as Names for a Number. For example, write the numeral 18 on the chalkboard and ask the children to give other names for the same number, including addition, subtraction, and multiplication phrases ($10 + 8$, $20 - 2$, 3×6, etc.).

Investigation

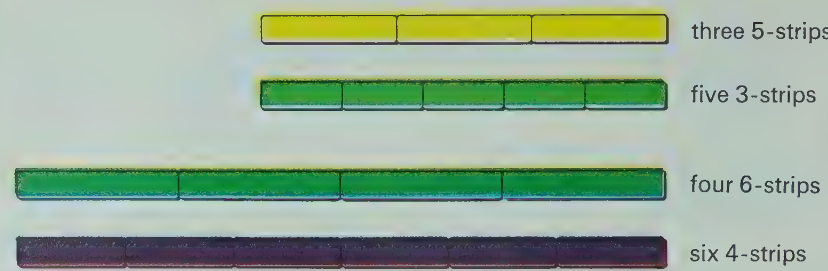
The children will need their complete set of colored strips for this investigation. It would even be suitable to have children combine some of their sets of strips and work together in groups of two or three. Whether they work singly or in groups, however, remind them that each child should make his own record of each pair that is found.

You might challenge some children to write multiplication equations for each pair that they find, though the central purpose of this activity is to have children explore the order principle concretely by finding "matching" trains. They will be asked to write equations in response to the discussion questions.

Let's explore the order principle.

Investigating the Ideas

Here are some special pairs of "matching" trains.

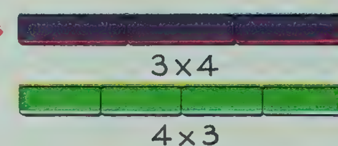


See Investigation.

? Can you make some more special pairs of trains like these?	Record each pair you make.

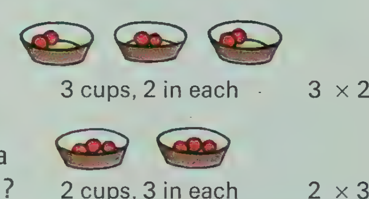
Discussing the Ideas

1. A Do these two trains match? Yes
   B Can you write an equation about this? $3 \times 4 = 4 \times 3$



2. Can you write an equation for each pair of trains in the Investigation? $3 \times 5 = 5 \times 3$ $4 \times 6 = 6 \times 4$

3. Think of cups and counters.
   A How does this show that $3 \times 2 = 2 \times 3$?
   B How would you use this idea to show that $5 \times 4 = 4 \times 5$?



4. When we change the order of the __?__, we get the same __?__.
   factors product

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Discussion

Use exercise 1 to help children realize that the order of factors does not influence the product; the product is the same even if the order of factors is changed.

Have some children display on the chalkboard the equations for each pair of trains pictured in the investigation. Exercise 3 continues development of the same concept but does so by considering sets of counters and cups rather than colored strips. Conclude the discussion by having children summarize the order principle in exercise 4.

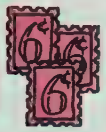
Using the Ideas

1. Answer the questions in the Short Stories for each part. Then write a multiplication equation for the exercise.

A Three 6-cent stamps cost how much? (Answer: 18¢)

Six 3-cent stamps cost how much? (Answer: 18¢)

(Equation: $3 \times 6 = 6 \times 3$)



C 5 marbles in a bag. 7 bags. How many marbles? 35

7 marbles in a bag. 5 bags. How many marbles? 35

$7 \times 5 = 5 \times 7$

B 4 girls. 2 books each. Total books? 8

2 girls. 4 books each. Total books? 8

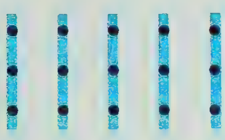
$4 \times 2 = 2 \times 4$

D 2 boys. 3 arrows each. How many arrows? 6

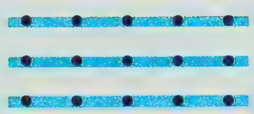
3 boys. 2 arrows each. How many arrows? 6

$2 \times 3 = 3 \times 2$

2. We can think about 15 dots in two different ways. Solve the equations.



$5 \times 3 = n$ 15



$3 \times 5 = n$ 15

3. Find the products. Use the table.

A 92×65 **D** 71×17
B 24×84 **E** 82×34
C 47×38 **F** 27×56

A 5980 D 1207
B 2016 E 2788
C 1786 F 1512

$56 \times 27 = 1512$	$65 \times 92 = 5980$
$34 \times 82 = 2788$	$17 \times 71 = 1207$
$38 \times 47 = 1786$	$84 \times 24 = 2016$

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Mathematics
The colored strips again serve as a useful device to illustrate a basic mathematical concept. Here children use them to explore the idea that the order of factors does not affect the product. The order, or commutative, principle for multiplication may be stated generally:

If a and b are whole numbers, then

$$a \times b = b \times a.$$

Follow-up
To give children practice with the order principle for multiplication, use some comparison stories like those which follow. Urge the class to decide which set is larger without multiplying.

1. 9 boxes: 7 pencils in each.
   7 cans: 9 pencils in each.
   Is the number of pencils in cans larger than the number of pencils in boxes?
2. Rabbit A jumped 8 times. Each jump was 1 metre long. Rabbit B jumped 9 times. Each jump was 1 metre long. Was the distance rabbit B jumped greater than or less than the distance rabbit A jumped?
3. Compare the number of pennies equivalent to 4 dimes with the number of pennies equivalent to 8 nickels. Which set has more pennies?
4. Jane: 3 sweaters, 5 skirts.
   Nan: 5 sweaters, 4 skirts.
   Who has a greater number of outfits?

Using the Exercises
On page 139, exercise 1 develops the order principle through short stories that are close to the child's real world and which provide experiences that prepare the child for the more abstract exercises which follow.

In exercise 3, emphasize for the children that they are not expected to know how to multiply such large numbers but that, by knowing and using the order principle, they can still solve the problems with the help of the information given in the table. You may wish to work through these exercises orally with some groups of children.

Assignments (page 139)
Minimum: 1, oral; 2. Average: 1–3.
Maximum: 1–3.

Objective

Given three factors, the child will understand that he can rearrange their grouping and order without changing the product.

Preparation

Materials

slips of paper, 10 × 15 centimetres (3 per child)

Because of the nature of the investigation in this lesson it would be best to begin immediately with the text without special preparation.

Investigation

This investigation allows the children to discover for themselves that rearranging factors will not change the product. Guide the children in putting the suggested digits on their slips of paper. Read the directions together. Remind them to record the product for each multiplication. Allow sufficient time for all to finish.

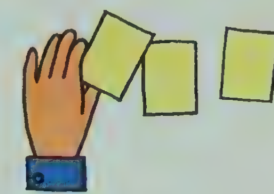
For children who finish the investigation quickly, you might write other factors, each less than six—for example, 2, 4, 5, or 2, 3, 5.

● *Let's explore rearranging factors.*

Investigating the Ideas

Make three slips of paper like these.
Then turn them over and mix them up.

Pick any **two** slips and multiply the numbers on them. Then multiply by the number on the other slip.



? If you do this five times, will you get the same final product each time? Yes
See Investigation.

Discussing the Ideas

With the factors 2, 3, and 4

A we could multiply these first.

$2 \times 3 \times 4$

$\underbrace{(2 \times 3)}_{6} \times 4$

or

B we could multiply these first.

$2 \times 3 \times 4$

$2 \times \underbrace{(3 \times 4)}_{12}$

or

C we could multiply these first.

$2 \times 3 \times 4$

$\underbrace{(2 \times 4)}_{8} \times 3$

1. Answer these questions for A, B, and C. Which two factors are grouped together? What is their product? What is the final product? 24

2. If we leave the order of three factors the same, we can state this principle about **grouping**:

> When we multiply, we can change the **grouping** and get the same product.

In which example above did we change both **order** and **grouping**? C

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Discussion

The development in this section of the text grows directly from the investigation. As you discuss the questions in exercise 1, lead the children to generalize the relationship between rearranging the factors and the product. Use other examples, such as the factors suggested in the preceding section for the more capable children.

For each example, guide the children to multiply the numerals in the parentheses first; for instance, for $(2 \times 3) \times 4$ we think $6 \times 4 = 24$.

It would be helpful also to include some exercises with numbers beyond the children's computational level, so that in order to solve the problems they must understand the principle. For example, $(35 \times 42) \times 70 = n \times (42 \times 70)$.

Exercise 2 summarizes the grouping principle. As was illustrated in example C, both the order and the grouping principle apply for multiplication, so we can conclude that rearranging the factors by order and by grouping does not change the product.

Throughout the discussion, be sure that children are using the terms factor and product properly.

Using the Ideas

1. Find each product. Use the groupings shown. In each part, check to see that the two different groupings give the same product.
   - A $(5 \times 1) \times 6$ 30 / $5 \times (1 \times 6)$ 30
   - B $(2 \times 2) \times 4$ 16 / $2 \times (2 \times 4)$ 16
   - C $(4 \times 2) \times 5$ 40 / $4 \times (2 \times 5)$ 40
2. Find each product. Choose the grouping that is most helpful. Do not change the order.
   - A $7 \times 5 \times 2$ 70
   - B $8 \times 10 \times 10$ 800
   - C $593 \times 497 \times 0$ 0
3. Solve the equations.
   - A $(3 \times 7) \times 5 = n \times (7 \times 5)$ 3
   - B $17 \times (4 \times 29) = (n \times 4) \times 29$ 17
4. Find the products.
   - A Since $(4 \times 6) \times 3 = 72$, we know that $4 \times (6 \times 3) = n$. 72
   - B Since $(5 \times 7) \times 2 = 70$, we know that $5 \times (7 \times 2) = n$. 70
5. Find the products. Arrange the factors any way you choose.
   - A $5 \times 8 \times 2$ 80
   - B $2 \times 9 \times 5$ 90
   - C $4 \times 2 \times 1$ 8
   - D $989 \times 7 \times 0$ 0
6. Find the products.
   - A Since $3 \times 4 \times 5 = 60$, we know that $5 \times 3 \times 4 = n$. 60
   - B Since $8 \times 7 \times 6 = 336$, we know that $7 \times 8 \times 6 = n$. 336
7. Find the products. Use the table on the right.
   - A $3 \times 27 \times 6$ 486
   - B $5 \times 8 \times 13$ 520
   - C $4 \times 65 \times 9$ 2340
   - D $17 \times 4 \times 6$ 408
   - E $4 \times 27 \times 5$ 540
   - F $65 \times 8 \times 4$ 2080
   - G $3 \times 7 \times 17$ 357
   - H $13 \times 9 \times 8$ 936

$5 \times 27 \times 4 = 540$
$17 \times 6 \times 4 = 408$
$8 \times 13 \times 9 = 936$
$6 \times 3 \times 27 = 486$
$17 \times 7 \times 3 = 357$
$8 \times 5 \times 13 = 520$
$65 \times 9 \times 4 = 2340$
$8 \times 4 \times 65 = 2080$

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Using the Exercises

Assign these exercises as independent work, but first work through one or two exercises of each part orally if that seems desirable. Exercise 6 may be treated as an oral discussion exercise, with children explaining why the first equation leads to the second in each exercise. The children should be reminded to use the table while doing exercise 7. When the children have finished, allow time for discussion and checking papers.

Assignments (page 141)*

Minimum: 1–2. Average: 1–2, 4–6.
Maximum: 1–7.

Mathematics

Below is a formal statement of the associative (grouping) principle for multiplication.

For all whole numbers a, b, and c,
$$(a \times b) \times c = a \times (b \times c).$$

The following list shows all the ways in which we can change the order or the grouping of three factors. Each step involves either a change of order or a change of grouping. To find the twelve different forms of the product of the three factors a, b, and c, we start with $(a \times b) \times c$ and alternately regroup and reorder until we come back to the original.

$(a \times b) \times c$	
$a \times (b \times c)$	$(c \times b) \times a$
$a \times (c \times b)$	$(b \times c) \times a$
$(a \times c) \times b$	$b \times (c \times a)$
$(c \times a) \times b$	$b \times (a \times c)$
$c \times (a \times b)$	$(b \times a) \times c$
$c \times (b \times a)$	$(a \times b) \times c$

Follow-up

To give the children practice in recognizing factors in different arrangements, prepare a worksheet similar to the one below.

Match the factors on the left to the factors on the right.

$2 \times (4 \times 5)$	35×70
$(3 \times 4) \times 2$	$92 \times 76 \times 84$
$35 \times (7 \times 10)$	$(2 \times 4) \times 5$
$429 \times 34 \times 205$	$21 \times 120 \times 50$
$50 \times 21 \times 120$	12×2
$84 \times 76 \times 92$	$34 \times 429 \times 205$

Solve the equations.

$(15 \times 3) \times 8 = n \times (3 \times 8)$
$42 \times (9 \times 75) = (42 \times 9) \times n$
$(3 \times 4) \times 8 = 12 \times n$
$72 \times 36 \times 94 = 94 \times 36 \times n$

Duplicator Masters, page 37
Workbook, page 49
Skill Masters, page 37

PAGES 142–143

Objective

Given an equation in which multiplication is distributed over addition, the child will be able to solve the equation.

Preparation

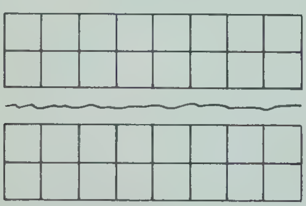
Materials

graph paper (1 or $\frac{1}{2}$ sheet per child)

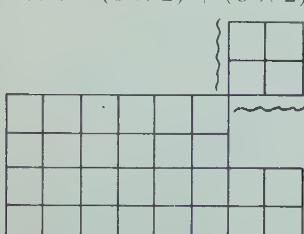
To prepare for this investigation you might want to review the term *rectangle* and how we indicate the dimensions of a rectangle. For example, you could draw a rectangle 30 cm by 24 cm on the chalkboard and explain that describing it as a 30-by-24 rectangle means that there are 30 units along one side and 24 along the other.

Investigation

This investigation allows children to explore the distributive principle by handling square arrays and "breaking apart" the first factor. The graph paper may be folded or colored instead of cut, but cutting is preferable. Read the directions with the class, but allow the children to show their own creativity in making cuts and writing equations. A couple of variations are shown below.



$8 \times 4 = (8 \times 2) + (8 \times 2)$



$8 \times 4 = (7 \times 4) + (1 \times 4)$

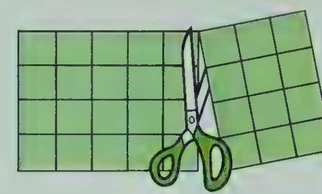
If a few children finish the investigation quickly, you might suggest that they follow the same directions for rectangles of other dimensions, such as 7 by 5 or 6 by 4.

● Let's explore the multiplication-addition principle.

Investigating the Ideas

Cut three 8 by 4 rectangles from graph paper. Color each one a different color.

One way to think about these 8 fours is shown by this cut.



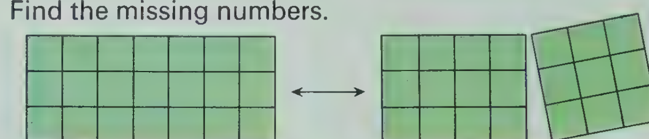
5 fours and 3 fours

See Investigation.

? Can you make different cuts in your rectangles to show other ways to think about 8 fours?	Record your results as shown above.

Discussing the Ideas

1. To help you understand the multiplication-addition principle, think of "breaking apart" a factor before you multiply. Find the missing numbers.



7 threes ⟷ 4 threes and ? threes 3

(7×3) equals $(4 \times 3) + (? \times 3)$ 3

2. These are other examples of the multiplication-addition principle. Give the missing numbers.
   - A 8 fives equals 6 fives and __?__ fives. 2
   - B 6 eights equals __?__ eights and 2 eights. 4
   - C 7 sixes equals 5 sixes and __?__ sixes. 2

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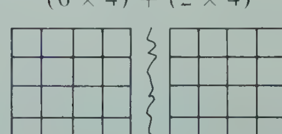
Discussion

Although variations, such as those mentioned in the investigation section, may describe the distributive principle, centre your discussion on cuts similar to the one in the text and those pictured below.

$(6 \times 4) + (2 \times 4)$



$(4 \times 4) + (4 \times 4)$

Channel the discussion in such a way as to include several equations like the following on the chalkboard:

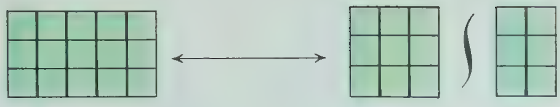
$$(3 \times 6) + (6 \times 6) = n$$
$$(5 \times 4) + (3 \times 4) = n$$

Let the children work several of these to make sure they understand that they should first multiply to find the products in the parentheses and then add to find the final product.

After the mechanics of this procedure have been made sufficiently clear, relate several of these equations to their equivalent expression before the principle is applied, as

Using the Ideas

1. Solve the equations.



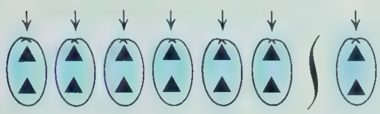
We think : 5 threes ⟷ 3 threes and 2 threes

We write : $5 \times 3 = n$ ⟷ $(3 \times 3) + (2 \times 3) = n$

15   15

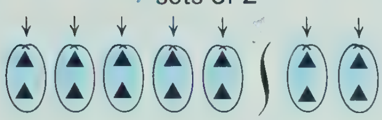
2. Give the missing number of twos.

A 7 sets of 2



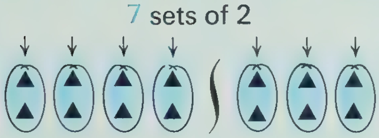
→ For 7 sets of two, we can think 6 twos and ▒ two. 1

B 7 sets of 2



→ For 7 sets of two, we can think 5 twos and ▒ twos. 2

C 7 sets of 2



→ For 7 sets of two, we can think 4 twos and ▒ twos. 3

3. Solve the equations.

A $7 \times 2 = (6 \times 2) + (n \times 2)$ 1
B $7 \times 2 = (5 \times 2) + (n \times 2)$ 2
C $7 \times 2 = (4 \times 2) + (n \times 2)$ 3
D $7 \times 2 = (3 \times 2) + (n \times 2)$ 4

4. Give the missing number. Then solve the equation.

A For 6 sets of three, we can think 4 threes and ▒ threes. 2
$6 \times 3 = (4 \times 3) + (n \times 3)$ 2

B For 5 sets of four, we can think 2 fours and ▒ fours. 3
$5 \times 4 = (2 \times 4) + (n \times 4)$ 3

C For 8 sets of two, we can think 4 twos and ▒ twos. 4
$8 \times 2 = (4 \times 2) + (n \times 2)$ 4

D For 4 sets of six, we can think 3 sixes and ▒ sixes. 1
$4 \times 6 = (3 \times 6) + (n \times 6)$ 1

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illustrated in the two examples which follow.

$$(9 \times 6) = (3 \times 6) + (6 \times 6)$$
$$(8 \times 4) = (5 \times 4) + (3 \times 4)$$

Emphasize that this new principle can help them find unknown products as well as products for larger numbers.

Using the Exercises

On page 143, work through exercises 1 and 2 orally, guiding the children's use of the multiplication-addition principle. They should be able to complete exercises 3 and 4 by themselves. Have more graph paper available and encourage its use by any child who would find it helpful. When you check answers together, discuss any parts of the assigned exercises which seem to be causing trouble.

Assignments (page 143)*

Minimum: 1–2. Average: 1–4.
Maximum: 1–4.

Mathematics

Below are formal statements of the left and right multiplication-addition (distributive) principle.

For all whole numbers a, b, c, $a \times (b + c) = (a \times b) + (a \times c)$ and $(a + b) \times c = (a \times c) + (b \times c)$.

Children use these principles when they "break apart" numbers for multiplication.

In this and the next lesson, we are working only with the *right* distributive principle (though work including the left distributive principle is suggested in the next lesson as a possible follow-up activity for the more capable children).

$$(a + b) \times c = (a \times c) + (b \times c)$$

In other words, in these two lessons we always break apart the *first* factor. Breaking apart the first factor is useful in developing multiplication facts. For example, 8 sevens are 7 sevens plus 1 seven.

$$8 \times 7 = (7 \times 7) + (1 \times 7)$$

Later, when we prepare for the multiplication algorithm, we will break apart the *second* factor.

$$a \times (b + c) = (a \times b) + (a \times c)$$

Breaking apart the second factor will be useful in algorithmic work. For example:

$$3 \times 24 = 3 \times (20 + 4) = (3 \times 20) + (3 \times 4)$$

The descriptive language "break apart" evolves from this type of exercise.

Follow-up/Chain Game

A chain game involving addition, subtraction, and multiplication will provide review of basic facts.

Start   End

3	×5		−10		+5		×2	20

Start   End

10	−8		×9		−7		−8	3

Start   End

7	+1		×2		−8		×3	24

Workbook, page 50

PAGES 144–145

Objective

Given two single-digit factors, the child will be able to apply the multiplication-addition principle and find the product.

Preparation

To prepare for this lesson, review the procedure developed in the previous lesson. Write equations like these on the chalkboard

$(3 \times 2) + (5 \times 2) = n$

$(5 \times 3) + (2 \times 3) = n$

$(4 \times 4) + (5 \times 4) = n$

and have children "think aloud," finding the products first and then adding.

Investigation

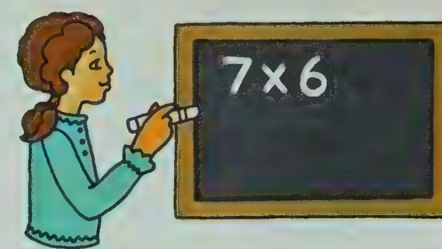
Encourage children to work independently on this investigation, for at least five minutes. Then suggest that they work with one or two classmates and compare the ways they found to show how they would use other multiplication facts to figure out 7×6.

If necessary, help children having difficulty by asking questions such as, "How would knowing 6×6 help you find 7×6?" "How would knowing 3×6 and 4×6 help you find 7×6?" Those still having trouble should be reminded about the multiplication-addition principle studied on the preceding pages.

- *Can the multiplication-addition principle help you find products?*

Investigating the Ideas

Suppose you don't know 7×6.



? Can you show a way to find this product by using other products that you do know? See Investigation.

Discussing the Ideas See Discussion.

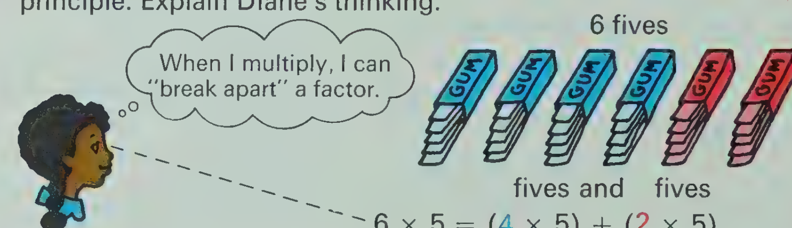
1. Here is the way Eric thought about using the multiplication-addition principle to find 7×6.

When you want to multiply by a number		you can multiply by part of it		and then by the rest of it
7×6	$=$	(4×6)	$+$	(3×6)

Explain how to use the same idea to find 8×5.

Sample answer: Multiply 4×5 and then 4×5 again and add the products.

2. Diane is thinking about the multiplication-addition principle. Explain Diane's thinking.



$6 \times 5 = (4 \times 5) + (2 \times 5)$

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Discussion

In the investigation, the children began with a multiplication fact, 7×6, and applied the multiplication-addition principle to find the product. Discuss the various methods children used to do this.

In exercise 1, help the children see that the 7 has been "broken apart" into 4 and 3. You might put additional equations on the chalkboard to show these other ways of breaking apart 7:

$7 \times 6 = (1 \times 6) + (6 \times 6)$

$7 \times 6 = (2 \times 6) + (5 \times 6)$

Have the class verify each equation by performing the indicated multiplications and additions. In all cases they will see that the final answers are the same, even though the partial products are different.

Develop the product 8×5 in a similar manner, and then, for exercise 2, encourage children to suggest other ways Diane could have "broken apart" 6×5. Include several other equations if you think your class would benefit. For example:

$8 \times 3 = (5 \times 3) + (3 \times 3)$

$7 \times 4 = (2 \times 4) + (5 \times 4)$

$9 \times 8 = (5 \times 8) + (4 \times 8)$

Finally write only the fact, such as 8×5, and ask for volunteers to explain how to find the product. Again, use other facts as needed.

Using the Ideas

1. Solve the equations.

A 7 fours ⟶ ? fours and 3 fours

4 $7 \times 4 = (n \times 4) + (3 \times 4)$

B 7 fours ⟶ 5 fours and ? fours

2 $7 \times 4 = (5 \times 4) + (n \times 4)$

C 7 fours ⟶ 6 fours and ? fours

1 $7 \times 4 = (6 \times 4) + (n \times 4)$

D 8 fives ⟶ ? fives and 6 fives

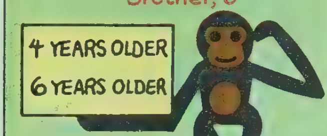
2 $8 \times 5 = (n \times 5) + (6 \times 5)$

E 8 fives ⟶ 7 fives and ? fives

1 $8 \times 5 = (7 \times 5) + (n \times 5)$

think

Tom is 4 years older than his younger sister and 6 years older than his younger brother. The sum of their ages is 26. How old is each child? Tom, 12 Sister, 8 Brother, 6



2. Find the products.

A $\left.\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 2 = 6 \end{array}\right\} 5 \times 2 = n$ 10

B $\left.\begin{array}{l} 2 \times 3 = 6 \\ 3 \times 3 = 9 \end{array}\right\} 5 \times 3 = n$ 15

C $\left.\begin{array}{l} 6 \times 5 = 30 \\ 2 \times 5 = 10 \end{array}\right\} 8 \times 5 = n$ 40

D $\left.\begin{array}{l} 3 \times 2 = 6 \\ 4 \times 2 = 8 \end{array}\right\} 7 \times 2 = n$ 14

E $\left.\begin{array}{l} 3 \times 3 = 9 \\ 4 \times 3 = 12 \end{array}\right\} 7 \times 3 = n$ 21

F $\left.\begin{array}{l} 3 \times 7 = 21 \\ 5 \times 7 = 35 \end{array}\right\} 8 \times 7 = n$ 56

3. Find the products.

A Since $5 \times 8 = 40$, we know that $6 \times 8 = n$. 48

B Since $5 \times 9 = 45$, we know that $6 \times 9 = n$. 54

C Since $6 \times 8 = 48$, we know that $7 \times 8 = n$. 56

D Since $7 \times 6 = 42$, we know that $8 \times 6 = n$. 48

More practice, page A-17, Set 23.

Follow-up

For more capable children, build a patterned exercise with equations showing both the right and left multiplication-addition (distributive) principles. You may wish to use higher multiplication facts than those shown in the sample.

Study the pattern and give the missing numbers.

If $6 \times 4 = (4 \times 4) + (2 \times 4)$,
then $4 \times 6 = (4 \times 4) + (4 \times \boxed{2})$.

If $8 \times 3 = (5 \times 3) + (3 \times 3)$,
then $3 \times 8 = (3 \times 5) + (3 \times \square)$.

If $9 \times 5 = (8 \times 5) + (1 \times 5)$,
then $5 \times 9 = (5 \times \square) + (5 \times 1)$.

If $7 \times 3 = (3 \times 3) + (4 \times 3)$,
then $3 \times \square = (3 \times 3) + (3 \times 4)$.

If $8 \times 6 = (5 \times 6) + (3 \times 6)$,
then $6 \times 8 = (6 \times \square) + (6 \times \square)$.

For children who had difficulty with the concept of the multiplication-addition principle, provide guidance in using activities similar to those suggested for the graph paper (page 142).

Duplicator Masters, page 38
Workbook, page 51

Using the Exercises

Assign the exercises to be completed independently, if the children are able. When necessary, however, do a sample of each kind of problem. Call attention to exercise 3A and explain that since 5 eights are 40, 6 eights are 40 plus 1 eight, or 48.

The *Think* problem is intended for more capable children. Expect them to use trial and error combinations of three numbers in the proper relationships to find this solution.

Assignments (page 145)*

Minimum: 1. Average: 1–3.
Maximum: 1–3.

PAGES 146–147

Objectives

Given a multiplication table, the child will know how to read it to find the product of two factors.

Given two single-digit factors one of which is 3 or less, the child will be able to use known multiplication facts to find the product.

Preparation

Materials

duplicated copies of multiplication tables with factors 0 to 9 for the children to use in this and succeeding lessons (*Duplicator Masters*, page 62)

To introduce this lesson, ask several children to name different multiplication facts that they know. After several facts have been given, ask the children how many facts they think there are. Explain that there is no end to the number of multiplications we can perform but that there are 100 basic facts, many of which they already know and 64 of which they will study in this lesson.

Let's look at some of the easier multiplication facts.

Discussing the Ideas See Discussion.

1. "0" and "1" facts
   A What do you know about multiplying by **0** and **1** that will help you fill in the **0** and **1** rows in a multiplication table?
   B Where are the "0" and "1" columns in the table? What makes it easy to fill in these columns?

×	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3										
4										
5										
6										
7										
8										
9										

2. "2" facts
   How can the "1" facts help you find the "2" facts?
   The picture may help.

2 sixes are 1 six and 1 six.
$2 \times 6 = 6 + 6$

×	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2					12			

3. Give the products for **A** through **H** in this table.
   See table.

4. A If you know the product for **B** it is easy to find the product for **I**. Why? Factors are the same.
   B Give the products for **I** through **O** See table.

×	0	1	2	3	4	5	6	7	8	9
0										
1			4	6	8	10	12	14	16	18
2			A	B	C	D	E	F	G	H
3		6	I							
4		8	J							
5		10	K							
6		12	L							
7		14	M							
8		16	N							
9		18	O							

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Discussion

The purpose of this lesson is (1) to show children ways to use the basic principles to figure out higher multiplication facts from those they already know and (2) to encourage them to memorize the multiplication facts for quick recall. The first goal is important in its own right, but in achieving it, children also gain a valuable tool to use when they cannot recall the memorized facts. The second goal is important because working out products each time is slow and awkward. Both goals need to be achieved, and neither should be abandoned in favor of the other.

Distribute blank tables with 11 rows and 11 columns for the children to begin filling in as these discussion exercises are treated. Display a blank multiplication table on the chalkboard or overhead projector, and show the children how to label the top row and the left-hand column of their tables. Then, after discussing exercise 1, guide the children in filling in the columns and rows for the "0" and "1" facts. If necessary, refer to pages 136 and 137 to review the properties of zero and one.

In exercise 2, the "think cloud" should suggest that the "2" facts may be figured by addition. As

Using the Ideas

1. Study the picture to see how the "1" and "2" facts can help you find the "3" facts.

×	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6				18			

3 sixes are 2 sixes and 1 six.
$3 \times 6 = 12 + 6$

21 A What are 2 sevens and 1 seven?
21 B What is 3×7?

2. Give the products A through G in the table. See table.

×	0	1	2	3	4	5	6	7	8	9
0										
1										
2				9	12	15	18	21	24	27
3				A	B	C	D	E	F	G
4			12	H						
5			15	I						
6			18	J						
7			21	K						
8			24	L						
9			27	M						

3. Use the order principle and the facts in the "3" row to quickly give the products for the "3" column (H through M).
Example: $3 \times 4 = 12$ (B),
so $4 \times 3 = 12$ (H).
See table.

4. Find the products.

A 6×1 6　C 4×3 12　E 3×8 24　G 5×3 15　I 7×3 21
B 2×8 16　D 3×0 0　F 0×9 0　H 1×8 8　J 0×5 0

5. Find the products.

A 6×2 = 12　B 1×7 = 7　C 4×0 = 0
D 4×3 = 12　E 3×3 = 9　F 8×2 = 16
G 3×6 = 18　H 2×7 = 14　I 9×3 = 27

More practice, page A-17, Set 24

6. Copy and give the missing numbers.

A 2, 4, 6, 8, ▮, ▮, ▮, 16, ▮ 10, 12, 14, 18
B 3, 6, 9, ▮, ▮, 18, ▮, ▮, 27 12, 15, 21, 24

7. Give the missing products.

×	5	
1	A	5
2	B	10
3	C	15

×	8	
1	D	8
2	E	16
3	F	24

×	9	
1	G	9
2	H	18
3	I	27

you work through exercise 3, you might write on the board statements such as, "Since $7 + 7 = ?$, we know that $2 \times 7 = ?$" In discussing exercise 4, remind the children of the order principle.

Using the Exercises
The class should be able to complete most of the exercises on page 147 without your help. If there are children who cannot, urge them to work as many exercises as they can. Then let them complete the assignment by using the table displayed on the chalkboard or overhead projector as you work through the remaining exercises with them.

Assignments (page 147)
Minimum: 1–5. Average: 1–6.
Maximum: 1–7.

Mathematics
Pages 146 through 154 help the children find unknown facts and lead them to use the basic principles to organize facts. Memorizing is kept at a minimum. However, you must decide how much additional practice is necessary for your particular group of children. The games and activities we suggest are intended to stimulate you to use or adapt them to the needs of your class. *Do not* abandon your own tried and proven techniques for helping children remember basic facts. *Do* keep the practice interesting and moving at a rapid pace.

Follow-up
The following worksheet might be used to help children review addition, subtraction, and multiplication.

Make the equations true by putting +, −, or × in each ○.	
8 ○ 3 = 11	7 ○ 2 = 9
8 ○ 3 = 24	27 = 9 ○ 3
8 ○ 3 = 5	6 ○ 3 = 18
8 ○ 2 = 16	12 = 3 ○ 4
3 ○ 7 = 10	12 = 6 ○ 2
7 ○ 2 = 14	32 = 28 ○ 4
7 ○ 2 = 5	12 ○ 3 = 9

Resources for Active Learning
Developmental Math Cards, F¹17. Addison-Wesley.
Mathex: Operations and Problem Solving No. 8, "Tables," pp. 15–16 (pupil page 19), Encyclopaedia Britannica Publications Ltd.
Nuffield Project: *Computation and Structure* 3, pp. 26–27, 30–34, Wiley. [All about "tables"]

Workbook, page 52

PAGES 148–149

Objective

Given 2 single-digit factors in which one is either 4 or 5, the child will be able to find the product.

Preparation

Materials

duplicated multiplication tables from previous lesson

Exercise 1 in the text would serve as suitable preparation for this lesson. However, if you prefer to begin with an oral warm-up, use a short review of the "2" and "3" facts, emphasizing those in which the second factor is greater than 5.

● *What are the facts when 4 and 5 are factors?*

Discussing the Ideas

1. If you have learned the "0," "1," "2," and "3" facts, how many more multiplication facts in the table do you have left to learn? 36

×	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3					16	20	24	28	32	36
4					A	B	C	D	E	F
5				20	G					
6				24	H					
7				28	I					
8				32	J					
9				36	K					

2. A Study the small table.

×	0	1	2	3	4	5	6	7	8	9
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4						20		28		

How can the "1" and "3" facts help you find the "4" facts? How can the "2" facts help you find the "4" facts? See Discussion.

B Find facts for A through F. Then find facts for G through K. See table above.

3. A Study the small table. How can "2" and "3" facts help you find the "5" facts?

×	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
5							30			

×	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3										
4						25	30	35	40	45
5						A	B	C	D	E
6					30	F				
7					35	G				
8					40	H				
9					45	I				

B Find the facts for A through E. Then find the facts for F through I. See table.

Discussion

In exercise 1, help the children figure that out of the 100 basic multiplication facts they need only 36 more to complete the table. Exercise 2 utilizes a twofold approach to learning the multiplication facts in which 4 is a factor. First, we depend upon the multiples of two, which nearly all children know from the doubles in addition. Thus, as illustrated in the table, $5 + 5 = 10$, so 2 fives are 10; and, from the multiplication-addition principle, they know $(2 \times 5) + (2 \times 5) = 10 + 10 = 20$, so $4 \times 5 = 20$. Second, if the children learned the multiplication facts using 3 as a factor, they can use the multiplication-addition principle again. Thus, as illustrated in the table:

$$4 \times 7 = (3 \times 7) + (1 \times 7) = 21 + 7 = 28$$

The multiplication-addition principle is used again in exercise 3 for the "5" facts.

$$5 \times 6 = (2 \times 6) + (3 \times 6) = 12 + 18 = 30$$

The children should be encouraged when you tell them that they have to learn only five new products and four repeated ones for the "5" facts. Because of the order principle they need to learn only those combinations in which the second factor is greater than or equal to 5. The rest of the combinations are the same facts with factors in different order. As you work through these exercises, add the facts to your demonstration table and direct the children to fill in their own copies.

1. Copy and complete.
   A 4, 8, 12, ___, ___, 24, ___, ___, 36
   16, 20 28, 32
   B 5, 10, ___, ___, ___, ___, 35, ___, ___
   15, 20, 25, 30 40, 45

2. Copy and complete each table.

A

×	6
1	6
3	18
4	24

B

×	8
1	8
2	16
4	32

C

×	9
1	9
3	27
4	36

D

×	7
1	7
4	28
5	35

E

×	8
2	16
3	24
5	40

F

×	9
2	18
4	36
5	45

3. Find the products.

A 1 × 4 4	F 6 × 4 24	K 4 × 8 32	P 4 × 5 20	U 9 × 5 45
B 2 × 4 8	G 7 × 4 28	L 4 × 6 24	Q 5 × 5 25	V 5 × 4 20
C 3 × 4 12	H 8 × 4 32	M 1 × 5 5	R 6 × 5 30	W 5 × 6 30
D 4 × 4 16	I 9 × 4 36	N 2 × 5 10	S 7 × 5 35	X 5 × 5 25
E 5 × 4 20	J 4 × 7 28	O 3 × 5 15	T 8 × 5 40	Y 5 × 9 45

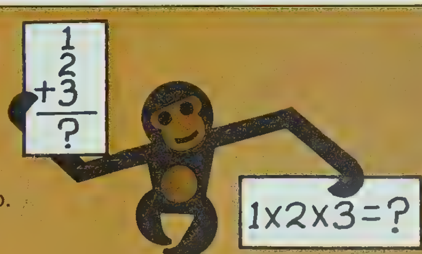
4. If you know the facts up through the "5" facts, how many facts do you have left to learn? 16

★ 5. Use the "5" and the "4" facts to find these "9" facts.
   A 9 × 6 54 C 9 × 7 63
   B 9 × 8 72

think

Add the numbers
one, two, three.
Now find their product too.
I'm the answer either way,
Whichever one you do. 6

WHO AM I?



More practice, page A-18, Set 25.

Using the Exercises

Assign the exercises on page 149 as independent work, but help the children check their answers carefully. The starred exercise is intended for more capable children. The approach again utilizes the multiplication-addition principle.

Most of the children should be able to solve the *Think* problem. Make sure they keep the answer to themselves until everyone has a chance to find it. As you discuss it, observe that it involves a *perfect number*. (When the sum of all the factors of a number, except the number itself, equals that number, we say it is a perfect number.) The first perfect number is 6, the next is 28 $(1+2+4+7+14=28)$. Faster children may enjoy searching for it.

Assignments (page 149)
Minimum: 1–3. Average: 1–4.
Maximum: 1–5.

Mathematics
It is not expected of all third graders that they learn all the facts and be able to recall them *with speed*. Based on the approach in this lesson and subsequent lessons on facts, however, they should have the *power skill* to figure out a fact they cannot remember by using facts they already know. If they know the facts through the "5" facts, they will be able to figure the facts greater than five. Therefore, stress the development of the *speed skill* for the "0" through "5" facts.

Follow-up/"Gold-and-Green Relay"
A competitive relay can stimulate children to learn multiplication facts for quick recall. Draw on the chalkboard a multiplication table similar to the one below.

×	3	8	4	9	2	6	1	7	5	0
5							5			
2										0
1										
4					8					
3										0

Divide the class into two evenly matched teams, and ask the teams to line up facing the chalkboard. Give each team a different-colored piece of chalk. Begin the game by having one player from each team go to the board at your signal and enter any product he chooses in its proper place on the table. He should then pass the chalk to the next teammate and go to the end of his line. Repeat this until all the blanks on the table are filled. Ask the children to judge the accuracy of the products as you read through the table with them. Using white chalk, circle all correct answers and cross out all incorrect ones. The team with the greater number of correct entries is the winner.

Resources for Active Learning
Nuffield Project: *Computation and Structure* 3, "Odd and Even," p. 50, Wiley.

Workbook, page 53

PAGES 150–151

Objectives

Given a function machine problem involving multiplication facts up through the "5" facts, the child will be able to supply the missing element: input number, function rule, or output number.

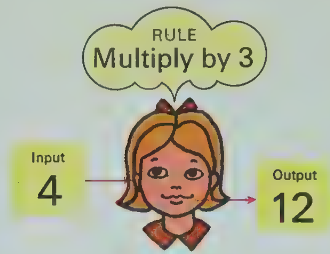
Given factors of the multiplication facts up through "5," the child will be able to find the products.

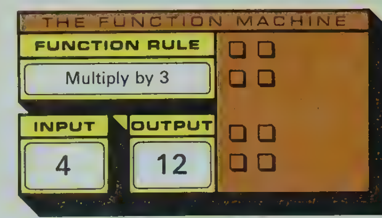
Preparation

For this lesson, use an oral warm-up activity such as "What's My Rule." (See page 69.) Think of rules which involve both multiplication and addition. Remind the children that, when a child thinks he knows your rule, he should fold his arms. When you call on a child with his arms folded, give him an appropriate number (less than ten) and he should respond according to your rule. If he is correct, caution him not to tell anyone your rule. If he is in error, suggest that he continue to listen to the numbers given and to your responses.

Then modify this so that it becomes a function machine game. Help the children agree on an appropriate function rule and then, for each input number you name, ask a child to give the output number. Or you can give the output number and ask a child to give the input number. For further variety, you can have the children suggest input numbers to which you respond with appropriate output numbers until the children guess the rule you are using. (In these adaptations of the game, children verbalize the rule, whereas in other forms of the game the rule is never actually stated.)

Function Machine Problems





Think about the function machine and give the missing number or rule.

1. Function Rule: Multiply by 3

	Input	Output	
	2	6	
	9	27	
A	8		24
B	5		15
C	7		21

2. Function Rule: Multiply by 4

	Input	Output	
	2	8	
A	4		16
B	9		36
C	6		24
D	8		32

3. Function Rule: Multiply by 5

	Input	Output	
A	3		15
B	7		35
	9	45	
C	6		30
D	8		40

4. Function Rule: A ______ Multiply by 5

	Input	Output	
	2	10	
	3	15	
	7	35	
B	6		30
C		20	4

★ **5.** Function Rule: Multiply by 2

	Input	Output	
	2	4	
A	5		10
B		8	4
C		14	7
D		12	6

★ **6.** Function Rule: Multiply the number by itself
A ______

	Input	Output	
	2	4	
	3	9	
	5	25	
B	4		16
C	1		1

150

Discussion

Although both pages can be assigned as independent work, the exercises on page 150 should be introduced by class discussion.

Call the children's attention to the illustrations at the top of page 150 and make sure they understand that the girl hears the input number (4), uses the rule (multiply by 3), and gives the output number (12). Point out that this corresponds to what is shown in the picture of the function machine; that is, when the input is 4 and the rule is multiply by 3, the output is 12. If you used the suggested preparation activities, the children should have no difficulty with the function machine.

Ask the children to study the table in exercise 1. Note that when the input is 2 and the rule is Multiply by 3, the output is 6. Using the same rule, an input of 9 produces an output of 27. Ask the children to tell the outputs for inputs 8, 5, and 7, and to write the answers.

Have the children do exercise 2 independently. When they have finished, discuss the four answers, allowing each child to check his paper. Give the class an opportunity to complete the page and check their answers. Notice that exercises 5 and 6 are designed primarily for faster children.

Multiplication Practice

1. Find the products.

"0" facts

A 6×0 0	E 3×0 0	I 7×0 0
B 0×3 0	F 0×5 0	J 5×0 0
C 4×0 0	G 0×1 0	K 1×0 0
D 9×0 0	H 0×0 0	L 0×8 0

"1" facts

A 1×4 4	E 7×1 7	I 9×1 9
B 3×1 3	F 4×1 4	J 1×6 6
C 6×1 6	G 1×1 1	K 1×7 7
D 1×8 8	H 1×0 0	L 5×1 5

"2" facts

A 2×7 14	E 8×2 16	I 2×3 6
B 4×2 8	F 0×2 0	J 1×2 2
C 2×5 10	G 2×2 4	K 2×6 12
D 3×2 6	H 5×2 10	L 2×9 18

"3" facts

A 6×3 18	E 3×3 9	I 3×2 6
B 3×1 3	F 3×0 0	J 3×4 12
C 3×6 18	G 4×3 12	K 5×3 15
D 7×3 21	H 3×8 24	L 3×9 27

"4" facts

A 4×5 20	E 4×9 36	I 4×2 8
B 1×4 4	F 4×0 0	J 0×4 0
C 4×8 32	G 6×4 24	K 4×4 16
D 5×4 20	H 3×4 12	L 7×4 28

"5" facts

A 5×4 20	E 5×9 45	I 5×2 10
B 5×3 15	F 3×5 15	J 6×5 30
C 2×5 10	G 1×5 5	K 5×5 25
D 4×5 20	H 5×8 40	L 7×5 35

2. Draw a table like this one.
Find the products not given.

×	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45

3. Find the products.

A $4 \times 4 = 16$	B $4 \times 5 = 20$	C $6 \times 2 = 12$	D $6 \times 3 = 18$
E $8 \times 3 = 24$	F $8 \times 4 = 32$	G $5 \times 5 = 25$	H $5 \times 6 = 30$
I $7 \times 2 = 14$	J $7 \times 3 = 21$	K $9 \times 2 = 18$	L $9 \times 4 = 36$

More practice, page A-18, Set 26

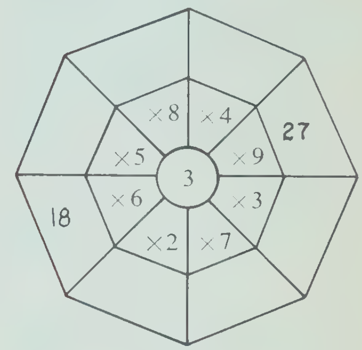
151

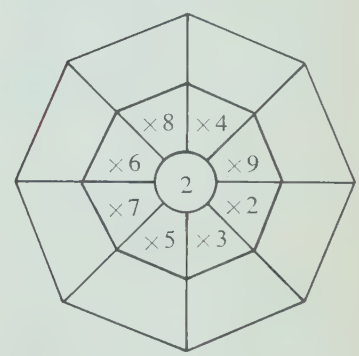
Using the Exercises
You may prefer to have the children give oral responses for exercise 1 on page 151. Some children may be able to fill in the table for exercise 2 on the basis of the patterns, without recalling each product specifically.

Exercise 3 could be used as a short quiz to determine the speed and accuracy with which the children recall the combinations using a factor of 5 or less. If they have difficulty with this exercise, you will probably want to spend additional time on these combinations.

Assignments (page 151)
Minimum: 1, 3. Average: 1–3.
Maximum: 1–3.

Follow-up/Practagons
To give the children practice in finding products (and later, missing factors), create worksheets consisting of six or eight "practagons." Four of them should be partially filled in as indicated in the samples below; the others should be blank. Instruct the children to fill in the missing products for the partially completed figures and to make up problems of their own for the blank practagons.





Practagons

Resources for Active Learning
Franklin Series: *Patterns and Puzzles* "Magic Stars," pp. 27–29, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)
Mathex: Operations and Problem Solving No. 8, "What's My Rule?—Activity 1," p. 17; "Function Machine—Activity 3," p 18, Wiley.

Duplicator Masters, page 39
Skill Masters, page 39

PAGES 152–153

Objective

Given two single-digit factors in which one factor is 6 or 7, the child will be able to find the product.

Preparation

Materials

duplicated multiplication tables from previous lesson

Devise some chain games to play mentally. (*Hint:* Writing the chains down before giving them aloud will save possible disagreement after you have given several long ones.) Include multiplication facts previously studied as well as addition and subtraction facts for review purposes. Examples of oral chain games follow.

"Start with 4 . . . Multiply by 3 . . . Add 4. What's my number?" (16)

"Start with 7 . . . Add 5 . . . Subtract 9 . . . Multiply by 5. What's my number?" (15)

"Start with 15 . . . Subtract 8 . . . Multiply by 4 . . . Add 2. What's my number?" (30)

● *What are the facts when 6 and 7 are factors?*

Discussing the Ideas

1. Study the small table.

   A How can the "2" and the "4" facts help you find the "6" facts? Sample answer: The "2" and the "4" facts can be added to give the "6" facts.

×	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
4	0	4	8	12	16	20	24	28	32	36
6									48	

   B Give the products for A through D in the "6" row of the large table. See table.

×	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3										
4										
5							36	42	48	54
6							A	B	C	D
7						42	E			
8						48	F			
9						54	G			

2. What other two rows could you use to help you find the products in the "6" row? Rows "1" and "5"

3. Use the order principle and give the products for E through G in the "6" column. See table.

4. Explain how to use the products given in the table to help find the products for H, I, and J in the "7" row. Sample answer: The "3" and the "4" facts can be added to give the "7" facts.

×	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3								21	24	27
4								28	32	36
5										
6								49	56	63
7								H	I	J
8							56	K		
9							63	L		

5. What other two rows could you have used to find the products in the "7" row? Explain. Rows "2" and "5" or "1" and "6"

6. Give the products for K and L. See table.

152

Discussion

Most children should make some effort toward learning the higher combinations using a factor of 6, 7, 8, or 9, but make certain the time devoted to this is appropriate to the children's needs and abilities. For some, any attempt to memorize these facts should be optional.

As you present pages 152 and 153, use the demonstration multiplication table begun previously to continue the development of multiplication facts. Point out that there are only four new facts using a factor of 6 and only three using 7.

Help children apply the multiplication-addition principle as suggested in exercise 1 on each page. For example, while discussing the table at the bottom of page 152, write the following equations on the chalkboard:

$$7 \times 7 = (3 \times 7) + (4 \times 7)$$
$$= 21 + 28 = 49$$
$$7 \times 8 = (3 \times 8) + (4 \times 8)$$
$$= 24 + 32 = 56$$
$$7 \times 9 = (3 \times 9) + (4 \times 9)$$
$$= 27 + 36 = 63$$

Also, be sure the children understand how the order principle helps them find the products in discussion exercises 3 and 6.

Using the Ideas

1. Solve the equations.

A $3 \times 7 = 21 \rightarrow 6 \times 7 = n$ 42 C $2 \times 6 = 12 \rightarrow 4 \times 6 = n$ 24

B $5 \times 8 = 40 \rightarrow 6 \times 8 = n$ 48 D $2 \times 6 = 12 \rightarrow 6 \times 6 = n$ 36

2. Copy and complete the tables.

A

×	5
1	5
5	25
6	30

B

×	6
2	12
4	24
6	36

C

×	7
3	21
4	28
6	42

D

×	8
4	32
2	16
6	48

E

×	9
5	45
1	9
6	54

3. Copy and complete the tables.

A

×	5
3	15
4	20
7	35

B

×	8
3	24
4	32
7	56

C

×	6
5	30
2	12
7	42

D

×	9
2	18
5	45
7	63

E

×	7
6	42
1	7
7	49

4. Find the products.

A 0×6 0 C 2×6 12 E 4×6 24 G 6×6 36 I 8×6 48

B 1×6 6 D 3×6 18 F 5×6 30 H 7×6 42 J 9×6 54

5. Find the products.

A $\begin{array}{r}3\\ \times 7\\ \hline 21\end{array}$ $\begin{array}{r}7\\ \times 3\\ \hline 21\end{array}$ B $\begin{array}{r}6\\ \times 7\\ \hline 42\end{array}$ $\begin{array}{r}7\\ \times 6\\ \hline 42\end{array}$

C $\begin{array}{r}5\\ \times 7\\ \hline 35\end{array}$ $\begin{array}{r}7\\ \times 5\\ \hline 35\end{array}$ D $\begin{array}{r}8\\ \times 7\\ \hline 56\end{array}$ $\begin{array}{r}7\\ \times 8\\ \hline 56\end{array}$

E $\begin{array}{r}9\\ \times 7\\ \hline 63\end{array}$ $\begin{array}{r}7\\ \times 9\\ \hline 63\end{array}$ F $\begin{array}{r}4\\ \times 7\\ \hline 28\end{array}$ $\begin{array}{r}7\\ \times 4\\ \hline 28\end{array}$

More practice, page A-19, Set 27

think

Whenever I'm a factor,
I think I'm quite a hero.
You'll always get a product
That ends in five or zero.

WHO AM I? 5

Using the Exercises

On page 153 the first exercise may be used as a basis for discussion. For example, in exercise 1A, point out that since 6×7 is double 3×7, 6×7 must equal $21 + 21$, or 42.

Assign the remaining exercises as independent work. Allow children to refer to their table to complete these exercises when they must, but urge them first to do all they can without help from the tables.

If some children complete the work quickly and accurately by themselves, assign a pattern activity such as the one suggested in the follow-up section. If the children need more practice, provide a drill or game activity such as Multiplication Relay or Combo.

The patterned practice suggested in the follow-up section may also help children answer the *Think* problem. Give them an opportunity to see the relationship for themselves.

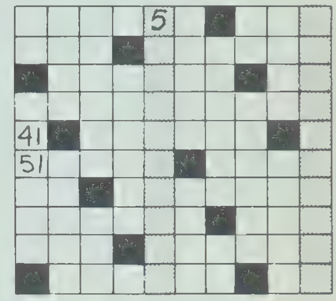
Assignments (page 153)

Minimum: 1–5. Average: 1–5.
Maximum: 1–5.

Follow-up/"Find the Pattern"

You may wish to use this as an enrichment activity to drill the "7" facts.

Find the numbers for the black squares and the shaded squares. Write in the white squares if you need to.



As another means of drilling on "5" facts and "6" facts, make a table like the following for the children to complete.

·	7	8	9	6	5	4
5	35					
6		48				

Resources for Active Learning

Developmental Math Cards, E[1]16, Addison-Wesley.

Franklin Series: *From Fingers to Computers*, "Finger Computation," pp. 13–16, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Mathex: Operations and Problem Solving No. 8, "Finger Multiplication – Activity 4," pp. 18–19, Encyclopaedia Britannica Publications Ltd.

Mathex: Numeration No. 2, "Patterns Using Number Cards," pp. 40–41 (pupil pages 32, 39), Encyclopaedia Britannica Publications Ltd.

PAGES 154–155

Objective

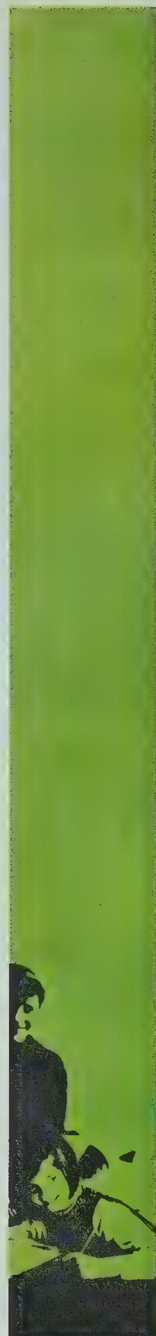
Given two single-digit factors in which one factor is 8 or 9, the child will be able to find the product.

Preparation

Materials

duplicated multiplication tables used in previous lessons

Motivation for this lesson should be high since it introduces the last of the basic facts. However, you may wish to warm up with a short oral game of "What's My Rule" (page 150) or with a chain game (page 152).



What are the facts when 8 and 9 are factors?

Discussing the Ideas

×	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3									24	27
4										
5									40	45
6										
7										
8									A	B
9									C	D

1. Explain how to use the products in the "3" row and the "5" row to find the products for A and B in the "8" row.
   See Discussion.
2. Explain how the fact $4 \times 8 = 32$ can be used to find 8×8.
   $8 \times 8 = (4 \times 8) + (4 \times 8) = 32 + 32 = 64$
3. What other two rows could you use to find the products in the "8" row. Explain. Sample: Rows 2 and 6
   $8 \times 6 = (2 \times 6) + (6 \times 6) = 12 + 36 = 48$
4. A How can the fact $7 \times 9 = 63$ be used to find 8×9? 63 and one more 9 equal 72.
   B What is the product for B in the table? 72
5. A Why is the product for C the same as the product for B? Order principle
   B What is the product for C? 72
6. A What two rows could you use to find the product for D in the "9" row? Sample: Rows 8 and 1 or 2 and 7
   B Find the product for D in the table. 81
7. How can the fact $10 \times 9 = 90$ be used to find 9×9?
   Subtract 1×9 or 9.
8. How can the fact that $3 \times 9 = 27$ be used to find 9×9?
   $9 \times 9 = (3 \times 9) + (3 \times 9) + (3 \times 9) = 81$

154

Discussion

Before beginning the discussion exercises in the text, it might be helpful to ask all the children to try to find 9×9 using any facts they already know. Then discuss the various methods they have used to find the product. Some may have thought of 10 nines less 1 nine; others, of 5 nines plus 4 nines, or of 8 nines plus 1 nine. A few may even have added together 9 nines.

As you discuss these exercises, again stress the many possible approaches for finding these last four products. Develop the approach suggested in exercise 1 by writing the following equation on the chalkboard:

$$8 \times 8 = (3 \times 8) + (5 \times 8)$$
$$= 24 + 40 = 64$$

Let the children suggest other approaches to the 8 factor and the 9 factor in exercises 2 and 4. For exercise 5, be sure they see that they should use the order principle to find the product. Complete your demonstration table, and give the children a chance to complete theirs.

Using the Ideas

1. Copy and complete the tables.

A

×	6
4	24
5	30
9	54

B

×	9
6	54
2	18
8	72

C

×	9
4	36
5	45
9	81

D

×	8
3	24
6	48
9	72

E

×	9
1	9
7	63
8	72

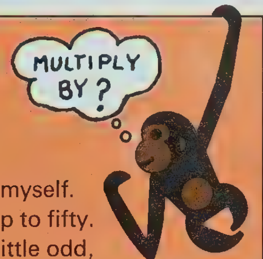
2. Find the products.

A 8×0 0 E 8×4 32 I 8×8 64 M 9×2 18 Q 9×6 54
B 8×1 8 F 8×5 40 J 8×9 72 N 9×3 27 R 9×7 63
C 8×2 16 G 8×6 48 K 9×0 0 O 9×4 36 S 9×8 72
D 8×3 24 H 8×7 56 L 9×1 9 P 9×5 45 T 9×9 81

3. Find the products.

A 4×5 20 I 5×3 15
B 3×8 24 J 7×3 21
C 2×9 18 K 7×8 56
D 0×7 0 L 8×8 64
E 6×1 6 M 6×7 42
F 5×6 30 N 6×6 36
G 4×4 16 O 9×7 63
H 8×4 32 P 9×9 81

think



Multiply me by myself.
You're almost up to fifty.
Though I am a little odd,
I think I'm pretty nifty.

WHO AM I?

4. Find the products.

A $6 \times 2 = 12$ B $2 \times 7 = 14$ C $8 \times 7 = 56$ D $4 \times 5 = 20$ E $8 \times 4 = 32$ F $6 \times 6 = 36$ G $4 \times 4 = 16$

H $6 \times 7 = 42$ I $3 \times 8 = 24$ J $5 \times 3 = 15$ K $9 \times 7 = 63$ L $5 \times 5 = 25$ M $5 \times 6 = 30$ N $6 \times 3 = 18$

O $6 \times 1 = 6$ P $9 \times 5 = 45$ Q $2 \times 9 = 18$ R $7 \times 3 = 21$ S $9 \times 9 = 81$ T $3 \times 4 = 12$ U $0 \times 8 = 0$

More practice, page A-19, Set 28

155

Follow-up

Have cards approximately 10 by 15 centimetres available and encourage children to make flash cards for the facts which they find troublesome. It would also be helpful to have a large copy of the multiplication table displayed somewhere in the classroom so that children can continue their study of multiplication without being handicapped by lack of knowledge of some facts. Parts of this table may be covered gradually, as the children all agree that they no longer need them.

Resources for Active Learning

Franklin Series: *From Fingers to Computers*, "Finger Computation," pp. 5–11, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Mathematics in Modules, S1, Addison-Wesley.

Duplicator Masters, page 40
Workbook, pages 54, 55
Skill Masters, page 40

Using the Exercises

On page 155, point out how the tables in exercise 1 suggest the use of the multiplication-addition principle in finding unknown or more difficult facts. Assign the remaining exercises as independent work, allowing children to use their tables if they want.

The *Think* problem requires children to inspect factors multiplied by themselves and to recall which ones are odd numbers.

This would be an appropriate time to use interviewing techniques to evaluate the children's ability to recall the facts and to figure out facts they cannot remember. Over a period of about two weeks, daily interview two or three children individually and ask each to give the products for a list of 5 or 6 basic facts, graduated from easier facts to the "7," "8," and "9" facts. Notice the child's response for accuracy, speed, and ability to figure a fact using basic principles. Give special help to any child who neither knows the facts with quick recall nor can figure an unknown fact from known facts.

Assignments (page 155)
Minimum: 1–4. Average: 1–4.
Maximum: 1–4.

PAGES 156–157

Objective

Given factors of basic multiplication facts in picture and story problems, the child will be able to solve the problems by finding the products.

Preparation

To prepare for this lesson, you might work through some oral stories patterned on those on page 156. For example, you might say: "If one row of desks has six desks, then in 5 of these rows there would be how many desks?" Or: "If 1 team has 9 players, then 7 teams have how many players?"

Short Picture Problems

156

Discussion

Although there is no discussion section as such in the child's text for these two pages, you will probably want to discuss parts of both of them. On page 156, have the children tell stories about the examples. When necessary, summarize their story using the if-then pattern suggested in the preparation section. Then have them write and solve an equation for each.

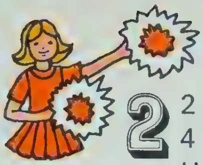
The two starred problems on page 156 are for the more capable children; however, a discussion of them will prepare the children for the missing-factor concept soon to be developed.

You might work through some of the problems on page 157 orally, showing the children how to write an equation for each problem. Remind them to ask of each answer: "Does it make sense? Does it fit the question asked?" Assign most of the problems as independent work, instructing the children to write and solve equations for each.

When the children have completed the page and you are discussing the problems, give them adequate opportunity to discuss any which have caused them difficulty. It might be helpful to have some of the children write the multiplication equations for spe-

Short Sport Stories

1 2 hockey teams.
6 players on each team.
How many players? 12
2×6=12

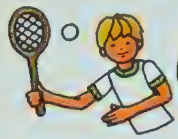


2 2 basketball teams.
4 cheerleaders for each team.
How many cheerleaders? 8
2×4=8

3 Baseball game.
6 outs each inning.
9 innings.
How many outs? 54
6×9=54

4 Red Sox.
3 outs each inning.
9 innings.
How many outs? 27
3×9=27

5 Hockey game.
3 periods.
8 penalties
each period.
How many
penalties? 24
3×8=24



6 Tennis.
9 courts.
4 players
on each
court.
How many
players? 36
9×4=36

7 Bowling. 8 balls in each
rack. 7 racks. How many
bowling balls? 56
8×7=56

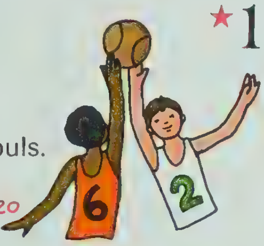
8 Baseball. 3 strikes, you're out.
8 strikeouts. How many strikes? 24
3×8=24

9 Football game.
6 points for
a touchdown.
5 touchdowns.
How many points? 30
6×5=30

10 Football game.
6 points for
a touchdown.
7 touchdowns.
How many points? 42
6×7=42

11 Softball. 9 players on each team.
7 teams. How many players? 63
9×7=63

12 Basketball.
5 fouls, you're out
of the game.
4 players out on fouls.
How many fouls
for these players? 20
5×4=20



★**13** Football.
6 points for
a touchdown.
Bulldogs scored
8 touchdowns
and 4 extra points.
What was their score? 52
(6×8)+4=52

157

cific problems on the chalkboard. Notice that exercise 13 is intended for the faster children.

Assignments (page 156)
Minimum: 1–4. Average: 1–5. Maximum: 1–7.

Assignments (page 157)
Minimum: Even-numbered problems. Average: 1–12. Maximum: 1–13.

Follow-up
Provide grocery sale advertisements or the sports sections from some recent Sunday papers for the children to use in making up word problems or short stories like those on pages 156 and 157. If they have difficulty devising problems which require multiplication, help them to get started by making up several short story problems like the following.

1. Baseball game.
   6 outs per inning.
   Game called after 7 innings.
   How many outs this game?
2. Relay race.
   4 runners per team.
   7 teams.
   How many runners in race?
3. Watermelon sale.
   9 cents per kilogram.
   6-kilogram melon.
   Cost of half the melon?
4. Soft drinks.
   9 cents per can.
   8 cans.
   Cost of all the drinks?

Resources for Active Learning
Math Workshop: Games and Enrichment Activities. "Three Boxes," pp. 10–11, Encyclopaedia Britannica Educational Corp.

Duplicator Masters, page 41

Objective

Given several of the basic multiplication facts, the child will have an opportunity to explore patterns which make recall easier.

Preparation

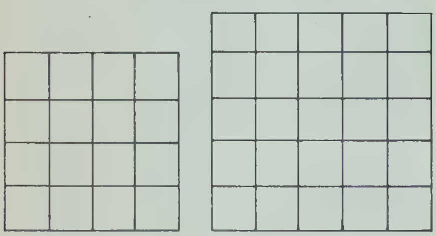
Materials

graph paper (1 sheet per child); scissors; glue and construction paper (optional)

Specific preparation for this lesson is not necessary, but you might use a short oral drill of multiplication facts if you wish.

Investigation

In this investigation, the children explore square numbers by cutting squares from graph paper, ranging in area from 16 squares to 81 squares. The children might see the progression of squares more clearly if you suggest that they glue each square on light-colored construction paper and write the equation for each figure under the corresponding square.



$4 \times 4 = 16$ $5 \times 5 = 25$

This display of "square facts" would then be suitable for use in the discussion.

Discussion

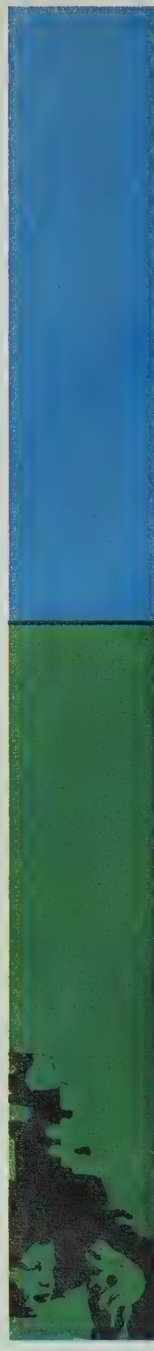
Encourage the children to share any observations they made during the investigation. In particular, be sure they notice the pattern for "square facts"—that the "square facts" are those in which both factors are the same.

During the discussion of exercises 3 and 4, accept any pattern which a child can justify. As you discuss exercise 4, have the children list the "9" facts vertically.

$$2 \times 9 = 18$$
$$3 \times 9 = 27$$
$$\vdots \quad \vdots \quad \vdots$$
$$9 \times 9 = 81$$

Thus, the children may notice the progression of the digits in the tens' place from 1 to 8 and in the ones' place from 8 to 1. Also, some might notice the relation between pairs of products such as 2×9 and 9×9, 3×9 and 8×9, etc., in which the order of the digits in the products of each pair is reversed. Some children might also notice that the digits in the tens' place of each product is one less than the factor other than the 9.

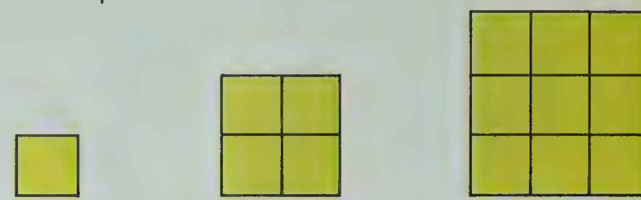
For exercise 5, you might list on the chalkboard in numerical sequence the answers as they are suggested by the children. After all suggestions have been listed, have the children review the list for possible omissions or the inclusion of numbers that should not have been listed. Finally, use the display table of multiplication facts to verify that none of the numbers listed is a product for any of the basic facts.



How can patterns help you with the facts?

Investigating the Ideas

Some "Square Facts"



$1 \times 1 = 1$ $2 \times 2 = 4$ $3 \times 3 = 9$

? Can you cut squares from graph paper to show the other square facts up to 9×9?

4x4=16 5x5=25 6x6=36 7x7=49 8x8=64 9x9=81

See Investigation.

Discussing the Ideas

1. Is $(3 \times 3) + (4 \times 4) = (5 \times 5)$? Yes

2. Find the number for **n**. $(6 \times 6) + (8 \times 8) = (\mathbf{n} \times \mathbf{n})$ 10

3. The "5" facts are easy. What pattern do you see for the ones digits for the "5" facts? The ones' digits are either 0 or 5.

0 × 5 =	0
1 × 5 =	5
2 × 5 =	10
3 × 5 =	15
4 × 5 =	20

4. What patterns do you notice in the "9" facts? List all the "9" facts. See Discussion.

2 × 9 = 18	1 + 8 = ?
3 × 9 = 27	2 + 7 = ?
4 × 9 = 36	3 + 6 = ?

5. Can you list all the whole numbers less than 9×9 that are not products for any of the basic facts? 11, 13, 17, 19, 22, 23, 26, 29, 31, 33, 34, 37-39, 41, 43, 44, 46, 47, 50-53, 55, 57-62, 65-71, 73-80

158

THE FORESTER

Using the Ideas

Last summer Frank spent a week with his Uncle Bill, who is a forester.

1. Frank helped his uncle plant some small pine trees. Frank planted 3 rows of trees. He put 8 trees in each row. How many trees did he plant? 24

2. Frank learned how to build a fire without matches. Uncle Bill let Frank build the fires for 3 meals each day. Frank stayed 7 days. How many fires did he build? 21

3. Uncle Bill showed Frank some fish that were to be put in Blue Lake. The fish were in cans. Frank counted 9 cans. There were 8 fish in each can. How many fish were there? 72

4. Frank helped Uncle Bill put signs on trees. They posted signs on trees along 4 different trails. They put 9 signs on each trail. How many signs did they post? 36

5. Frank looked at Bald Mountain through a telescope. His uncle said, "It is 7 kilometres from this tower straight to the top of Bald Mountain. But since you can't fly, it is 5 times as far by the trail." How many kilometres long is the trail from the tower to the top of the mountain? 35 km

More practice, page A-20, Set 29

Using the Exercises

If necessary, help the children read the word problems on page 159. Urge them to analyze each problem, write down an equation using the correct operation, and solve it.

When the children have finished, discuss the exercises with them and have them check their papers. Allow freedom in discussing the ideas in some of the exercises. In problem 6, for example, a child may speculate about how the distance by trail to the top of Bald Mountain can be five times as great as it is straight from the tower.

Assignments (page 159)
Minimum: 1–5, oral. Average: 1–5.
Maximum: 1–5.

Follow-up

Some children may need more practice in analyzing problems than in multiplying. For these children, you might present problem situations and ask them to write **A, S,** or **M** to denote which operation(s) can be used to solve each problem. For more capable children, include problems whose solutions require several steps. These children might also be asked to write and solve the equations, if you wish. Sample problems follow.

1. 5 ice cream cones, 2 dips each. How many dips? (**M** $5 \times 2 = 10$)
2. 8-cent apple, 5-cent candy bar, a quarter for snacks. How much change? (**A, S** 8¢ + 5¢ = 13¢; 25¢ − 13¢ = 12¢)
*3. 5 sticks of gum per pack. 6 packs of gum. How many sticks? (**M** $5 \times 6 = 30$) 6 packs for 25¢. How many packs for 50¢? (**M** 50¢ = 2 × 25¢, and $2 \times 6 = 12$)

Resources for Active Learning

Developmental Math Cards, C¹14, D²5, E¹9, F¹15, Addison-Wesley.
Discovery, Section I, Activity 9, p. 11 Encyclopaedia Britannica Educational Corp. [Work with square numbers]
Math Workshop: Games and Enrichment Activities, "Square Numbers," pp. 22–23, Encyclopaedia Britannica Educational Corp.
Nuffield Project: *Computation and Structure* 3, "Square Numbers," pp. 53–55, Wiley.

Workbook, page 56

PAGES 160–161

Objective

Given the product and one factor of a basic multiplication fact, the child will be able to find the missing factor.

Preparation

To prepare for this lesson, use a mental arithmetic game to find missing factors. Say, for example: "I'm thinking of 28. If one factor is 7, what is the other?"

Practice in Multiplication Facts

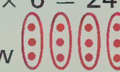
1. Give the product for n in each equation.

A $3 \times 4 = n$ 12	D $3 \times 7 = n$ 21	G $5 \times 3 = n$ 15	J $7 \times 1 = n$ 7
B $6 \times 3 = n$ 18	E $4 \times 6 = n$ 24	H $6 \times 5 = n$ 30	K $0 \times 8 = n$ 0
C $4 \times 5 = n$ 20	F $2 \times 9 = n$ 18	I $5 \times 2 = n$ 10	L $3 \times 8 = n$ 24

2. How can you use your answers in the equations above to find the missing factors in these equations? Find the missing factors.

A $n \times 2 = 10$ 5	D $n \times 5 = 30$ 6	G $n \times 9 = 18$ 2	J $n \times 8 = 0$ 0
B $7 \times n = 7$ 1	E $3 \times n = 12$ 4	H $5 \times n = 15$ 3	K $n \times 8 = 24$ 3
C $6 \times n = 18$ 3	F $4 \times n = 20$ 5	I $n \times 7 = 21$ 3	L $n \times 6 = 24$ 4

3. Draw a set of 12 dots on your paper. Ring sets of 3 to find how many threes in 12. Write a multiplication equation about this.



$4 \times 3 = 12$

4. Draw a set of 20 dots on your paper. Ring sets of 4 to find how many fours in 20. Write a multiplication equation about this.

$5 \times 4 = 20$

5. Find the missing factors.

A $3 \times n = 9$ 3
B $n \times 4 = 16$ 4
C $n \times 1 = 7$ 7
D $n \times 5 = 15$ 3
E $n \times 9 = 0$ 0
F $n \times 5 = 25$ 5
G $4 \times n = 4$ 1
H $n \times 3 = 12$ 4
I $2 \times n = 14$ 7
J $n \times 2 = 12$ 6
K $9 \times n = 18$ 2

think

When you multiply by me,
I really am quite tame.
No matter what the factor is,
The product stays the same.

WHO AM I? 0

6. Find the missing factors.

A	B	C	D	E	F	G	H
▥ 4	3	5	4	4	▥ 3	▥ 9	▥ 5
×3	× ▥ 3	× ▥ 2	× ▥ 0	× ▥ 3	×5	×2	×5
12	9	10	0	12	15	18	25

160

Discussion

Just as some earlier lessons stressed finding the difference by finding the missing addend, this lesson begins the development of finding quotients by finding a missing factor. On the chalkboard write several equations like these:

$5 \times n = 15$, $3 \times n = 18$, $n \times 7 = 28$, $8 \times n = 40$.

Instruct the children to think of the multiplication facts and thus find the missing factor. Allow children to use their multiplication tables if they wish.

After working through exercises 1 and 2 with the children, present some chalkboard examples in vertical notation, as in exercise 6 at the bottom of page 160. When written in vertical notation, missing-factor problems are commonly called "reconstruction problems"; these are generally more difficult for children.

The *Think* problem on page 160 is more difficult than some, but most children will understand it when the correct answer is given.

Assignments (page 160)

Minimum: 1, 3–4. Average: 1–5. Maximum: 1–6.

Solving Story Problems

THE POST OFFICE

1. Jill bought eight 6-cent stamps. How much did they cost? 48¢

2. She bought six 8-cent stamps. How much did she spend for them? 48¢

3. Jill saw some 1-cent stamps. She counted 3 rows and 9 stamps in each row. How many 1-cent stamps did she see? 27

4. 6-cent stamps come to the post office in sheets with 10 rows and 10 columns. How many stamps are in each sheet? 100

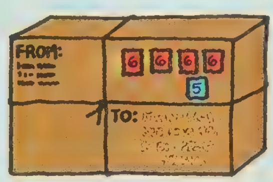
5. Jill bought 24 airmail stamps. She bought 4 times as many as Kay. How many airmail stamps did Kay buy? 6

6. Kay pasted 15 stamps in her collection book. She had 5 rows. How many were there in each row? 3

7. Which cost more, nine 5-cent stamps or seven 6-cent stamps?

8. Jan spent 32 cents for 8-cent stamps. How many stamps did she buy? 4

★ 9. Bill needed 29 cents' worth of stamps to mail his package. He had only 5-cent, 6-cent, 7-cent, and 8-cent stamps. He wanted to use only **2 different kinds** of stamps. One way to stamp the package is given in the picture.



A Find another way, using 7-cent and 8-cent stamps. Three 7-cent stamps and one 8-cent stamp

B Find another way, using 5-cent and 8-cent stamps. Three 8-cent stamps and one 5-cent stamp

C Find another way. Three 5-cent stamps and two 7-cent stamps

More practice, page A-21, Set 30

Mathematics

Multiplication and division are related, just as addition and subtraction are related. Finding the missing addend is like finding the difference, and finding the missing factor is like finding the quotient. If you know the solution of $4 \times n = 20$, then you know the solution of $20 \div 4 = n$.

Follow-up

A worksheet which emphasizes comparisons can be used to provide a review of combinations. You might follow this example.

Use <, >, or = in each ○ to make a true statement.	
$4 + 6 \bigcirc 6 \times 4$	$5 + 7 \bigcirc 17 - 5$
$3 \times 3 \bigcirc 3 + 3$	$18 - 8 \bigcirc 2 + 6$
$8 \times 5 \bigcirc 40 + 2$	$5 \times 9 \bigcirc 8 \times 6$
$7 + 3 \bigcirc 7 - 3$	$12 + 7 \bigcirc 30 - 9$
$9 \times 2 \bigcirc 20 - 5$	$6 \times 6 \bigcirc 5 \times 7$

The Combo cards made for missing addends in subtraction combinations may be used here and later for missing factors. Games like this will encourage children to memorize the facts.

Duplicator Masters, page 42
Workbook, page 57
Skill Masters, page 42

Using the Exercises

Before assigning the word problems on page 161, you might work through one or two, reviewing problem-solving guidelines (What do I know? What must I find? What do I do? Does my answer make sense?). Since some of these problems involve finding a missing factor, urge the children to analyze each problem independently. Also note that the value of the stamp given in problem 4 is irrelevant.

Some children may need help reading through the problems on page 161, but urge them to try to analyze most of the problems for themselves. You may want to allow time for the children to write the equation as each problem is read, and then allow more time after the reading is completed for solving and checking.

Assignments (page 161)

Minimum: 1–8, oral. Average: 1–8. Maximum: 1–9.

PAGES 162–163

Objective

Given two sets (each containing less than 10 elements), the child will be able to find the elements in the Cartesian product of the sets and write and solve a corresponding multiplication equation.

Preparation

Materials

crayons; scissors; paper; flannelboard (optional)

You may omit formal preparation for this lesson and begin immediately with the text. However, if you prefer, you could review the multiplication combinations with a brief oral game.

Investigation

In this investigation, the child explores the pairings that are possible between a set of 4 and a set of 2. By actually cutting out and coloring the possibilities, the child will discover that he needs 4 circles (one for each color) and 4 squares (one for each color). Thus, he sees that he has 2 × 4, or 8, possibilities.

If some children finish this activity quickly, you might suggest other sets, such as a triangle, square, and circle, and ask them to see how many possibilities there are with 4 colors and 3 figures; or have them introduce a new color and find 5 × 2 or 5 × 3. The pairing is complete only when each crayon has been matched with both (or all) the figures. The figures may be traced, or they may be drawn freehand even though freehand figures would not be very accurate squares and circles. But since the purpose here is pairing, the perfection of the figures is not important.

How are pairing and multiplication related?

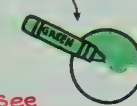
Investigating the Ideas

4 crayons

square circle

2 types of figures

You are to choose **1 crayon** and **1 figure** to color.
One choice might be green, **circle**. You would do this.



?	How many different choices do you think there are? 8	Show them.

See Investigation.

Discussing the Ideas

Sue is getting her mother a birthday present.
She will get her an apron or gloves.
She will put the gift in one of the 3 boxes shown below.

What might Sue's mother find when she unwraps this package?

1. Which gift and box would you choose? Answers will vary.
2. Sue finally decided to get the gloves. She put them in the box with the stars. What would your choices have been? Give as many different choices as you can. See Discussion.
3. Solve: Number of gifts to choose from | Number of boxes to choose from | Number of different choices possible

$$2 \times 3 = n \quad 6$$

162

Discussion

One of the main points of this discussion is to show the relation of the number of possible pairings to multiplication. Have children use large crayons and squares and circles on the flannelboard, or colored chalk on the chalkboard, to demonstrate the pairings they found. On the chalkboard write the equation 4 × 2 = 8 and relate it to the sets of colors and figures. Follow a similar procedure to develop the pairings children found by using other sets.

For an especially effective discussion of the textual presentation, display three gift-wrapped boxes the same size and an apron and a pair of gloves. As the children make choices in response to exercise 3, record their selections on the chalkboard. Make sure they understand that there are 2 choices of presents and 3 choices of boxes, or 6 possible combinations:

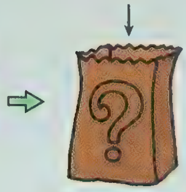
(1) Apron, flowered box; (2) Apron, starred box; (3) Apron, striped box; (4) Gloves, flowered box; (5) Gloves, starred box; (6) Gloves, striped box

Using the Ideas

1. You can have one piece of fruit. One piece of candy. One of each.

Apple
Banana
Pear
Orange

Candycane
Kiss
Lollipop



A How many different pieces of fruit are there? 4

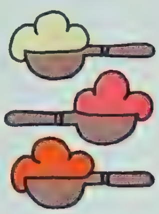
B How many different pieces of candy are there? 3

C Name all the possible choices that could be in the sack. Use the red letters to stand for each object. AC BC PC OC AK BK PK OK AL BL PL OL

D Solve: $4 \times 3 = n$ 12

2. 3 flavors of ice cream. 3 kinds of syrup. One of each.

Vanilla
Strawberry
Orange

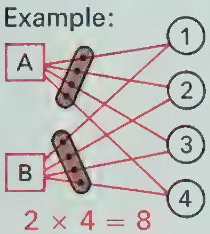


A How many different sundaes can you make? List them. 9 VCh SCh OCh VM SM OM VCa SCa OCa

B Solve: $3 \times 3 = n$ 9

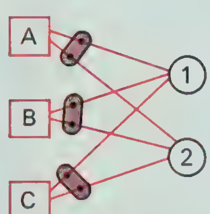
3. Write a multiplication equation for each picture. The small dots help you count the red lines that pair the squares with the circles.

Example:



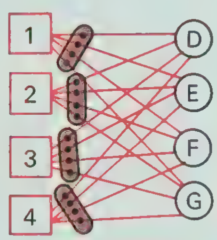
$2 \times 4 = 8$

A



$3 \times 2 = 6$

B



$4 \times 4 = 16$

Using the Exercises

The children should enjoy making the combinations suggested in exercises 1 and 2. You may wish to assign a symbol to each item to make the pairing easier. Paper or felt cutouts for demonstrating the choices will help show the number of different combinations possible.

In exercise 3, some children may have difficulty counting the small dots because they are relatively close together. When you discuss this exercise, ask the children if they can find the number of red lines without counting the small black dots. In this way, you can encourage them to discover that from each red box there is one red line to each circle. Therefore, each red box will have as many red lines as there are large black dots.

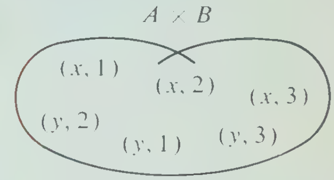
Assignments (page 163)*

Minimum: 1–2. Average: 1–2.
Maximum: 1–3.

Mathematics

Two sets, A and B, are illustrated below.

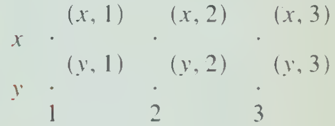
Consider the set of all pairs of objects such that the first object in each pair is from set A and the second is from set B. We call this set of pairs the *Cartesian product* of A and B, written $A \times B$ and read "A cross B." The set $A \times B$ consists of six ordered pairs.



Now relate these set concepts to the equation

$$2 \times 3 = 6.$$

The Cartesian product can also be conveniently represented in a figure called an array, as shown.



Resources for Active Learning

Developmental Math Cards, E-17, Addison-Wesley.

Mathex: Operations and Problem Solving No. 8, "Counting Pairs," pp. 16–17 (pupil pages 29–33), Encyclopaedia Britannica Publications Ltd.

Workbook, page 58

PAGES 164–165

Objective

Given word problems involving pairings and product sets, the child will be able to solve the problems by multiplication.

Preparation

Although both of these sets of problems are designed for independent work by the children, you will probably want to base a discussion on at least a few of them. Thus, you might omit any formal preparation and begin immediately with a discussion. However, if you prefer, you could use a short oral game to review multiplication facts.

Solving Story Problems

At the CARNIVAL

1. In the Fun House, there are 3 doors (A, B, and C) leading from the Noise Room to the Mirror Room. There are 4 doors (D, E, F, and G) leading from the Mirror Room to the Dark Room.

- **A** There are 12 ways to get from the Noise Room to the Dark Room. See if you can find them all.

AD BD CD
AE BE CE
AF BF CF
AG BG CG

- **B** Write a multiplication equation about this. $3 \times 4 = 12$

2. There are 2 sizes of ice cream cones. There are 6 flavors to choose from.

- **A** How many different ice cream cones could you buy? 12
- **B** Write a multiplication equation about this. $2 \times 6 = 12$

★**3.** David and Michael each had 1 ticket for rides. There were 4 things they could ride: the merry-go-round, ferris wheel, airplanes, and roller coaster.

- **A** Give all the ways David and Michael could use their tickets.
- **B** Write a multiplication equation about this. $4 \times 4 = 16$

Dmgr Mmgr	Dfw Mmgr	Da Mmgr	Drc Mmgr
Dmgr Mfw	Dfw Mfw	Da Mfw	Drc Mfw
Dmgr Ma	Dfw Ma	Da Ma	Drc Ma
Dmgr Mrc	Dfw Mrc	Da Mrc	Drc Mrc

164

Discussion

To enable the children to approach the problems on page 164 independently, work through a similar one with them. For instance, give them an opportunity to make several choices by letting them pretend they are spending a 15-cent allowance by choosing one nickel treat and one dime treat. Adjust the following list of choices to include the children's favorite treats.

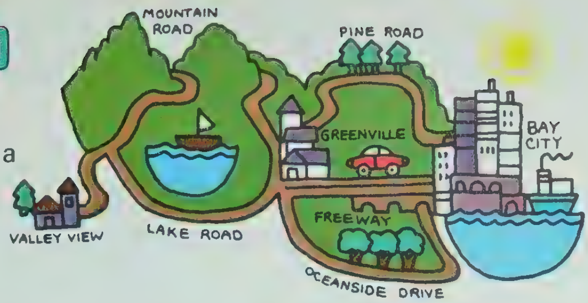
5¢	10¢
Gum (G) Candy Bar (CB) Sucker (S) Peanuts (P)	Frozen Bar (FB) Ice Cream (IC) Potato Chips (PC)

Have the children write the possible selections on the chalkboard. Arranging them in the familiar rectangular array is an efficient way to list them. You may wish to use initials for each choice to facilitate the pairing operation. The following array shows clearly that the number of choices is 4×3.

	FB	IC	PC
G	•	•	•
B	•	•	•
S	•	•	•
P	•	•	•

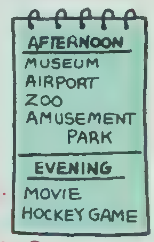
To introduce page 165, discuss exercise 1. Talk about the possible routes from Valley View to Bay City. Be sure that all children un-

Planning a Trip

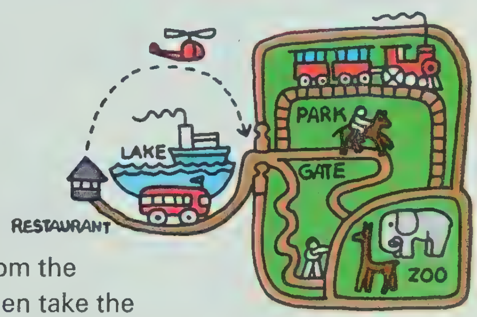


1. Stuart lives in Valley View. When he was helping plan a family trip to Bay City, he thought about these questions. See if you can answer them.
   - A In how many ways can we drive from Valley View to Greenville? 2
   - B In how many ways can we drive from Greenville to Bay City? 3
   - C What are the 6 ways to drive from Valley View to Bay City? See Answers.
   - D What multiplication equation can we write about these ideas? 2×3=6

2. Stuart and his sister made plans for the day in Bay City. Here is the list of things they wanted to do.
   - A How many afternoon choices are on the list? 4
   - B How many evening choices are on the list? 2
   - C Use the list to give all the possible ways they could spend the day. (You should find 8 ways.) See Answers.
   - D Write a multiplication equation about these ideas. 4×2=8



3. Stuart and his family can take the **helicopter,** the **boat,** or the **bus** from the restaurant to the park gate.
   To get from the gate to the zoo, they can ride the small **train,** ride the **pony,** or **walk.**



   - A Stuart wanted to take the boat from the restaurant to the park gate and then take the train from the gate to the zoo. Brenda wanted to take the helicopter and then the pony. Find as many more ways as you can to get from the restaurant to the zoo. See Answers.
   - B Write a multiplication equation about these ideas. 3×3=9

165

Follow-up
The children might enjoy making their own map and having group discussions of ways of travelling from one place to another; then they could relate possibilities to a multiplication equation. These maps and the corresponding equations might then be displayed around the classroom.

Answers, exercises 1C, 2C, and 3A, page 165

1. C Lake Road, Pine Road
   Lake Road, Freeway
   Lake Road, Oceanside Drive
   Mountain Road, Pine Road
   Mountain Road, Freeway
   Mountain Road, Oceanside Drive

2. C museum, movie
   museum, hockey game
   airport, movie
   airport, hockey game
   park, movie
   park, hockey game
   zoo, movie
   zoo, hockey game

3. A bus, train
   bus, pony
   bus, walk
   helicopter, train
   helicopter, walk
   boat, pony
   boat, walk

Workbook, page 59

derstand what the correct multiplication equation is.

When children have finished the problems, allow ample time for discussion. It would be helpful to develop an array for problems with which children had difficulty.

Assignments (page 164)
Minimum: 1–2, oral. Average: 1–2. Maximum: 1–3.

Assignments (page 165)
Minimum: 1–3, oral. Average: 1–3. Maximum: 1–3.

PAGES 166–167

Objective

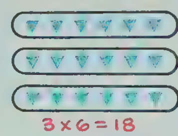
The child will demonstrate his ability to work with the concepts presented in this chapter.

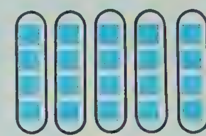
Preparation

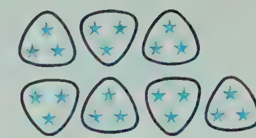
The preparation for this review lesson should depend upon the needs of your class. If the children have had difficulty with a particular aspect of multiplication, use this time to review it.

Reviewing the Ideas

1. Write multiplication equations for each exercise.

A 
$3 \times 6 = 18$

B 
$5 \times 4 = 20$

C 
$7 \times 3 = 21$

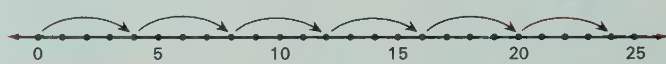
$6 \times 5 = 30$ D $5 + 5 + 5 + 5 + 5 + 5$

$4 \times 8 = 32$ E $8 + 8 + 8 + 8$

$7 \times 4 = 28$ F $4 + 4 + 4 + 4 + 4 + 4 + 4$

$5 \times 2 = 10$ G $2 + 2 + 2 + 2 + 2$

H $2 \times 3 = 6$

I 

$6 \times 4 = 24$

J 

$4 \times 8 = 32$

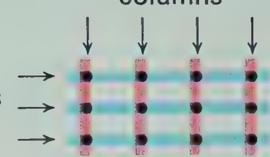
2. Write a multiplication equation about this set by

A thinking about the rows. $3 \times 4 = 12$

B thinking about the columns. $4 \times 3 = 12$

columns

rows



3. Find the missing factors.

A $6 \times 7 = n \times 6$ 7

B $9 \times 8 = 8 \times n$ 9

C $27 \times 56 = n \times 27$ 56

D $98 \times 76 = 76 \times n$ 98

4. Complete each statement.

A The product of any number and one is __?__. That number

B The product of any number and zero is __?__. Zero

5. Solve the equations.

A $5 \times 4 = (3 \times 4) + (n \times 4)$ 2

B $8 \times 3 = (4 \times 3) + (n \times 3)$ 4

C $7 \times 5 = (6 \times 5) + (n \times 5)$ 1

166

Discussion

Pages 166 and 167 may be used either as evaluation or as review. In either case, when you discuss the exercises with the children, review step by step each of the interpretations which were introduced for multiplication. First, go over multiplication associated with groups of sets, then repeated addition, the number line, skip counting, and product sets. Also, review the association of multiplication with rectangular arrays of objects. Use suitable exercises from page 166 to review the grouping, order, and multiplication-addition principles.

Provide any necessary reteaching before proceeding to the next chapter, on division, but do not expect children at this level to have complete mastery of the multiplication facts.

Treat the *Think* problem (page 167) as enrichment material for more capable children.

5. Find the products.

A 6×0 0	I 2×7 14	Q Since $9 \times 6 = 54$, we know $6 \times 9 = n$. 54
B 1×8 8	J 5×4 20	R Since $6 \times 7 = 42$, we know $7 \times 6 = n$. 42
C 5×3 15	K 2×5 10	S Since $8 \times 6 = 48$, we know $6 \times 8 = n$. 48
D 2×6 12	L 5×6 30	T Since $9 \times 8 = 72$, we know $8 \times 9 = n$. 72
E 0×9 0	M 2×9 18	U Since $5 \times 5 = 25$, we know $6 \times 5 = n$. 30
F 5×5 25	N 5×7 35	V Since $6 \times 6 = 36$, we know $7 \times 6 = n$. 42
G 7×1 7	O 2×8 16	W Since $7 \times 7 = 49$, we know $8 \times 7 = n$. 56
H 4×3 12	P 6×2 12	X Since $8 \times 8 = 64$, we know $9 \times 8 = n$. 72

6. Find the products.

A 3×3 9	E 4×5 20	I 4×7 28	M 3×8 24	Q 4×6 24	U 6×9 54
B 7×8 56	F 7×7 49	J 6×8 48	N 4×9 36	R 7×9 63	V 3×7 21
C 3×4 12	G 3×9 27	K 5×8 40	O 3×6 18	S 3×5 15	W 4×8 32
D 6×7 42	H 8×8 64	L 4×4 16	P 6×6 36	T 9×9 81	X 8×9 72

7. Complete the sentence.
Then find the product.

A For 3 sets of 10, we write ||||. 30
$3 \times 10 = n$ 30

B For 5 sets of 10, we write ||||. 50
$5 \times 10 = n$ 50

C For 8 sets of 10, we write ||||. 80
$8 \times 10 = n$ 80

D For 7 sets of 10, we write ||||. 70
$7 \times 10 = n$ 70

If it's products you must find,
You'll think I'm quite a guy.
I don't cause a single change
When I just multiply.

WHO AM I? One

8. Solve the equations.

A $(2 \times 7) + (3 \times 7) = n$ 35
B $(8 \times 7) + (2 \times 7) = n$ 70
C $(5 \times 9) + (5 \times 9) = n$ 90
D $(6 \times 8) + (4 \times 8) = n$ 80
E $(7 \times 7) + (3 \times 7) = n$ 70
F $(9 \times 8) + (1 \times 8) = n$ 80

Follow-up

A coded exercise is usually an exciting way for children to do necessary practice problems. Make a bulletin board display or worksheet which capitalizes on this interest in codes. Consider the following code and exercises.

E	G	H	I	K	M	N	O	R	T	W	!
12	13	14	15	16	17	18	19	20	21	23	25

Solve the following exercises and write the answers in the proper squares. Then decode the message according to the code given.

1. $(9 \times 2) - 0 = \square$
2. $(2 \times 9) + 1 = \square$
3. $7 \times 2 = \square$
4. $(5 \times 4) - 1 = \square$
5. $1 + (2 \times 8) = \square$
6. $(3 + 9) - 0 = \square$
7. $20 + 3 = \square$
8. $25 - 6 = \square$
9. $30 - 10 = \square$
10. $(8 \times 3) - 8 = \square$
11. $1 + 20 = \square$
12. $(9 \times 3) - 8 = \square$
13. $(10 + 10) - 2 = \square$
14. $(2 \times 5) + (1 \times 5) = \square$
15. $(2 \times 6) + (1 \times 1) = \square$
16. $10 + 4 = \square$
17. $(3 \times 7) + (0 \times 7) = \square$
18. $(5 \times 3) + (5 \times 2) = \square$

MESSAGE

Exercises:
1 2 3 4 5 6 7 8 9 10
Answers:
_18_ _19_ _ _ _ _ _ _ _ _
Letters:
N O H O M E W O R K

Exercises:
11 12 13 14 15 16 17 18
Answers:
_ _ _ _ _ _ _ _
Letters:
T O N I G H T !

(*Note:* The letters of the "message" are provided for your convenience. Do not include them in the chart or worksheet for the children.)

Resources for Active Learning

Developmental Math Cards, F#13, Addison-Wesley.

Math Activity Cards, "Chart," B2, Macmillan. [A multiplication practice grid]

Nuffield Project: *Computation and Structure* 3, "Dominoes," p. 34, Wiley.

Workbook, page 60

PAGES 168–169

Objective

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

To prepare for this lesson, use an oral chain game for mental practice (see page 152). For this lesson, however, the emphasis should be on the addition and subtraction facts rather than the more recently studied multiplication facts.

Keeping in Touch with Addition, Subtraction, Place value

1. Find the sums.

A	B	C	D	E	F	G	H
3	8	5	7	8	4	4	9
+9	+3	+7	+7	+5	+9	+7	+6
12	11	12	14	13	13	11	15

I	J	K	L	M	N	O	P
7	8	6	9	7	9	8	7
+8	+9	+8	+5	+9	+9	+8	+6
15	17	14	14	16	18	16	13

2. Find the differences.

A	B	C	D	E	F	G	H
11	12	16	11	12	13	14	14
−9	−8	−7	−8	−3	−8	−9	−7
2	4	9	3	9	5	5	7

I	J	K	L	M	N	O	P
11	17	13	12	15	16	18	15
−6	−8	−7	−5	−7	−8	−9	−9
5	9	6	7	8	8	9	6

3. Find the sums.

A	B	C	D	E	F
62	35	16	328	457	615
+23	+24	+51	+160	+132	+204
85	59	67	488	589	819

4. Find the differences.

A	B	C
89	68	586
−23	−51	−212
66	17	374

5. Find the sums.

A	B	C
38	65	73
+27	+25	+18
65	90	91

D	E	F
75	38	67
+86	+95	+67
161	133	134

think

The product is even
When I multiply.
On words like bicycle,
My nickname is "bi."

WHO AM I? 2

168

Discussion

Before assigning these pages, it would be helpful to present on the chalkboard several addition and subtraction problems that involve regrouping. Ask volunteers to give the solutions and then explain their techniques to the rest of the class.

This is also a good time to review place value for large numbers. You might extend exercises 8 and 9 by reading aloud numbers such as "fifty-nine thousand, six hundred twenty-eight." The children should write the proper numerals on their papers and then write the place-value notation (e.g., 59 628; 50 000 + 9000 + 600 + 20 + 8). They could take turns working at the chalkboard, while those at their seats check for accuracy.

The chalkboard is also a good place for children to work reconstruction problems like those in exercise 10, page 169. It may help to pair a more capable child with one who needs practice on this kind of problem and to allow them to make up problems for each other. Exercise 10, which is starred, and the *Think* problems are enrichment material for more capable children.

6. Find the differences.

A 36 − 17 = 19	B 43 − 24 = 19	C 52 − 17 = 35	D 58 − 29 = 29	E 64 − 27 = 37	F 80 − 68 = 12
G 123 − 47 = 76	H 144 − 58 = 86	I 163 − 76 = 87	J 150 − 68 = 82	K 131 − 75 = 56	L 155 − 67 = 88

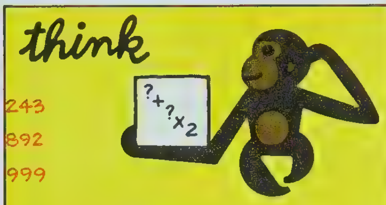
7. Find the sums.

69 A 60 + 9 E 500 + 40 540
247 B 200 + 40 + 7 F 40 + 200 + 3 243
614 C 600 + 10 + 4 G 90 + 2 + 800 892
902 D 900 + 2 H 9 + 900 + 90 999

8. Find the sums.

A 3000 + 600 + 20 + 8 3628
B 4000 + 200 + 80 + 9 4289
C 200 + 6 + 4000 + 30 4236
D 50 + 6000 + 200 + 6 6256

think



If you add me to myself
Then multiply by two,
It's 12 plus 4 you're
sure to get.
You need no other clue.

WHO AM I? 4

9. Study the example. Then write each number as shown in the example.
Example: 8295 = 8000 + 200 + 90 + 5

A 654	B 892	C 4347	D 8126	E 87 265	F 27 615
600+50+4	800+90+2	4000+300+40+7	8000+100+20+6	80 000+7000 +200+60+5	20 000+7000 +600+10+5

★ 10. Copy each problem. Give the missing digit for each ▥.

A 3▥▥9	B 9▥3	C 84	D 8▥4▥9	E 63▥5
−27	−36	−3▥7	−237	−207
12	5▥7	4▥7	6▥2 1	▥▥8 42

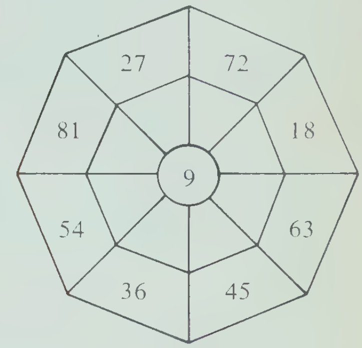
You are invited to explore

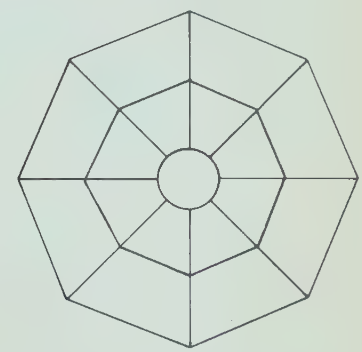
ACTIVITY CARD 7
Page 312

169

Follow-up/Practagons

For more practice in finding missing factors, rerun the masters suggested in the follow-up section for pupil pages 150 and 151. Fill them in as suggested by the following figure, making sure to leave at least one practagon empty so that children can make up problems on their own.





Resources for Active Learning

Developmental Math Cards, C18, E17, E18, G9, Addison-Wesley.

Mathex: Operations and Problem Solving No. 8, "Hexagon Drill—Activity 2," p. 18 (pupil page 21), Encyclopaedia Britannica Publications Ltd.

Pages 170–195

General Objectives

To introduce division concepts
To relate division to repeated subtraction
To relate division to multiplication
To relate division to the number line
To introduce word problems involving division
To develop skills in finding quotients related to basic multiplication facts

Like the other operations, division is introduced first in terms of concepts of sets. In order to relate division to the idea of repeated subtraction, the first interpretation deals with partitioning a given set into a number of equivalent subsets. Later in the chapter, another set interpretation of division (the number in each set) is presented and emphasized primarily as a key for solving certain types of word problems.

Although the initial introduction to division is in terms of sets, the working definition is in terms of multiplication. After a relationship is established between division and multiplication, the set concepts associated with division are used primarily as aids in relating division to certain problem situations which the children may encounter. For example, it may turn out that a child will use more than one concept of division in working a single problem. If he must find the number of sets or the number of objects in each of several equivalent sets, he may decide that division is the correct operation for the particular problem. After the child has written a division equation, he need not use ideas of sets to solve it; he may solve the equation either by repeated subtraction or by thinking of the quotient as a missing factor. The important thing is that he is able to interpret word problems and the set concepts, determine the correct operations, and then consider the problems with an understanding of the operation. The method used to solve the equation can be a method other than the one suggested in the problem.

The relationship between division and repeated subtraction is developed for two reasons. First, it provides an excellent opportunity to relate certain ideas of division to those of multiplication. (In earlier chapters, multiplication was related to repeated addition, and now division is related to repeated subtraction.) Second, and perhaps more important, relating division to repeated subtraction prepares the children for a thorough understanding of the long-division process, which will be approached in a later chapter.

Children should be led to understand why the subtraction process is involved in long division. A careful development of the relation between division and repeated subtraction will form the necessary background for such an understanding. Relating division to operations demonstrated on the number line is nothing more than relating division to repeated subtraction or to "backward skip counting."

Mathematics

A formal definition of division follows.

If a, b, and c are whole numbers such that

$b \neq 0$ and $a \times b = c$,

then

$$a = c \div b.$$

Note in particular that $c \div b$ is the first factor in the equation $a \times b = c$.

$$\boxed{\underbrace{a}_{c \div b} \times b = c \qquad (b \neq 0)}$$

For example:

$2 \times 3 = 6 \qquad 2 = 6 \div 3$

The relationship between division and multiplication can be brought into clearer focus with the following example:

$n \times 4 = 12 \qquad 12 \div 4 = n$

These numbers are the same.

That is, finding the quotient is equivalent to finding a missing factor. This idea parallels the addition-subtraction relationship, with the single exception that in division one of the factors (the second in our definition) cannot be zero.

Teaching the Chapter

Materials

Boxes (to hold counters)
Counters (at least 24 per child)
Felt objects
Flannelboard
Graph paper (1-cm for children; larger grid for demonstrations)
Objects for set demonstrations and manipulation
Overhead projector (optional)
Rulers (centimetre)
Scissors
Yarn

Vocabulary

divide	missing factor
division	quotient
function rule	

In the early part of the chapter, the use of set materials in children's investigations and in your demonstrations should be restricted to finding the number of subsets of a given size within a set. For example, in the first lesson you could exhibit 18 objects and have the children circle or remove 3 at a time to show that there are 6 sets of 3 in 18, or $18 \div 3 = 6$.

In the latter part of the chapter, however, you will want to use set materials to demonstrate partitioning a set of objects into a given number of equivalent subsets. For example, you could demonstrate a set of 18 partitioned into 3 subsets

of the same number and encourage the children to make the observation that each set would contain 6 objects; thus, $18 \div 3 = 6$.

As mentioned in other sections of this manual, the use of manipulative materials and activities should be stressed as long as the children seem to benefit from them. It is recommended that you have the materials in a convenient location so that the children who wish to may use them to work out any of the exercises. Although it is hoped that most children at this level will gradually dispense with them for the basic operations, you should make no effort to discourage their use by any child still dependent on the materials for the newer operations such as multiplication and division. Under normal circumstances, the child will dispense with them naturally when he is ready to work without them.

Each word in the vocabulary list is important in the development of this chapter. The list is brief, so the children should have little difficulty mastering the meaning and use of these words.

Lesson Schedule

Plan to cover this material in about three-and-a-half weeks, with an upper limit of four-and-a-half weeks. Again, you should adjust the coverage of this chapter to the abilities and previous achievements of your children.

Evaluation of Progress

In evaluating children's progress in this chapter, one of the main items to consider is their ability to find quotients by thinking about missing factors. When they are asked to find the quotient $48 \div 6$, it is desirable that they think, "What number times six gives forty-eight? Six eights are forty-eight; therefore, forty-eight divided by six equals eight." Since we do not emphasize memorization of subtraction or division facts in this series, it is vital that the children become proficient in this method of thinking.

We also want children to understand the various set concepts related to division. They should be able to see the relationship between division and the process of finding both the number of equivalent sets and the number in each of so many equivalent sets.

As we pointed out earlier, set concepts in word problems merely trigger the division operation. Once the child recognizes that he should divide to get the correct answer, we hope he thinks about missing factors to find the correct quotient. However, do not penalize slower children if they continue to use repeated subtraction in order to find quotients.

When you design tests, your main concern should be threefold: first, to evaluate the children's understanding of the general concept of division; second, to determine their skill in finding the quotients related to basic multiplication facts; and third, to test the children's ability to interpret various kinds of word problems.

Pages 194 and 195 provide a cumulative review of concepts and skills thus far developed in Book 3. The chapter review on pages 192 and 193 can serve either as an evaluation or as a guide in designing tests of your own.

Resources for Active Learning

General Activities

[In addition to the activities listed below, refer again to those listed for Chapter 6.]

Mathex: Operations No. 3, "Mul-Div Hundred Square," pupil pages 39–45, Encyclopaedia Britannica Publications Ltd.

Math Workshop: Games and Enrichment Activities, "Multiplication and Division Facts," pp. 62–79, Encyclopaedia Britannica Educational Corp.

Nuffield Project: *Computation and Structure* 3, "Simple Sharing," pp. 61–64, Wiley

Manipulative Devices

Cuisenaire Cubes, Squares, and Rods (Cuisenaire Co.)

Dienes Multibase Arithmetic Blocks (Herder and Herder)

"Invicta" Math Balance (Math Media; Selective Educational Equipment)

Pegboards (school supplier)

SEE Calculator (Selective Educational Equipment)

Sigma Chips (Sigma, Scott Scientific)

Unifix Math Lab Kit (Educational Teaching Aids; Math Media; Responsive Environments Corp.)

Commercial Games

(Repeat any of the games from Chapters 3, 5, and 6 that help to develop competence in the basic facts.)

Twin Choice (Holt, Rinehart and Winston). A game somewhat like Hearts that sharpens computational skills.

Objective

Given a set of objects such as 12 counters, the child will be able to divide the set into equivalent sets and write a corresponding division equation.

Preparation

Materials

paper cups; counters (at least 24 per child); boxes

To prepare for this lesson, plan a short oral review of multiplication facts and missing factors. For example, say: "I'm thinking of 30. If one factor is 5, what is the other?" Include factors of 24 in your review.

Investigation

This investigation might be called the inverse of the first investigation in the preceding chapter. The children start with a collection of counters and separate them into equivalent sets and then determine the number of equivalent sets. Children should interpret what they are to do from the illustration in the text, but make sure they realize that they must put the same number of counters in each cup and have none left over. It would be helpful to suggest other numbers in a chart on the chalkboard, such as:

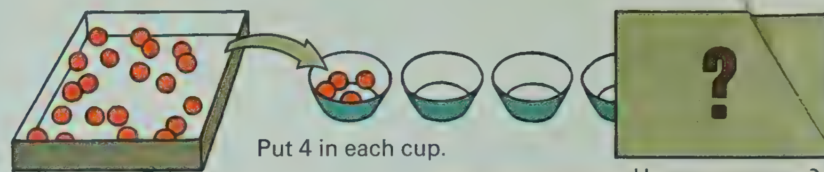
Total	*Number in each*	*Number of cups*	*Division equation*
18	3	?	18 ÷ 3 = ?
18	9	?	18 ÷ 9 = ?
27	3	?	27 ÷ 3 = ?
20	?	?	20 ÷ ? = ?

Allow the children to investigate any number of counters in a cup and then, if necessary, struggle with the problem of leftover counters. However, observe the children carefully so that you can aid any child who appears confused.

7 Division

● *What is division?*

Investigating the Ideas



24 counters

Put 4 in each cup.

How many cups?

Can you use counters and containers to help you complete the chart?

See Investigation.

Number of counters	Counters in each cup	Number of cups	Division equation
24	4	6 ?	24 ÷ 4 = ? 6
24	2	12 ?	24 ÷ 2 = ? 12
24	3	8 ?	24 ÷ 3 = ? 8
24	?	?	

Answers will vary.

Discussing the Ideas

1. Give the missing numbers. Write a division equation for B.

We see — We think — We write

A 12 tomatoes

4 in each box

There are 3 fours in 12. → 12 ÷ 4 = 3

(Read: 12 divided by 4 equals 3.)

B 20 books

5 in each stack

There are 4 fives in 20. → ? 20÷5=4

2. Make up an example like those above.
Sample: 30 chairs; 6 in each row; 5 sixes in 30
30÷6=5

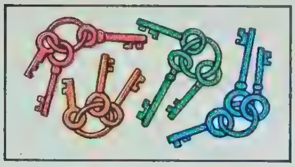
Discussion

Spend ample time discussing the sets and equations discovered in the investigation. Use the counters or other objects to demonstrate a few examples. For instance, place 18 felt objects on the flannelboard. Ask the children how many sets of 3 they can find in 18. Let volunteers circle groups of 3 with yarn until all 6 groups are circled. Write on the chalkboard "6 sets of 3 in 18," and beside it put the division equation, 18 ÷ 3 = 6. Call attention to the new operation *division;* the phrase *divided by;* the symbol for this phrase, ÷ ; and the term *quotient.*

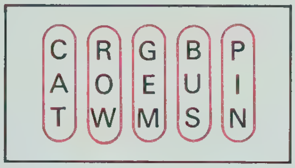
As you read and work through the discussion exercises, emphasize again the words *division* and *divided by,* and be sure the children understand that the symbol ÷ means *division,* and is read *divided by.* It would be helpful to use demonstration sets as you discuss exercises 1A and 1B. You may let several volunteers give an example for exercise 2.

Using the Ideas

1. A How many keys in all? 12
   B How many on each ring? 3
   C How many rings? 4
   D How many threes in 12? 4
   E Solve: $12 \div 3 = n$ 4

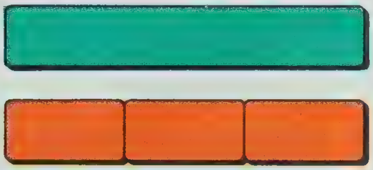


2. A How many letters in all? 15
   B How many in each set? 3
   C How many sets? 5
   D How many threes in 15? 5
   E Solve: $15 \div 3 = n$ 5



3. A How many shoes in all? 10
   B How many in each pair? 2
   C How many pairs? 5
   D How many twos in 10? 5
   E Solve: $10 \div 2 = n$ 5

4. A How long is the green strip? 6
   B How many of the red strips are needed to match the green strip? 3
   C How many twos in 6? 3
   D Solve: $6 \div 2 = n$ 3



When you solved the equations above, you found the **quotient**.

5. Draw 21 dots. Ring as many sets of 3 as you can.
   
   A How many sets of 3 did you find? 7
   B How many threes in 21? 7
   C Find the quotient: $21 \div 3 = n$ 7

6. A Draw 24 dots. Ring as many sets of 4 as you can.
   
   B Solve: $24 \div 4 = n$ 6

Using the Exercises

On page 171, ask the children to answer the questions in exercises 1 through 6 independently if they can. Otherwise, continue to work through the exercises with them, letting the children supply the answers. When they finish, point out the screened statement, which says they have been finding the *quotient* when they solved the equations for *n*. Discuss any exercises with which the children had difficulty.

Assignments (page 171)
Minimum: 1–6. Average: 1–6.
Maximum: 1–6.

Mathematics

When we have defined division in terms of multiplication by treating quotients as missing factors, the relationships between division and sets, as well as between division and repeated subtraction, should become apparent.

In this lesson we associate division with finding a number of equivalent sets of a given size that can be formed from a given set. To see how this idea relates to multiplication, we need only look at our set interpretation of multiplication. That is, the equation $3 \times 4 = 12$ is associated with 3 sets, each containing 4 elements. In division, when we attempt to find the number of sets of 4 in a set of 12, we are actually finding the factor 3 in the multiplication equation $3 \times 4 = 12$. Note that 3 is associated with the number of sets and 4 with the number in each set.

Follow-up/Practice Cards

To help the children review multiplication, give each child a 10 by 15 centimetre piece of tagboard or an index card. Tell him to choose six multiplication facts that are hard for him, to write them on one side of the card, and to label them as shown. Then have him write the answers for *A* through *F* on the back.

Practice Cards

(front)

A	B	C	D	E	F
8	4	9	7	6	3
×3	×7	×5	×6	×8	×9

JANE

(back)

A	B	C	D	E	F
24	28	45	42	48	27

After each child has put his name on his card for identification, everyone may exchange cards and work a classmate's problems, writing the answers on a separate sheet; or each may choose a partner to study the facts with him.

Resources for Active Learning

Mathex: Operations No. 3, "Later Multiplication and Division Activities – Activity 1," pp. 30–31 (pupil pages 37–38), Encyclopaedia Britannica Publications Ltd.

Workbook, page 61

PAGES 172–173

Objective

Given a rectangular array which has been divided into equivalent sets, the child will be able to solve simple division equations by relating each equation to the array.

Preparation

Materials

graph paper; scissors

To prepare for this investigation, review the term *rectangle,* pointing out examples of objects which have rectangular shapes. It would also be helpful to display a rectangle divided into squares. The term *unit* as used in the investigation refers to units of length.

Investigation

After you have read the directions with the class and they have shown their ability to perform the activity suggested by the investigation, write on the chalkboard a list of some other rectangles for them to cut out, such as:

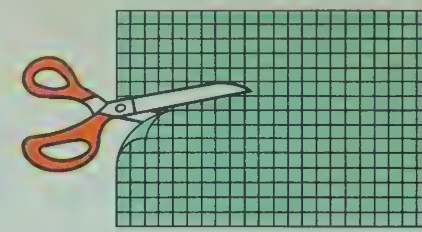
24-unit rectangle, 4 units wide
32-unit rectangle, 8 units wide
18-unit rectangle, 3 units wide
27-unit rectangle, 3 units wide

In this investigation, the sets the children investigate are the sets of rows or sets of columns in each rectangle. You might challenge the more capable children to write a division equation for each rectangle they cut.

Can "rectangular sets" help you find quotients?

Investigating the Ideas

How long do you think a rectangle with 28 squares would be if it were 4 units wide? 7 units long



? Can you cut from graph paper a rectangle with 30 squares that is 5 units wide? See Investigation.

Discussing the Ideas

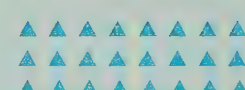
1. What division equation can you write about the rectangle you cut out? $30 \div 5 = 6$

2. A Can you find some other rectangles that have 30 squares?
   B Write division equations for each one you find. $30 \div 15 = 2$ $30 \div 2 = 15$ $30 \div 10 = 3$ $30 \div 3 = 10$ $30 \div 6 = 5$

3. Give the missing numbers and solve the equations.

	Rectangle	Width	Length	Division equation
A	12 squares	2 units	6 units	$12 \div 2 = n$ 6
B	15 squares	3 units	5 units	$15 \div 3 = n$ 5
C	18 squares	3 units	6 units	$18 \div 6 = n$ 3
D	16 squares	4 units	4 units	$16 \div 4 = n$ 4

4. Explain how you can use this "rectangular set" to get 2 division equations.



$3 \times 8 = 24$

Sample answer: Ring 8 sets of 3 to get $24 \div 3 = 8$, and ring 3 sets of 8 to get $24 \div 8 = 3$.

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Discussion

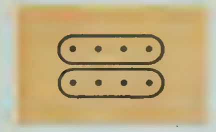
For an effective discussion of the results in this investigation, use 2-cm graph paper to demonstrate a few rectangles the children might have cut. For each rectangle, give the corresponding division equation, stressing that as soon as we know the total number of units and the number of units in each row we can write a division equation to find the number of units in each column.

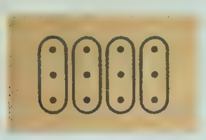
It would also be helpful to present a set demonstration similar to the one suggested on page 170. Place 15 felt objects on the flannelboard, and ask a child first to group them into sets of 3 and then to count the number of sets as he removes them or circles them with yarn.

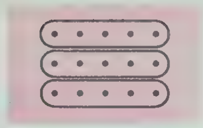
The exercises at the bottom of page 172 may also be used for demonstrations or assigned as independent work if the children are capable.

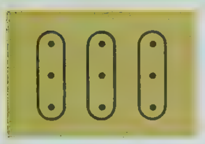
Using the Ideas

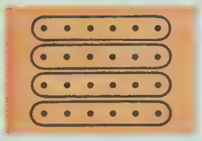
1. Look at the set. Then answer the question.

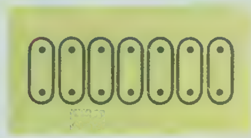
A 
How many sets of 4 in a set of 8? 2

B 
How many sets of 3 in a set of 12? 4

C 
How many sets of 5 in a set of 15? 3

D 
How many threes are in 9? 3

E 
How many sixes are in 24? 4

F 
How many twos are in 14? 7

2. Solve the equations. Your work in exercise 1 will help you.

A $8 \div 4 = n$ 2
B $12 \div 3 = n$ 4
C $15 \div 5 = n$ 3
D $9 \div 3 = n$ 3
E $24 \div 6 = n$ 4
F $14 \div 2 = n$ 7

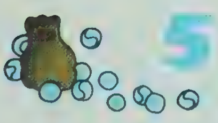
Short Stories

1 15 chocolates. 3 in each row. How many rows? 5

2 20 boys. 5 on a team. How many teams? 4

3 18 letters. 3 in each word. How many words? 6

4 24 books. 6 in each stack. How many stacks? 4



5 15 marbles. 5 in a sack. How many sacks? 3

6 16 white rats. 4 per cage. How many cages? 4

7 30 bottles of pop. 6 bottles in a carton. How many cartons? 5

8 10 kilograms sugar. 2-kilogram bags. How many bags? 5

Using the Exercises

Work through the first one or two problems on page 173 with the children. Before you ask them to do the short story problems, remind them that they should use the problem-solving guidelines if they find them helpful. (What do I know? What must I find? What do I do? Does my answer make sense?) Then ask the children to try to work the rest of the problems independently. You might ask those who finish quickly to write an appropriate division equation for each of the short stories.

Assignments (page 173)
Minimum: 1–2; odd-numbered short stories.
Average: 1–2; short stories.
Maximum: 1–2; short stories.

Follow-up
Give the children a worksheet from time to time, asking them to list the operation(s) needed to solve each problem.

1. Write +, ×, −, or ÷ for each ○.
   A. 3 ○ 3 = 27 ○ 3
   B. 7 ○ 7 = 55 ○ 6
   C. 12 ○ 4 = 15 ○ 12
   ⋮
2. Thirty children went on a field trip to the museum. If five children rode in each car, how many cars were needed for the trip?

Resources for Active Learning
Mathex: Operations No. 3, "Array Building – Activity 4," pp. 33–34, Encyclopaedia Britannica Publications Ltd.

Duplicator Masters, page 43
Workbook, page 62

Objective

Given a division problem, the child will be able to find the quotient by repeated subtraction of a single-digit divisor.

Preparation

Materials

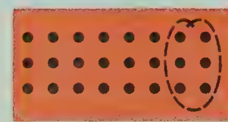
felt discs (28)

To prepare for this lesson, write $3 + 3 + 3 + 3 + 3 = 15$ on the chalkboard. Ask the children to suggest another way of thinking about this equation. When someone suggests multiplication, write $5 \times 3 = 15$. Then mention that subtraction and division may be related to each other similarly, and proceed to the demonstration suggested in the discussion section.

Can you use subtraction to help you find quotients?

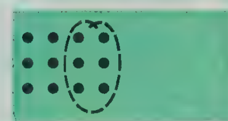
Discussing the Ideas

1. Think about removing the dots in the ring.
Then solve the subtraction equations.

A

$24 - 6 = n$ 18

B

$18 - 6 = n$ 12

C

$12 - 6 = n$ 6

D

$6 - 6 = n$ 0

2. A Study exercise 1 and tell how many sixes are in 24. 4
   B Solve this equation. $24 \div 6 = n$ 4

3. Study each example. Solve and explain the equation.

A
We can use addition to help us find products. $\boxed{4 \times 5}$
Since $5 + 5 + 5 + 5 = 20$, we know that $4 \times 5 = n$. 20

B
We can use subtraction to help us find quotients. $\boxed{18 \div 6}$

$$\begin{array}{r} 18 \\ -6 \\ \hline 12 \end{array} \quad \begin{array}{r} 12 \\ -6 \\ \hline 6 \end{array} \quad \begin{array}{r} 6 \\ -6 \\ \hline 0 \end{array}$$

Since we subtracted 6 three times, we know that $18 \div 6 = n$. 3

4. Explain how you could find $36 \div 3$ by subtracting. Subtract 3 twelve times.
Solve: $36 \div 3 = n$ 12

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Discussion

Exhibit a set of 28 discs on the flannelboard. Have a child come to the flannelboard and remove the discs, 7 at a time. Each time a set of 7 is removed, write a subtraction equation which illustrates what happened. For example, the first time a child removes 7 discs, write $28 - 7 = 21$. The next time, write $21 - 7 = 14$, and so on to $7 - 7 = 0$. Then have the children count aloud the number of times they are able to remove sets of 7. Observe with them that this shows that there are 4 sevens in 28, or that $28 \div 7 = 4$. Repeat this activity with other sets, first writing subtraction equations to illustrate what happens and then writing the corresponding division equation.

Relate this procedure to the first two discussion exercises. Then, for exercise 3, review the concept of multiplication as repeated addition and relate it to the interpretation of division as repeated subtraction.

If time permits, you might let the children work in groups of two or three to try to work out exercise 4 by using repeated subtraction. You may include other easier equations such as $27 \div 3$, $32 \div 4$, $45 \div 9$. Then use $36 \div 3$ as a basis for summarizing how division may be thought of as repeated subtraction.

Using the Ideas

1. A Find these differences.

24	20	16	12	8	4
−4	−4	−4	−4	−4	−4
20	16	12	8	4	0

B How many times did you subtract 4 ? 6
C How many fours are in 24 ? 6
D Write a division equation about this. 24 ÷ 4 = 6

2. A Find these differences.

21	18	15	12	9	6	3
−3	−3	−3	−3	−3	−3	−3
18	15	12	9	6	3	0

B How many times did you subtract 3 ? 7
C How many threes are in 21 ? 7
D Write a division equation about this. 21 ÷ 3 = 7

3. A Find these differences.

30	25	20	15	10	5
−5	−5	−5	−5	−5	−5
25	20	15	10	5	0

B How many times did you subtract 5 ? 6
C How many fives are in 30 ? 6
D Write a division equation about this. 30 ÷ 5 = 6

4. Solve the equations.

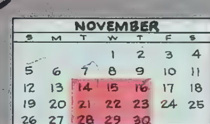
A
$15 - 5 = n$ 0
$10 - 5 = n$ 5
$5 - 5 = n$ 0
$15 \div 5 = n$ 3

B
$28 - 7 = n$ 21
$21 - 7 = n$ 14
$14 - 7 = n$ 7
$7 - 7 = n$ 0
$28 \div 7 = n$ 4

★ **5.** Find the quotients.

A $28 \div 2$ 14
B $42 \div 3$ 14
C $48 \div 4$ 12
D $75 \div 5$ 15

think



1. Find the sum along each arrow. 66
2. Try this with any 3-by-3 square on any calendar.

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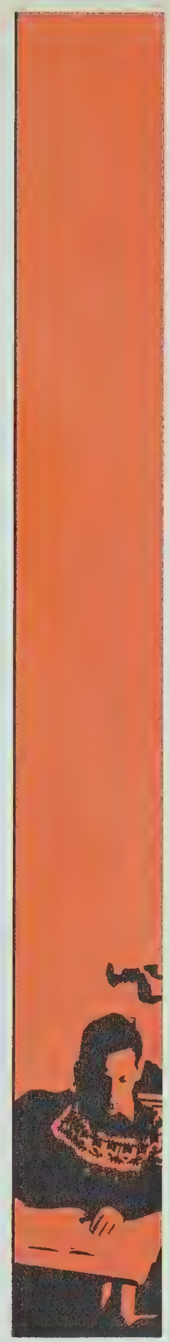


Follow-up

To keep children on their toes with respect to skills in operations, create some chain games which include all four operations. Remind the children that each answer is used in the next part of the chain. Make the games more difficult to challenge capable children.

Start — End

5	×5		−5		+10		÷3	10

Start — End

8	+9		−5		÷2		×9	54

Start — End

12	−4		×5		÷8		+20	25

Resources for Active Learning
Developmental Math Cards, G[1]16, Addison-Wesley. [An approach to division]

Workbook, page 63

Using the Exercises

Assign the exercises on page 175 as independent work. You might want to write other division equations on the chalkboard to be solved by children who finish the exercises quickly. Allow time for questions and checking papers when the children have finished.

All the children will enjoy an opportunity to discover the pattern for the calendar dates shown in the *Think* problem. If time permits, allow volunteers to check on whether the pattern holds for 3-by-3 squares for various months on your classroom calendar.

Assignments (page 175)
Minimum: 1–4. Average: 1–4.
Maximum: 1–5.

PAGES 176–177

Objective

Given a division equation, the child will be able to solve it by using a number line.

Preparation

Materials

centimetre rulers (1 per child)

Have the children take out their centimetre rulers and study the scaled edge. Help them to understand that they can think of this edge as a number line. You might ask them to describe the ways they have used number lines to illustrate addition, subtraction, and multiplication. Then direct their attention to the investigation.

Investigation

Read through the directions with the class, making sure the children see that to get to zero they must make backward jumps (subtractions) of 3. When the children understand $24 \div 3 = 8$, they should be ready to try other equations. Avoid giving the children explicit directions on how to proceed with the division equations; rather, if a child has difficulty, review the first equation with him and help him relate it to another equation, such as $24 \div 4 = 6$.

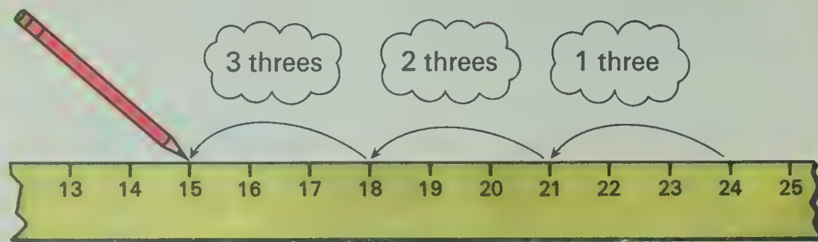
One of the main reasons we use a number line here is that it provides an excellent visual model to show division as repeated subtraction. Thus, it would be helpful for those having difficulty to review how subtraction is illustrated on the number line, and then extend that understanding to division as repeated subtraction.

Can the number line help you think about division?

Investigating the Ideas

Use your centimetre ruler as a number line. Starting at 24, it takes 8 jumps of three to get to zero.
There are 8 threes in 24.

$$24 \div 3 = 8$$



?

How many different division equations can you write to show other jumps you can make to get from 24 to zero?

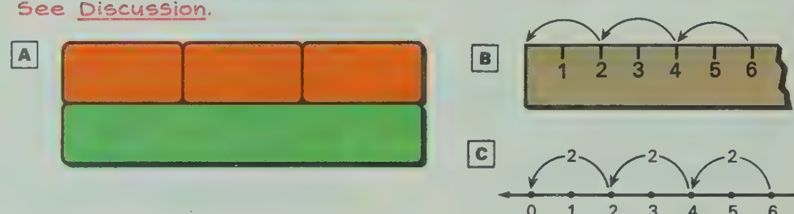
$24 \div 8 = 3$
$24 \div 4 = 6$
$24 \div 6 = 4$
$24 \div 2 = 12$
$24 \div 12 = 2$
$24 \div 1 = 24$
$24 \div 24 = 1$

Discussing the Ideas

1. What division fact will you discover if you start at 18 on your ruler and jump back 2 at a time? $18 \div 2 = 9$

2. Explain why starting at 16 and jumping by threes will not give you a division fact.
After 5 jumps of three, you would be at 1 on the number line, so 16 is not evenly divisible by 3.

3. Explain how each figure below helps you solve $6 \div 2 = n$.
See Discussion.



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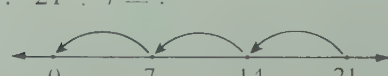
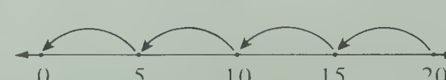
Discussion

Display a number line on the chalkboard or overhead projector and let volunteers explain how they used their ruler to find the answers for the investigation. Then present other equations to explain how the jumps to the left illustrate division. Work through equations such as the following examples with the class (include division equations presented in the context of word problems).

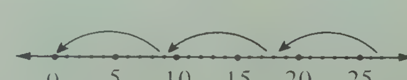
1. $21 \div 7 = ?$



2. 20 pieces of gum. 5 boys. How many pieces of gum for each?
$20 \div 5 = ?$



3. 27 players. 9 on a team. How many teams?
$27 \div 9 = ?$



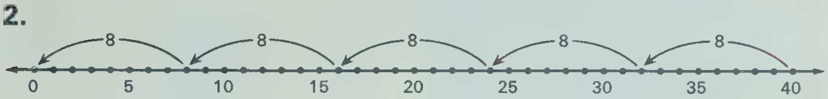
The three parts of discussion exercise 3 show three different methods of representing $6 \div 2$.

Using the Ideas

1.

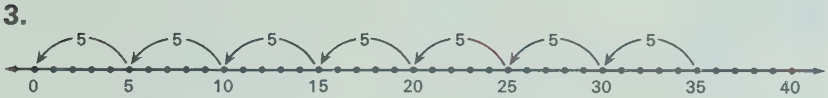
A How many nines in 36 ? 4 B $36 \div 9 = n$ 4

2.



A How many eights in 40 ? 5 B $40 \div 8 = n$ 5

3.



A How many fives in 35 ? 7 B $35 \div 5 = n$ 7

4. Draw a number line to help you find each of these.
A $14 \div 2$ B $12 \div 4$ C $20 \div 5$ D $16 \div 2$
See Answers at right.

Solving Story Problems

1. Mr. Smith paid \$15 for the children's baseball tickets. They were \$3 each. How many children went to the ball game with Mr. Smith ? 5

2. The Blue Sox scored 2 runs in each inning until the scoreboard read How many innings had they played then ? 4

★ 3. Ted lived 36 kilometres from the ball park. Mr. Smith drove at the rate of 6 kilometres each 5 minutes. How long did it take to get from the ball park to Ted's house ? 25 minutes

More practice, page A-22, Set 31

There is a gradual progression in the degree of sophistication of the representations, from the strip interpretation in part A to the number-line representation in part C. In A, the 6-strip is spanned by three 2-strips. In B, $6 \div 2$ is shown by jumps along a ruler. Finally, part C shows the familiar number-line interpretation.

Using the Exercises

On page 177, assign exercises 1 to 4 as independent work.

Have the children take turns reading the remaining exercises orally. Then give them time to write the equations and solve the problems. You may wish to let some children write the equations for the problems as you proceed, going back to solve them after the reading has been completed.

Assignments (page 177)

Minimum: 1–3. Average: 1–4, story problems 1–2. Maximum: 1–4, story problems 1–3.

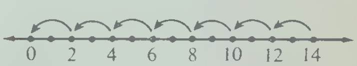
Follow-up

Unless you have several metre sticks or commercial number lines, you might have the children make individual number lines. For this activity, give each of the children about 1 metre of adding-machine tape (or cut strips of construction paper about 6 centimetres wide and tape them together to form a long strip). Show the children how to fold the strip in half *lengthwise* by matching the edges. Next direct them to make a crease along the fold and then to open the strip and use the crease as a guide in making a number line. If they make the dots about a centimetre apart, they can show about 90 points. You may wish to have them label only even numbers, or the multiples of five, to simplify the task. These number lines can be used to help them understand division exercises such as those on page 177 or to complete a chart like the one below.

Start	Size of Jumps	Number of Jumps	End
30	6		0
18	9		0
15		5	0
42		7	0
63	7		0
56	8		0
	8	8	0
	9	8	0

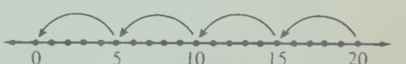
Answers, exercise 4, page 177

A



B

C



D

Workbook, page 64

PAGES 178–179

Objective

Given a division equation, the child will be able to find the quotient by thinking of missing factors.

Preparation

Materials

scissors; graph paper

If you are sure that the children are sufficiently familiar with the shape of a rectangle, begin immediately with the investigation. For some, you may want to review the term *rectangle* and explain what is meant by rows and number of squares in a row.

Investigation

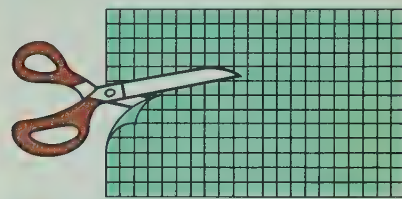
Read the directions with the children and then allow them freedom to cut many different rectangles. If a child incorrectly thinks he has found all he can, direct him to study the chart and find another. If some children cut a 5 x 7 rectangle by mistake, help them realize that this has 35 squares, not 36. For those who finish very quickly, you might write a similar chart for another area on the chalkboard; for example:

Number of squares in a row	*Number of rows*	*Area of rectangle*
?	8	24
?	4	24
12	?	24
?	1	24

Can you find quotients by finding missing factors?

Investigating the Ideas

Cut from graph paper as many different rectangles that have 36 squares as you can.



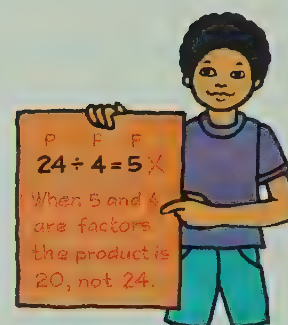
Can you complete this table with the help of your rectangles?

Number of squares in a row		Number of rows		Number of squares in the rectangle
6	×	? 6	=	36
9	×	? 4	=	36
? 12	×	3	=	36
18	×	? 2	=	36
? 36	×	1	=	36

Discussing the Ideas

See Discussion.

1. On Friday, the children checked their arithmetic papers. Peter checked Judy's paper. Under one exercise, he wrote →



   A Explain what Peter was trying to tell Judy.

   B What do you think Peter would say about this equation?

   $18 \div 3 = 7$

   When 3 and 7 are factors, the product is 21, not 18.

2. Find the missing factors and quotients. Explain your answers.

   A Think 4 $? \times 5 = 20$ — $20 \div 5 = n$ 4

   B Think 1 $? \times 6 = 6$ — $6 \div 6 = n$ 1

   C Think 2 $? \times 7 = 14$ — $14 \div 7 = n$ 2

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Discussion

As the children discuss the rectangles they cut, have volunteers write the multiplication equations from the chart, filling in the missing numerals as they do. Next to each multiplication equation, write the corresponding division equation. Emphasize the missing-factor term and its position in both equations. Write **P** for product and **F** for factor over several of the terms.

F F P $6 \times 6 = 36$	P F F $36 \div 6 = 6$
F F P $9 \times 4 = 36$	P F F $36 \div 9 = 4$
F F P $? \times 3 = 36$	P F F $36 \div 3 = ?$
$18 \times ? = 36$	$36 \div 18 = ?$

Work through several equations until the children see that the answer in the division equation is the missing factor in the multiplication equation.

In the discussion exercises themselves, stress the relation between division and multiplication and point out that division can be checked by multiplying the known factor (the divisor) by the missing factor (the quotient). In exercise 2, emphasize that a division equation may be thought of as a related multiplication equation. It would be helpful to extend exercise 2 by writing other division equations on the chalkboard and having children

Using the Ideas

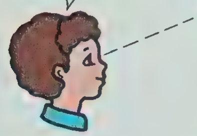
1. Find the missing factors.

A To find this quotient, I think 3 $? \times 5 = 15$.

$15 \div 5$

B To find this quotient, I think 4 $? \times 3 = 12$.

$12 \div 3$



2. Find the quotients.

A Think 3 $? \times 2 = 6$
$6 \div 2 = n$ 3

B Think 3 $? \times 3 = 9$
$9 \div 3 = n$ 3

C Think 2 $? \times 4 = 8$
$8 \div 4 = n$ 2

3. Find the missing factors and quotients.

A Since $n \times 4 = 8$, we know that $8 \div 4 = n$. 2
B Since $n \times 3 = 9$, we know that $9 \div 3 = n$. 3
C Since $n \times 5 = 15$, we know that $15 \div 5 = n$. 3
D Since $n \times 3 = 15$, we know that $15 \div 3 = n$. 5
E Since $n \times 5 = 20$, we know that $20 \div 5 = n$. 4

4. Find the quotients. Use multiplication to check your answers.

A $24 \div 8 = n$ 3
B $30 \div 6 = n$ 5
C $27 \div 9 = n$ 3
D $12 \div 3 = n$ 4
E $27 \div 3 = n$ 9
F $21 \div 3 = n$ 7
G $16 \div 8 = n$ 2

think

Our names you're sure to find
If you use this little clue.
Our product is 18,
While our quotient's only 2.
WHO ARE WE? 6, 3.

More practice, page A-23, Set 32

think aloud about a related multiplication equation. For example, "We see $20 \div 5 = n$, and we think, What number times 5 equals 20?"

Using the Exercises

Work through a sample part of exercises 1 and 3 with the class, and then assign the rest of the exercises as independent work. If any children have difficulty, work through several exercises with them in a small group. Afterward, allow time for further discussion of the exercises and of the multiplication-division relationship.

Most children will be able to solve the *Think* problem by a trial-and-error examination of the factors of 18.

Assignments (page 179)
Minimum: 1–4. Average: 1–4.
Maximum: 1–4.

Mathematics

This lesson begins the more formal treatment of division, that of relating division to multiplication, and constitutes one of the key lessons of the chapter. As noted previously, the working definition for division is expressed in terms of multiplication. That is, if we have three whole numbers a, b, and c, such that $b \neq 0$ and $a \times b = c$, then $a = c \div b$. In the mathematics section that introduces this chapter, you will find a more complete explanation of the relationship between quotient and missing factor.

Follow-up

To help the children relate multiplication and division, play an oral "train" game. If the class does not sit in rows, assign teams according to the seating plan so that members of a team may interchange seats easily.

Have the children appoint a captain and note where he is sitting. Begin by saying a division equation to which a member of the first team responds by giving the related multiplication equation. If the response is correct and quick, all members of that team move forward one seat, with the person who answered going to the end. If the response is incorrect or very slow the team may not change seats. The first team (after all teams have had equal chances) whose "captain" returns to his original seat wins.

Resources for Active Learning

Mathex: Operations and Problem Solving No. 8, "Arrays—Activity 2," pp. 14–15, Encyclopaedia Britannica Publications Ltd.

Duplicator Masters, page 44
Workbook, page 65
Skill Masters, page 44

PAGES 180–181

Objective

Given a division problem presented in terms of sets, repeated subtraction, or a missing factor, the child will be able to write and solve the appropriate division equation.

Preparation

Use an oral warm-up activity to review division and its relation to multiplication. For example, say: "I'm thinking of a number. If I multiply my number by 8, I will get 24. What's my number?" Or: "The product is 35. One factor is 5. What is the quotient?"

Discussing the Ideas

See Discussion.

1. For each example in the chart, solve the equation. Explain your answer.

[1] Sets

$15 \div 3 = n$ 5

How many sets of 3 can we get from a set of 15? 5 →

[2] Subtraction

$15 \div 3 = n$ 5

Starting with 15, how many times can we subtract 3? 5 →

$$\begin{array}{r}15\\-3\\\hline 12\end{array}\quad\begin{array}{r}12\\-3\\\hline 9\end{array}\quad\begin{array}{r}9\\-3\\\hline 6\end{array}\quad\begin{array}{r}6\\-3\\\hline 3\end{array}\quad\begin{array}{r}3\\-3\\\hline 0\end{array}$$

[3] Missing factor

$15 \div 3 = n$ 5

What number times 3 gives 15? → n (5) $\times 3 = 15$

2. Explain how you could use your 2-strip and 10-strip to find $10 \div 2$.

3. How could you use a centimetre ruler to find $27 \div 3$?

4. If you know $7 \times 8 = 56$, what two division facts can you give?
   $56 \div 7 = 8$ $56 \div 8 = 7$

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Discussion

As you read and study the discussion material, expand each interpretation with a short demonstration and several examples. For instance, to show sets, put a divisible number of felt objects on a flannelboard and have a child show division by circling equivalent sets with yarn. Present example 2 similarly, directing a child to remove the objects rather than showing the separation with yarn. Finally, present several examples of missing-factor interpretation: write pairs of division and multiplication equations and guide the class to think them through aloud.

If you prefer, or think it necessary, have numerous sets of objects available for the children to use to work through the examples, either individually or in groups. You might even divide the whole class into groups and let each group choose a different example and explain, by means of felt objects and demonstration sets, their interpretation to the class.

Using the Ideas

1. Write a division equation for each exercise.

A 10÷2=5

B 12÷6=2

C 18÷3=6

2. Write a division equation for each exercise.

A $\begin{array}{r} 27 \\ -9 \\ \hline 18 \end{array}$ $\begin{array}{r} 18 \\ -9 \\ \hline 9 \end{array}$ $\begin{array}{r} 9 \\ -9 \\ \hline 0 \end{array}$

27÷9=3

B $\begin{array}{r} 35 \\ -7 \\ \hline 28 \end{array}$ $\begin{array}{r} 28 \\ -7 \\ \hline 21 \end{array}$ $\begin{array}{r} 21 \\ -7 \\ \hline 14 \end{array}$ $\begin{array}{r} 14 \\ -7 \\ \hline 7 \end{array}$ $\begin{array}{r} 7 \\ -7 \\ \hline 0 \end{array}$

35÷7=5

3. Solve the equations.		4. Now find the quotients.
A $n \times 6 = 12$ 2	→	A $12 \div 6 = n$ 2
B $n \times 3 = 9$ 3	→	B $9 \div 3 = n$ 3
C $n \times 4 = 8$ 2	→	C $8 \div 4 = n$ 2
D $n \times 2 = 10$ 5	→	D $10 \div 2 = n$ 5
E $n \times 3 = 12$ 4	→	E $12 \div 3 = n$ 4
F $n \times 5 = 25$ 5	→	F $25 \div 5 = n$ 5

5. In each exercise, tell how many bags of marbles.

A	15 marbles	3	3		?
B	20 marbles	5	5		?
C	14 marbles	2	2		?
D	18 marbles	6	6		?
E	35 marbles	7	7		?
F	28 marbles	4	4		?

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Follow-up/"Challenge"

"Challenge" is an oral game designed to check the children's understanding of the inverse relation between division and multiplication. Begin by telling the children that you are going to give some division equations orally to which they must listen carefully. After saying each equation, call on one member of the class to say "True" if the equation is true, or "Challenge" if he thinks it is incorrect. If a child challenges a particular equation, he must justify the challenge by giving a multiplication combination that is correct for the given product. For example, if you say, "Twenty-one divided by seven is four" and a child says "Challenge," he should justify the challenge by saying, "That's false because three times seven is twenty-one."

Workbook, page 66

Using the Exercises

Depending on the ability and needs of your class, work through some of these exercises together or assign all of them as independent work.

Some children may need extra help with exercise 5. If so, suggest that they think of each bag in a row as containing the same number of marbles. The bags they can see on the page are labelled with a numeral, and the children should assume that all the other bags behind the gray screen have that same number. This exercise gives children an opportunity to extend their thinking about division.

When the class completes this page, allow time for checking the exercises and discussing the ideas, particularly those which review the various interpretations of division.

Assignments (page 181)

Minimum: 1–2, oral; 3–5. Average: 1–5. Maximum: 1–5.

PAGES 182–183

Objective

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

Since this lesson is a cumulative review, an oral game such as "What's My Rule" would be appropriate. Stress rules which review the multiplication facts as well as addition and subtraction. (See page 69 for directions for the game.)

Keeping in Touch with Addition Subtraction Inequalities

1. Write 2 addition and 2 subtraction equations for each figure.

A 8+9=17 9+8=17

17−8=9 17−9=8

B 8+7=15 7+8=15

15−8=7 15−7=8

2. Give the sums as quickly as you can.

A 5 + 4 9	E 5 + 5 10	I 8 + 9 17	M 5 + 6 11	Q 7 + 6 13
B 6 + 2 8	F 8 + 3 11	J 3 + 7 10	N 7 + 5 12	R 9 + 9 18
C 9 + 1 10	G 7 + 7 14	K 8 + 2 10	O 8 + 8 16	S 6 + 8 14
D 4 + 6 10	H 6 + 7 13	L 3 + 5 8	P 4 + 9 13	T 7 + 8 15

3. Give the differences as quickly as you can.

A 8 − 3 5	E 13 − 8 5	I 15 − 9 6	M 14 − 6 8	Q 12 − 7 5
B 9 − 2 7	F 15 − 7 8	J 13 − 5 8	N 9 − 4 5	R 15 − 8 7
C 6 − 5 1	G 17 − 9 8	K 14 − 5 9	O 16 − 7 9	S 10 − 3 7
D 11 − 4 7	H 16 − 8 8	L 10 − 6 4	P 18 − 9 9	T 13 − 7 6

4. List these numbers in order, from smallest to largest.

68 423	100 000	16 842	50 000	68 433
49 375	674 897	49 365	684 230	68 300

16 842; 49 365; 49 375; 50 000; 68 300; 68 423; 68 433; 100 000; 674 897; 684 230

5. Do not try to find the correct answer. Just tell whether each answer is more than 70 or less than 70.

A 37 + 38 More	D 17 + 58 More	G 93 − 18 More	J 95 − 26 Less	M 50 + 21 More
B 27 + 38 Less	E 83 − 8 More	H 93 − 28 Less	K 93 − 19 More	N 80 − 11 Less
C 27 + 48 More	F 83 − 18 Less	I 46 + 27 More	L 15 + 54 Less	O 90 − 19 More

6. Find the sums.

A	B	C	D	E	F
35	35	48	57	68	73
+24	+25	+26	+19	+24	+14
59	60	74	76	92	87

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Discussion

You might assign these pages as independent work and then review the concepts as you help the children check their work. Alternatively, you might work through some exercises with the children before assigning independent work. In either case, you will probably find that some children will need help with exercises 8 and 9, which include subtraction and regrouping. Note that starred exercise 10 constitutes a special challenge in that it involves finding sums and differences for 3- and 4-digit numbers and requires regrouping over several places.

7. Find the sums.

A $66 + 27$ = 93 | B $99 + 4$ = 103 | C $88 + 7$ = 95 | D $76 + 66$ = 142 | E $35 + 48$ = 83 | F $87 + 32$ = 119

G $93 + 27$ = 120 | H $69 + 25$ = 94 | I $46 + 58$ = 104 | J $57 + 48$ = 105 | K $76 + 64$ = 140 | L $19 + 27$ = 46

8. Find the differences.

A $83 - 22$ = 61 | B $92 - 33$ = 59 | C $57 - 34$ = 23

D $61 - 59$ = 2 | E $122 - 34$ = 88 | F $74 - 68$ = 6

G $182 - 93$ = 89 | H $65 - 37$ = 28 | I $136 - 48$ = 88

think

Dave is three times as old as Sue. In three years, he will be twice as old as Sue. How old are Dave and Sue now?

Dave, 9; Sue, 3

9. Find the sums and differences.

A $35 + 28$ = 63 | B $72 - 43$ = 29 | C $68 + 78$ = 146 | D $39 - 19$ = 20 | E $92 - 15$ = 77

F $67 + 67$ = 134 | G $95 + 78$ = 173 | H $60 - 17$ = 43 | I $34 - 19$ = 15 | J $78 + 62$ = 140

★10. Find the sums and differences.

A $345 + 167$ = 512 | B $6291 + 1963$ = 8254 | C $642 - 285$ = 357 | D $1354 - 678$ = 676 | E $1876 + 695$ = 2571 | F $3604 - 538$ = 3066

You are invited to explore ACTIVITY CARD 8 Page 313

Follow-up

After more capable children have had an opportunity to work on the *Think* problem independently, you may want to follow through by building a structured pattern and then guiding the children through this approach to solving a problem like this one. As you build the table, draw answers from the children and ask questions which force them to compare and evaluate results.

Now		3 years from now	
Sue's age	Don's age	Sue's age	Don's age
1	3	$1 + 3 = 4$	$3 + 3 = 6$
2	6	$2 + 3 = 5$	$6 + 3 = 9$
3	9	$3 + 3 = 6$	$9 + 3 = 12$

To review the operations introduced so far, the children might write their own problems. For example, on the chalkboard write a group of equations which use the same numerals but show different operations. Then ask the children to make up and solve a story problem that fits each equation.

Sample equations

1. $12 = 4 \times$ ____
2. $12 - 4 =$ ____
3. $12 \div 4 =$ ____

Sample stories

1. Jill's mother bought a dozen cookies. If she gives each of her 4 children the same number of cookies, how many does each one get?
2. There are 12 boys playing work-up baseball. If 4 of them decide to play four-square, how many will be left playing baseball?
3. If 12 children want to run relays and there are 4 teams, how many children can be on a team?

Resources for Active Learning

Mathex: Numeration No. 2, "Patterns for Any Number," p. 37, Encyclopaedia Britannica Publications Ltd.

[See also, Chapters 3 and 5 (Resources for Active Learning) for activities and games relevant to the concepts indicated in this lesson.]

Objective

Given a division equation, the child will be able to find the quotient by thinking of the missing factor in the related multiplication equation.

Preparation

To prepare for this lesson, conduct a brief oral review of the multiplication facts. Include division equations, but relate them to corresponding multiplication equations and missing factors.

Can multiplication facts help you with division facts?

First find the products in exercise 1. You can then use these facts to help you do exercise 2.

1. Find the products.

A $2 \times 7 = n$ 14	F $4 \times 4 = n$ 16	K $6 \times 6 = n$ 36	P $5 \times 3 = n$ 15
B $9 \times 2 = n$ 18	G $7 \times 5 = n$ 35	L $3 \times 5 = n$ 15	Q $4 \times 7 = n$ 28
C $5 \times 4 = n$ 20	H $2 \times 6 = n$ 12	M $1 \times 9 = n$ 9	R $6 \times 0 = n$ 0
D $6 \times 3 = n$ 18	I $5 \times 5 = n$ 25	N $4 \times 6 = n$ 24	S $5 \times 8 = n$ 40
E $8 \times 4 = n$ 32	J $9 \times 3 = n$ 27	O $3 \times 8 = n$ 24	T $6 \times 5 = n$ 30

2. Find the missing factors. Use exercise 1 to check your answers.

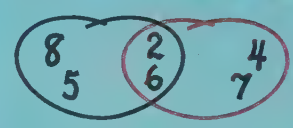
A $n \times 8 = 40$ 5	I $n \times 7 = 28$ 4	Q $n \times 3 = 15$ 5	S $6 \times n = 18$ 3
B $n \times 3 = 27$ 9	J $6 \times n = 36$ 6	R $n \times 5 = 30$ 6	T $n \times 5 = 15$ 3
C $4 \times n = 24$ 6	K $8 \times n = 32$ 4		
D $6 \times n = 0$ 0	L $n \times 9 = 9$ 1		
E $n \times 2 = 18$ 9	M $n \times 5 = 25$ 5		
F $n \times 8 = 24$ 3	N $5 \times n = 20$ 4		
G $4 \times n = 16$ 4	O $n \times 5 = 35$ 7		
H $2 \times n = 14$ 7	P $2 \times n = 12$ 6		

3. Find the quotients.

A $14 \div 2 = n$ 7	K $18 \div 2 = n$ 9
B $35 \div 5 = n$ 7	L $36 \div 6 = n$ 6
C $18 \div 6 = n$ 3	M $40 \div 8 = n$ 5
D $12 \div 2 = n$ 6	N $30 \div 5 = n$ 6
E $27 \div 3 = n$ 9	O $28 \div 7 = n$ 4
F $16 \div 4 = n$ 4	P $0 \div 6 = n$ 0
G $32 \div 8 = n$ 4	Q $25 \div 5 = n$ 5
H $15 \div 5 = n$ 3	R $24 \div 8 = n$ 3
I $24 \div 4 = n$ 6	S $15 \div 3 = n$ 5
J $9 \div 9 = n$ 1	T $20 \div 5 = n$ 4

think

$$\begin{array}{r} 2 \\ 6 \\ 4 \\ 7 \\ \hline 19 \end{array}$$



Set A Set B

Find the sum of

1. the numbers in set A. 21
2. the numbers in B but not A. 11
3. the numbers in both A and B. 8
4. the numbers in A or B but not both. 24

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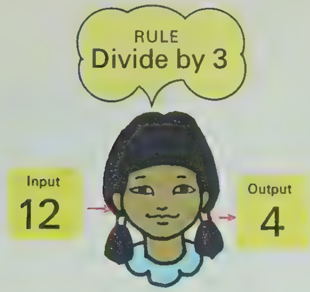
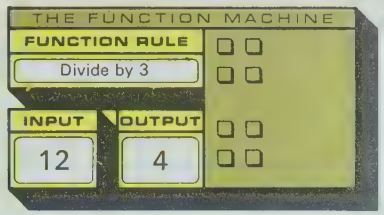
Discussion

One of the purposes of this lesson is to help the children realize that the best way to become skillful at finding quotients is to know the multiplication facts very well. Give children time to find the products in exercise 1. When they finish, check their answers, and ask someone to explain how knowing these facts can help them find the missing factors in exercise 2. It would be helpful to write the related division equation for a few of these. For example, write $n \times 8 = 40$ on the chalkboard and beside it write $40 \div 8 = n$.

Encourage the children to do exercise 3 on page 184 as rapidly as possible. Remind them that if they have difficulty finding any quotient, they can find the answer by checking one of the multiplication exercises in the upper part of the page.

Function Machine Problems





Think about the function machine and give the missing numbers.

1. Function Rule: Divide by 2

	Input	Output	
	8	4	
	6	3	
A	10		5
B	4		2
C	12		6

2. Function Rule: Divide by 3

	Input	Output	
	6	2	
A	15		5
	12	4	
B	9		3
C	3		1

3. Function Rule: Divide by 4

	Input	Output	
	12	3	
A	8		2
B	16		4
C	4		1
	20	5	

4. Function Rule: A Divide by ____ 5

	Input	Output	
	5	1	
B	10		2
	15	3	
	20	4	
C	25		5

5. Function Rule: Divide by 10

	Input	Output	
	20	2	
A	60		6
	50	5	
B	80		8
C		4	40

6. Function Rule: Divide by 2

	Input	Output	
	12	6	
A		5	10
B		7	14
C		4	8
	16	8	

More practice, page A-23, Set 33

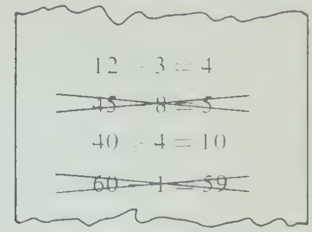
Using the Exercises

After the children have completed page 184, refer to the illustration at the top of page 185. Tell the children that the girl hears the number 12, uses the rule "Divide by 3," and then says 4. Discuss how the picture at the left is related to the function machine; explain that 12 is the input, the machine divides 12 by 3, and the output is 4.

Discuss exercise 1 and have the children give the answers for parts A, B, and C. Then instruct them to finish the page independently. When they have done this, allow time for further discussion and checking papers.

Follow-up/"You Goofed!"

Outputs from function machines are sometimes printed on a tape. Suppose the illustration below is part of an output tape for a function machine that is programmed to divide. Then suppose that if the machine cannot perform an operation or if the program is incorrect, it will print a large YOU GOOFED! Prepare worksheets containing a variety of division statements, some true and some false, and direct the children to examine the "tape" and use a bright crayon to cross out the spots where they think the machine would print YOU GOOFED!



Sample Marked Tape

Alternatively, you might create function tables for a worksheet. Vary the function rules to include different operations.

Function Rule: Subtract 9

Input	Output
?	12
15	?

Function Rule: ?

Input	Output
9	20
16	27

Assignments (page 185)

Minimum: 1, 3, 5. Average: 1–6. Maximum: 1–6.

Resources for Active Learning

Franklin Series: *Making and Using Graphs and Nomographs*, "Nomographs: Multiplication," pp. 71-72, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Mathex: Operations No. 3, "A Multiplication Wheel," pupil page 54, Encyclopaedia Britannica Publications Ltd.

Mathex: Operations No. 3, "Game 1," p. 35, Encyclopaedia Britannica Publications Ltd.

Mathex: Operations and Problem Solving No. 8, "What's My Rule?—Activity 1"; "Function Machine—Activity 3," pp. 17–18, Encyclopaedia Britannica Publications Ltd.

Duplicator Masters, page 45
Workbook, pages 67, 68
Skill Masters, page 45

PAGES 186–187

Objective

Given a set of objects (such as 12 checkers), the child will be able to divide them equally into a specified number of sets (such as 3) and write a corresponding division equation.

Preparation

Materials

counters; paper cups; boxes

This lesson requires no specific preparation, but, if you wish, you might use a short oral review of multiplication and division facts.

Investigation

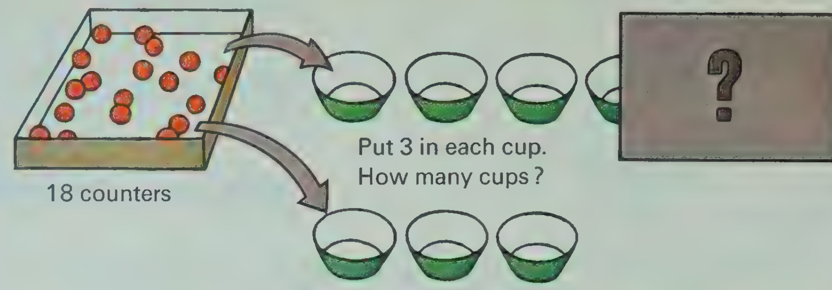
In the investigation for page 170, the children put a specific number of counters in each cup. In this investigation they review that procedure but also find how many counters can be put into a given number of cups if each cup contains the same number of counters. The illustrations and directions in the text should provide sufficient guidance for the children, but some may need help getting started. It would be helpful to read together and differentiate the two questions they are to investigate, namely, "How many cups?" and "How many in each cup?" You might write other examples on the chalkboard for those who finish more quickly.

Total number	*Number in each*	*Number of cups*	*Division equation*
12	4	?	$12 \div 4 = ?$
12	?	4	$12 \div ? = ?$
12	2	?	$12 \div ? = ?$
12	?	2	$12 \div ? = ?$

How are division and sets related?

Investigating the Ideas

Get some counters, paper cups, and a collection box.



Total number	Number in each cup	Number of cups	Division equation
18	3	? 6	$18 \div 3 = ?$ 6
18	? 6	3	$18 \div 3 = ?$ 6
18	? 3	6	
18	? 9	2	

$18 \div 6 = 3$
$18 \div 2 = 9$

? Can you use your counters to help you complete the chart?

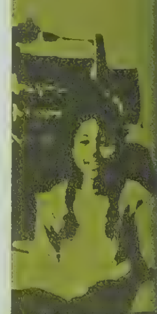
Discussing the Ideas

1. You can use division to find how many sets. Solve the equation to find out.

12 books → How many stacks of 4 can you make? → $12 \div 4 = n$ 3

2. You can use division to find how many in each set. Solve the equation to find out.

15 cookies → If the cookies are divided equally among 3 children, how many does each child get? → $15 \div 3 = n$ 5



186

Discussion

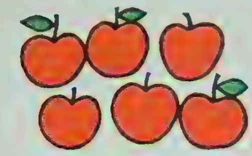
The main idea to stress throughout this lesson is that division may be used *both* to find *how many sets* of objects and to find *how many in each set*. Allow ample time for a discussion of the results of the investigation. Write the chart on the chalkboard and have the children complete it. Point out that the same division equation is used in either of the two set interpretations. That is, $18 \div 3 = n$ is the equation when 3 is the *number in each set* and the question is *how many sets*, as well as when 3 is the *number of sets* and the question is *how many in each set*. To help the children understand this more completely, point out that $18 \div 3 = n$ can be related to $3 \times 6 = 18$ or $6 \times 3 = 18$ because $3 \times 6 = 6 \times 3$.

Stress these ideas again as you discuss exercises 1 and 2 at the bottom of page 186.

Using the Ideas

1. Write a division equation for each exercise.

A 6 apples

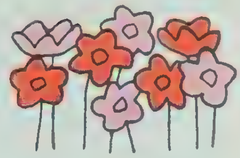


How many does each child get if you divide the apples equally among 3 children?
$6 \div 3 = 2$

B 10 cents

How many children can have 2 cents each?
$10 \div 2 = 5$

C 8 flowers



How many are in each vase if they are divided equally in 2 vases?
$8 \div 2 = 4$

2. In each part, the marbles are equally divided in the bags.

A	18 marbles	3 3 ?	How many bags? 6
B	16 marbles	? ? ? ?	How many in each bag? 4
C	24 marbles	4 4 ?	How many bags? 6
D	30 marbles	? ? ? ? ? ?	How many in each bag? 5
E	28 marbles	4 4 ?	How many bags? 7
F	27 marbles	9 9 ?	How many bags? 3
G	40 marbles	? ? ? ? ?	How many in each bag? 8

More practice, page A-24, Set 34

Using the Exercises

You might want to work through exercise 1 on page 187 together, having volunteers write each equation on the chalkboard while the other children do so at their desks or tables.

In exercise 2, call attention to the two types of exercises: those in which children are to find the number in each bag, as in exercise B, and those in which they are to find the number of bags, as in exercise A. The children should realize that it is helpful to think about the division operation in both cases. Some children might find it helpful to use small paper bags and marbles to perform the operations physically for some of the exercises.

Assignments (page 187)
Minimum: 1, oral; 2. Average: 1–2.
Maximum: 1–2.

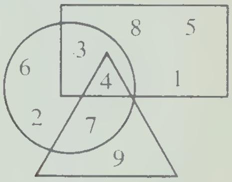
Mathematics

Since our working definition of division is in terms of multiplication, we can use set interpretations for multiplication to arrive at two different set interpretations for division. Because multiplication is commutative, we can think of division as finding either missing factor in a multiplication equation. For example, if we choose to associate 4 sets of 5 with 4×5, we can think of the equation $20 \div 5 = n$ to find the number of sets and we can think of $20 \div 4 = n$ to find the number in each set.

If children read a word problem which requires finding the number of equivalent sets of a given size or the number of objects in each of so many equivalent sets, they should think of division. In each case, however, once the idea of the division operation has been introduced, the children should be encouraged to find the quotient by thinking about the missing factor.

Follow-up

Create worksheets like the one below and instruct the children to find sums, differences, or products.



1. Find the sum of the numbers that are *in* the triangle. $(4 + 7 + 9 = 20)$
2. Find the sum of the numbers *in* the circle but *not in* the triangle. $(6 + 2 + 3 = 11)$
3. Find the sum of the numbers which are *in both* the circle *and* the rectangle. $(3 + 4 = 7)$
4. Find the sum of the numbers that are *in either* the circle *or* the rectangle. $(6 + 2 + 7 + 3 + 4 + 8 + 5 + 1 = 36)$

Resources for Active Learning

Math Workshop: Games and Enrichment Activities, "And-Or Games," pp. 28–29, Encyclopaedia Britannica Educational Corp.

Workbook, pages 69, 70

PAGES 188–189

Objectives

Given a simple division problem, the child will be able to solve it by using sets and/or a number line.

Given short picture problems of division, the child will be able to read and solve them.

Preparation

Materials

counters (at least 30 per child)

You may choose to begin immediately with the investigation. However, if your prefer, use a *short* oral drill to review finding missing factors. For example: "I'm thinking of a number. If I multiply it by 3, I get 15. What's my number?" It would be helpful to include factors of 30 in the drill.

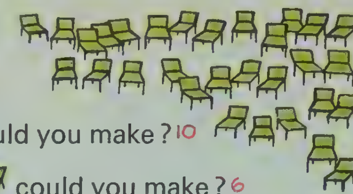
Investigation

Read through the text material with the children, emphasizing that they are to use their counters to represent desks. Then move around the room as the children work independently. If necessary, point out that a row means across, not up and down. You may encourage a spirit of enthusiasm by asking children questions such as, "If these chairs were in a theatre would you rather have 10 rows or 3 rows? Why? Would the same number of persons be able to sit in the 10 rows you made as in the 3 rows? If the room were long and narrow, how would you arrange the chairs?"

You might ask the children who finish very quickly to work with 24 counters and show how to arrange 24 counters in rows of 3, of 4, of 6, etc.

How well do you understand division?

Investigating the Ideas



Suppose there are 30 chairs in a room.

1. How many rows of 3 chairs could you make? 10
2. How many rows of 5 chairs could you make? 6
3. How many rows of 6 chairs could you make? 5
4. How many chairs in each row if you divide them equally in ten rows? 3

?	Can you use a set of counters and show your answer to each question? See Investigation.	Draw pictures to show how you arranged your counters.

Discussing the Ideas

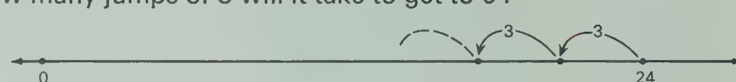
1. What division fact can you give for each of the questions above?
   1. 30÷3=10 2. 30÷5=6 3. 30÷6=5 4. 30÷10=3
2. Answer the questions about the set of children.

18 children

   A How many groups of 9 can be found? 2
   B How many at each table if they are divided equally among 6 tables? 3
   C How many girls are there if there are as many boys as girls? 9
   D How many teams of 3 can be formed for a spelling contest? 6

3. How long is each jump? 4

4. How many jumps of 3 will it take to get to 0? 8



188

Discussion

This lesson reviews the set interpretation and the use of the number line in division. Have volunteers explain the results of the investigation, showing arrangements of counters on a flannelboard.

If some children experienced difficulty, form a small group to work with you on flannelboard demonstrations. For example, place 20 felt squares on the flannelboard, asking how many teams of 5 could be made if the 20 squares represent children. Have a child divide the squares into groups to show that there would be 4 teams. Work through another example similarly.

Then ask if we could make up 3 teams of 7 each for races. When someone responds that we cannot form 3 equal teams from 20 children, ask for a volunteer to prove this with the felt squares.

In discussing the exercises at the bottom of page 188, for exercise 3 stress that the number of jumps is shown and that the children should find how many units are covered in each jump. In exercise 4, the size of the jumps is shown and the children should find how many jumps must be made in order to return to zero.

Using the Ideas

Solve the picture problems.

More practice, page A-25, Set 35

189

Follow-up

This activity may be used either for follow-up or for those children who did not need to participate in the special flannelboard work suggested in the discussion section.

Write six to eight multiplication and division equations on the chalkboard. Then direct the children to make up corresponding short stories for each equation. For example:

Equation	*Short Story*
$32 \div 4 = n$	32 marbles. 4 boys. How many marbles each?
$36 \div 9 = n$	36 children. 9 per team. How many teams?

Resources for Active Learning

Math Workshop: Games and Enrichment Activities, "Three Boxes," pp. 10–11, Encyclopaedia Britannica Educational Corp.

Duplicator Masters, page 46
Workbook, page 71
Skill Masters, page 46

Using the Exercises

Introduce page 189 by having children tell stories about some of the picture problems. Then encourage them to read each problem correctly, following this pattern: "If 6 candy bars cost 30 cents, then 1 candy bar costs how many cents?" and "If 1 carton holds 6 bottles, then how many cartons are needed to hold 24 bottles?"

Depending on the ability of your class, you may treat this entire page orally or you may have the children write and solve equations for each problem.

Assignments (page 189)
Minimum: 1–4. Average: 1–8.
Maximum: 1–8.

Objective

Given word problems involving division concepts, the child will be able to write an appropriate division equation and solve the problem.

Preparation

Use a short oral warm-up to practice finding missing factors. You might play "What's My Rule" by writing a row of multiples of a number and then applying a rule of division until the children can supply the missing factor. For example:

Rule:

$n \div 8 = ?$	48	24	32	16	8	40	56
	6	3	4	?	?	?	?

Rule:

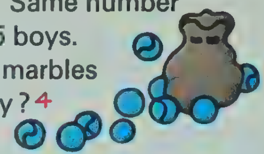
$n \div 7 = ?$	42	28	14	35	49	56	21
	6	4	2	?	?	?	?

Keep the game moving at a rapid pace, and use it only for a short time.

Solving Story Problems

1. Answer the question. Then write a division equation about the story.

A 20 marbles. Same number to each of 5 boys. How many marbles for each boy? 4
$20 \div 5 = 4$



B 15 cookies. 3 cookies to each child. How many children get cookies? 5
$15 \div 3 = 5$

C 30 cents. Apples 6 cents each. How many apples can we buy? 5
$30 \div 6 = 5$

D 24 cents spent for 3 oranges. How much per orange? 8¢
$24 \div 3 = 8$

E 24 people. Same number in each of 6 cars. How many people in each car? 4
$24 \div 6 = 4$

F 14 children. Same number of girls as boys. How many of each? 7
$14 \div 2 = 7$

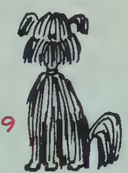
G 20 players for 4 teams. How many players for each team? 5
$20 \div 4 = 5$

H 18 players. 3 on each team. How many teams? 6
$18 \div 3 = 6$

I 28 days. 7 days per week. How many weeks? 4
$28 \div 7 = 4$

J 16 minutes. 2 minutes per kilometre. How many kilometres?
8 $16 \div 2 = 8$

K 36 feet. 4 per dog. How many dogs? 9
$36 \div 4 = 9$



2. Answer the question. Then write a division equation.

A 12 dots. 4 in each set. How many sets? 3
$12 \div 4 = 3$

B 12 dots. 2 sets. How many in each set? 6
$12 \div 2 = 6$

C 15 dots. 3 in each set. How many sets? 5
$15 \div 3 = 5$

D 10 dots. 5 sets. How many in each set? 2
$10 \div 5 = 2$

E 15 dots. 3 sets. How many in each set? 5
$15 \div 3 = 5$

F 18 dots. 3 in each set. How many sets? 6
$18 \div 3 = 6$

G 16 dots. 4 sets. How many in each set? 4
$16 \div 4 = 4$

H 24 dots. 6 in each set. How many sets? 4
$24 \div 6 = 4$

I 24 dots. 8 sets. How many in each set? 3
$24 \div 8 = 3$

190

Discussion

Depending on the needs of your class, work through several of the problems on page 190 together. Remind the children to apply these questions to each problem: "What do I know?" (That is, "What does the problem tell me?") "What am I asked to find?" "What do I do—multiply, divide, add, or subtract?" "Does my answer make sense?"

Exercise 1F may puzzle the children until they realize that they should think of the girls and boys as two equivalent sets and divide by 2. Some children may benefit from actually drawing the dots specified in each part of exercise 2.

Assignments (page 190)

Minimum: 1. Average: 1–2.
Maximum: 1–2.

At the Dairy

1. Cartons of milk are put in boxes, with 6 cartons in each row. If a box holds 24 cartons of milk, how many rows are there? $24 \div 6 = 4$

2. A machine fills 28 litre milk cartons every 4 minutes. How many cartons does it fill in one minute? $28 \div 4 = 7$

3. Cartons of ice cream are placed in large wire racks before they are put in the freezer. Each rack holds 48 cartons in 6 rows. How many cartons are in each row? $48 \div 6 = 8$

4. When Susan's class went to the dairy, they went into the pasteurizing room in groups of 6. There are 30 children in Susan's class. How many groups of 6 did they have? $30 \div 6 = 5$

5. On the way back to school, 5 children rode in each car. How many cars did they need? $30 \div 5 = 6$

6. Miss Smith, the teacher, asked the children to write a story about their trip to the dairy. Don decided to make up some problems for his story. See if you can work them.

A

How many cartons are in each row? 5
$20 \div 4 = 5$

B

How many cartons are in each row? 7
$42 \div 6 = 7$

More practice, page A-26, Set 36

191

Using the Exercises
Direct the children to write and solve a division equation for each problem on page 191. If necessary, help the children read the problems, but encourage them to do their own thinking. If they are capable, have the children do this page as independent work. When they are finished, ask several children to display their work on the chalkboard and explain their solutions to the rest of the class.

Follow-up
To give the children further opportunity to analyze word problems, write some short stories. Beneath each story, list the numbers it involves without indicating operation or relation symbols. Ask the children to read the problem and then complete the equation by writing in the proper symbols. For example:

Fran and Bill on a hike. Walked 3 miles per hour. How many miles in 4 hours?

$3 \bigcirc 4 \bigcirc n$

The children would benefit from writing their own short stories. You might divide the class into groups and ask each group to create story problems based on a theme that they select for themselves. Allow the children to use their multiplication table to help them choose factors and products correctly. When they finish you might duplicate some of each group's problems for the whole class to solve, or you might have the various groups work out the solutions as a group activity.

Resources for Active Learning
Mathex: Operations and Problem Solving No. 8, "Pupil-prepared Problems – Activity 3," pp. 29–30, Encyclopaedia Britannica Publications Ltd.

Duplicator Masters, page 47
Workbook, page 72
Skill Masters, page 47

Assignments (page 191)
Minimum: 1–6, oral. Average: 1–6.
Maximum: 1–6.

PAGES 192–193

Objective

The child will demonstrate his ability to work with the concepts presented in this chapter.

Preparation

Since this is a chapter review lesson, any oral warm-up which the children enjoy might be used to review multiplication and division facts. For example, you could say: "I'm thinking of the product eight times seven. What's my number?" Or: "I'm thinking of two factors of forty-eight. One of them is six. What is the other?" and so on.

Reviewing the Ideas

1. Find the quotients.

A Since $4 \times 9 = 36$, we know that $36 \div 9 = n$. 4 and $36 \div 4 = n$. 9

B Since $6 \times 8 = 48$, we know that $48 \div 8 = n$. 6 and $48 \div 6 = n$. 8

C Since $9 \times 7 = 63$, we know that $63 \div 9 = n$. 7 and $63 \div 7 = n$. 9

D Since $5 \times 8 = 40$, we know that $40 \div 5 = n$. 8 and $40 \div 8 = n$. 5

2. Find the missing factors.

A $n \times 3 = 12$ 4 C $n \times 6 = 24$ 4 E $n \times 3 = 15$ 5 G $n \times 3 = 27$ 9

B $7 \times n = 14$ 2 D $3 \times n = 24$ 8 F $8 \times n = 32$ 4 H $6 \times n = 36$ 6

3. Find the quotients.

A $27 \div 3 = n$ 9 C $14 \div 7 = n$ 2 E $12 \div 3 = n$ 4 G $32 \div 8 = n$ 4

B $24 \div 3 = n$ 8 D $24 \div 6 = n$ 4 F $36 \div 6 = n$ 6 H $15 \div 3 = n$ 5

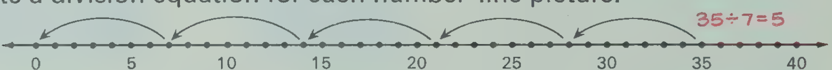
4. $24 - 3 = 21$, $21 - 3 = 18$, $18 - 3 = 15$, $15 - 3 = 12$, $12 - 3 = 9$, $9 - 3 = 6$, $6 - 3 = 3$, $3 - 3 = 0$

A How many times was 3 subtracted? 8 B How many threes are in 24? 8

C Write a division equation about this. $24 \div 3 = 8$

5. Write a division equation for each number-line picture.

A



$35 \div 7 = 5$

B

$32 \div 8 = 4$

192

Discussion

As with all chapter reviews, you may choose to use these pages as a basis for review or as a chapter evaluation. In either case, whether you discuss the exercises as the children work through them or when you return their checked papers, review the interpretations of division treated in them. For example, point out the relation of multiplication and division in exercise 1. In exercise 4, remind the children that, just as multiplication may be thought of as repeated addition, division may be thought of as repeated subtraction. Exercise 5 reviews the use of a number line in division.

The problems on page 193 provide an opportunity to evaluate the children's ability to solve word problems involving division.

Direct the children to read each short story and to write and solve an equation in order to answer the question. If you use this page for evaluation and check the papers yourself, allow the children to ask questions when you return their papers. The results of your evaluation should guide the kind and amount of review work you develop during the next several weeks.

Short Stories

1 18 dots.
3 in each column.
How many columns?
18÷3=6

2 15 children.
5 in each car.
How many cars?
15÷5=3

3 48 books. 8 shelves
(same number on each).
How many books on each shelf?
48÷8=6

4 Rode bicycle 32 kilometres.
8 kilometres each hour.
How many hours? 32÷8=4

6 50 marbles. 5 sacks
(same number in each).
How many marbles
in each sack? 50÷5=10

5 14 pieces of candy in 2 hands.
Same number in each hand.
How many pieces in each hand?
14÷2=7

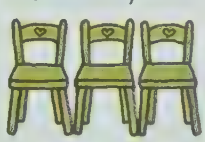
7 27 sails.
3 sails on each boat.
How many boats? 27÷3=9

8 36 players. 6 on each team. How many teams?
36÷9=4

9 Have 35 cents.
Apples 7 cents each.
How many apples can we buy? 35÷7=5

10 36 pieces of pie.
6 pieces in each pie.
How many pies?
36÷6=6

11 6 rows of chairs. 42 chairs.
How many chairs in each row?
42÷6=7



12 28 jet plane engines.
4 engines on each plane.
How many jet planes?
28÷4=7

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Follow-up

If the children need more work with multiplication and division, prepare duplicating masters like the ones below or write the tables on the chalkboard. You might challenge the more capable children by including tables which contain two rules, as illustrated below by the rule "$(\times 2) + 1$," which means "Multiply the input by 2 and then add 1."

Function Rule: $\div 9$

Input	Output
81	?
45	?
36	?
?	8
?	6

Function Rule: $(\times 2) + 1$

Input	Output
4	9
8	17
?	21
30	?
?	41

Also, inequalities can be used to review many operations and principles. Hence, you might create a worksheet like the one below.

Put <, >, or = in each ○ to make the equation true.

49 378 ○ 43 789
36 100 ○ 36000 + 1
\$1.00 ○ 25¢ + 25¢ + 25¢
132 − 42 ○ 45 + 35
746 + 19 ○ 19 + 746

(7 × 9) + (2 × 9) ○ 10 × 9
(75 × 4) − (5 × 4) ○ 80 × 4
62 × 3 ○ (2 × 3) + (60 × 3)
(27 − 9) − 9 ○ 9 × 1
79 × 36 ○ 39 × 76
14 ÷ 7 ○ 20 ÷ 10

Resources for Active Learning

Mathex: Operations No. 3, "Su-Quo-Game 3," pp. 35–38 (pupil page 56), Encyclopaedia Britannica Publications Ltd.

Workbook, pages 73, 74

PAGES 194–195

Objective

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

Plan a short oral activity or drill to practice concepts which have been troublesome. For instance, if your class has difficulty with place value and large numerals, give numerals in their expanded form and have the children name the number (e.g., if you say, "3000 + 200 + 40 + 5," a child should respond "3245"). If regrouping remains difficult for some, you might say, "3 tens and 13 ones is the same as . . ." and expect a response of "43."

Keeping in Touch with Addition, Subtraction, Multiplication, Division, Place value, Inequalities

1. For each pair of numbers, write the larger one on your paper.

A 84	B 807	C 8263	D 4003	E 8327
64	811	7379	4010	8309
F 43 005	G 40 040	H 60 003	I 648 356	J 260 009
43 004	40 050	60 012	647 356	260 101

2. Find the sums and differences.

A 38	B 72	C 65	D 39	E 56	F 120	G 148
+46	−39	+43	−19	+9	−7	+32
84	33	108	20	65	113	180
H 74	I 93	J 65	K 127	L 88	M 702	N 803
−56	+87	−28	−68	+88	−24	−56
18	180	37	59	176	678	747

3. Answer **T** (true) or **F** (false).

T A 7 + 6 is one more than 6 + 6.
T B 7 × 6 is six more than 6 × 6.
T C 26 + 27 > 50
T D Ten hundreds are one thousand.
F E One hundred thousand is one million.
T F 80 ÷ 8 = 10
F G 90 000 × 9 = 10 000
T H One thousand is one hundred tens.
T I 56 − 28 < 30
F J 9 × 5 is nine more than 8 × 5.
T★ K 12 × 14 is one less than 13 × 13.
T★ L 15 × 15 is one more than 14 × 16.
F★ M 37 + 35 > 70 and 48 + 54 < 100
T★ N 3486 + 3486 = 3485 + 3487
T★ O 7 + 5 < 12 or 7 + 5 = 12 or 7 + 5 > 12

think

A missing factor
I cannot be
Whenever the product
Is larger than me.

WHO AM I? Zero

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Discussion

Use appropriate parts of the text material to review any concepts with which the children have had previous difficulty. Now is the time to reinforce understanding of place value, inequalities, addition and subtraction with and without regrouping, and use of the basic principles. It is also the time to increase understanding of the multiplication and division concepts emphasized in the last two chapters.

Have available many of the materials used in the investigations, such as counters and cups, number lines (rulers), graph paper, strips, felt objects, and flannelboard. Encourage children to check their written work by using these materials.

Note that exercises 3K through 3O are designed primarily for children who are stimulated by an added challenge. Have the children write only the answers for exercises 4 and 5. In exercise 4A accept 2345 or 2000 + 300 + 40 + 5, as suggested by the cans.

The *Think* problem is intended as enrichment material. If some children do not understand the answer, explain it by writing several equations which have zero as a factor. Point out that when zero is a factor, the product is not larger than zero. Then write some equations to

4. Tell how many beads in each exercise. Pretend that the large red cans hold 1000 beads each, the gray cans hold 100 beads each, and the small red cans hold 10 beads each.

5. Find the products.

A 2×6 12	D 7×2 14	G 6×5 30	J 8×4 32	M 5×9 45	P 4×7 28
B 3×4 12	E 9×3 27	H 2×8 16	K 3×8 24	N 9×2 18	Q 3×6 18
C 4×9 36	F 6×6 36	I 5×5 25	L 7×3 21	O 5×7 35	R 8×5 40

6. No numbers are given in these exercises. You are to tell whether you would add, subtract, multiply, or divide to find each answer.

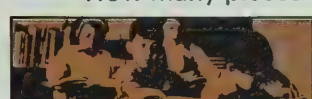
A John has ||||| marbles and Bill has ||||| marbles.
How many marbles do they have together? Add

B In exercise A, how many more marbles does John have than Bill? Subtract

C Mike arranged ||||| chairs in ||||| rows. How many chairs in each row? Divide

D Jim has ||||| pages of stamps with ||||| stamps on each page.
How many stamps does he have on these pages? Multiply

E There are ||||| pieces of candy to be passed among ||||| children.
All the children get the same number of pieces.
How many pieces does each child get? Divide



You are invited to explore

ACTIVITY CARD 9 Page 313

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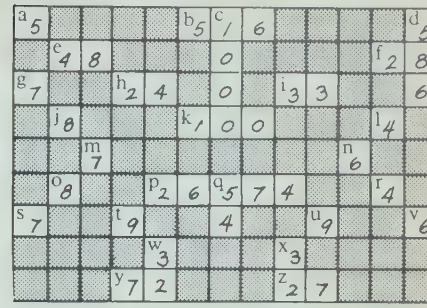
illustrate that for other numbers the product might be larger than the number. Therefore, zero is the only number that correctly answers this problem.

Follow-up

Children will enjoy number puzzles which review operations and concepts they have studied thus far.



Across	*Down*
b) $373 + 143$	a) $25 \div 5$
e) 6×8	c) $10 \times 10 \times 0$
f) 4×7	d) $1348 - 762$
h) 3×8	g) $56 \div 8$
i) $80 - 47$	j) $40 \div 5$
k) 10×10	m) $63 \div 9$
l) $36 \div 9$	o) $64 \div 8$
n) $24 \div 4$	q) 9×6
p) $20\,000 + 6000 + 500 + 70 + 4$	s) $49 \div 7$
	s) $49 \div 7$
r) $32 \div 8$	t) $72 \div 8$
u) $81 \div 9$	w) 4×8
v) $48 \div 8$	x) $24 + 8$
y) 9×8	
z) 9×3	

(One duplicating master of a puzzle may be used for several puzzles; simply change the problems in the *Down* and *Across* columns to meet your purposes.)

Resources for Active Learning

[Refer to activities from Chapter 2 relevant to place value and inequalities, and Chapter 5 for adding and subtracting (algorithms).]

Developmental Math Cards, G17, G19, F10, Addison-Wesley.

Franklin Series: *Patterns and Puzzles*, "Operations," pp. 93–94; "Without a Word," p. 46, Lyons and Carnahan. [A game and a puzzle which involve the four operations using basic facts] (Available from McGraw-Hill Ryerson)

Teaching Aids for Elementary Mathematics, "Puzzle," p. 78, Holt, Rinehart and Winston.

Pages 196–213

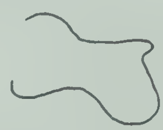
General Objectives

To introduce parallel lines
To provide an intuitive introduction to the relationships between the angles formed by two parallel lines cut by a third line
To introduce quadrilaterals, parallelograms, and other polygons
To introduce simple closed curves
To provide experiences in drawing circles
To introduce symmetry
To provide experience in making symmetrical figures

In this chapter, children will have an opportunity to explore the world of physical geometry. They will draw parallel lines, construct quadrilaterals and circles, cut polygons along the diagonals, and make symmetrical figures. Pertinent questions in the text will help the children to make important observations and relationships. However, the purpose of the chapter is to develop geometric concepts *on an intuitive level;* for children at this age, formal geometry would be more detrimental than helpful. Hopefully, the children will enjoy the investigations and find excitement in the many discoveries that are available to them. Your approach to the chapter will determine the attitude of your children toward the material. Although this chapter deals with many important geometric concepts, the student will benefit most from your enthusiasm for the kind of uninhibited exploration which will help him develop intuitively the important concepts he will later deal with formally.

Mathematics

Many of the lessons of this chapter deal with polygons of various kinds. Polygons are special kinds of *simple closed curves,* which is another topic of study in this chapter. A curve can be thought of in various ways: as a "string of points" or as "the path of a moving point," for example. If the curve does not cross itself, it is a simple curve. If the curve is traceable from one point completely around the curve and back to the starting point, it is a closed curve. Lines and segments can be thought of as special kinds of "straight curves." Hence polygons, whose sides are segments, can be thought of as special types of simple closed curves.



Simple curve

Closed curve

Simple closed curves

Every simple closed curve separates the plane into three sets of points—points in the exterior of the curve, points in the interior of the curve, and points on the curve itself. Children will gain much from an early awareness that a simple closed curve, such as a parallelogram or a circle, is separate and distinct from the set of points exterior and interior to it.

The concept of *midpoint* is used occasionally throughout the chapter, though it is never explicitly developed in the text. The midpoint of a line segment is the point equidistant from the endpoints. The line segment formed by connecting the midpoints of two sides of a triangle is parallel to the third side. When the midpoints of the sides of *any* four-sided polygon are connected, the resulting quadrilateral is a parallelogram.

The final lesson of the chapter deals with symmetry. A figure has a line of symmetry if it can be "folded" along that line in such a way that one half of the figure falls exactly on the other half. When a figure has one or more lines of symmetry it is said to be symmetrical.

Teaching the Chapter

Materials

Colored strips
Compass (1 per child, if possible)
Construction paper
Paper clips
Paper cups or equivalent circular objects
Paste or rubber cement
Ruler or straightedge
Scissors
String
Unlined paper
Basic geometric shapes for class discussion:
Quadrilaterals
Square—all sides of equal length, right angles
Rectangle—opposite sides of equal length, right angles
Rhombus—all sides of equal length
Parallelogram—opposite sides of equal length
Trapezoid—two sides parallel
Other Polygons
Pentagon—5 sides
Hexagon—6 sides
Octagon—8 sides

Vocabulary

angle	regular hexagon
centre point	regular pentagon
circle	regular polygon
closed	regular quadrilateral
diagonal	regular triangle
hexagon	rhombus
inside	simple closed curve
midpoint	simple curve
outside	square
parallel	symmetrical
parallelogram	symmetry
pentagon	trapezoid
polygon	vertex
quadrilateral	vertices
rectangle	

Chapter 8, like the other geometry chapters, should be above all an enjoyable and exciting experience for both teacher and children. As you work through the lessons, you will notice that investigative activities often appear in the "Using the Ideas" exercises. Depending on your class and your own method of teaching, you might occasionally depart from the outline suggested in the manual by combining the introductory investigation and the independent study activities, postponing the discussion until both activity sections have been completed. Also, encourage the children to try other investigative activities which they think of themselves or which you suggest. If geoboards are available, encourage their use in a variety of appropriate investigations. A few lessons may spark an idea for a class project, such as labelling models of various geometric shapes that are found in the school building or on the grounds. However, for each such activity or project, be sure to allow enough time for children to gain maximum benefit from their involvement. Teach the vocabulary, but do not expect correct usage by the children immediately; if your own usage is consistently correct, the children will follow your example with minimal conscious effort. Use models of geometric plane figures and pictures of parallel lines to help the children recognize and name them, but avoid leading the children to think that such an ability is the main goal of the chapter.

Evaluation of Progress

As you observe the children's work, keep in mind that the purpose of this chapter is to develop *intuitive* concepts. Try to correct any gross misconceptions, but do not expect mastery of all the concepts studied. Be patient with children's use of the vocabulary. If you do evaluate the children's efforts, such as their drawings, constructions, and the like, be mindful of each child's individual learning maturity. You may quite justly praise one child for work you would think poor from another.

Lesson Schedule

There can be no rigid schedule for a chapter such as this. Plan to cover it in approximately two weeks, but let your schedule be flexible enough to reflect children's special interests and enthusiasm. You may want to extend some lessons considerably, especially if you include many class projects.

Resources for Active Learning

GENERAL ACTIVITIES

[Refer to the introductory section and lessons in Chapter 4 for activities that are appropriate for this chapter as well. If the children had "favorites," use them again. Some are repeated in the list below because of their relevance to specific objectives of this chapter.]

Franklin Series: *Learn to Fold . . .*, Lyons and Carnahan (Available from McGraw-Hill Ryerson)

Franklin Series: *Pencil and Paper Geometry*, "Rectangles and Squares," pp. 38–41, Lyons and Carnahan (Available from McGraw-Hill Ryerson)

Freedom to Learn, "Tessellations or Tiling," pp. 95, 130–131, Addison-Wesley

Math Activity Cards, A15, 16, 17, 18, Macmillan. More tiling activities.

Nuffield Project: *Shape and Size* 3, "Tile Patterns," pp. 27–28; "Angles and Turning," pp. 29–42, Wiley

Paper Folding for the Mathematics Class, National Council of Teachers of Mathematics

MANIPULATIVE DEVICES

Geo Blocks (Selective Educational Equipment; Webster, McGraw-Hill)

Geoboards (Addison-Wesley)

Geo Strips (Math Media; Selective Educational Equipment)

Mirror Card (Selective Educational Equipment; Webster, McGraw-Hill) A study in symmetry.

Pattern Blocks (Selective Educational Equipment; Webster, McGraw-Hill)

Spirograph (Cuisenaire Co.)

Tangram Cards and Pieces (Selective Educational Equipment; Webster, McGraw-Hill)

COMMERCIAL GAMES

Beeline (Selective Educational Equipment) A game of strategy combined with measurement.

Psyche-Paths (Cuisenaire Co.)

Symmetry Dominoes (Selective Educational Equipment)

PAGES 196–197

Objective

Given appropriate material, the child will be able to identify and draw parallel lines.

Preparation

Materials

ruler or straightedge; full sheets, and strips, of unlined paper

To introduce this chapter on geometry, you might review some of the terms studied in the last geometry chapter (Chapter 4, pages 74–89). For example, ask children to name objects which make them think of a point, a ray, a line, a plane, and an angle. Then tell them that in this lesson they will have a chance to explore some special kinds of lines and segments.

Investigation

Guide the children in folding their strips of paper according to the instructions in their text. Allow volunteers to guess how many folds there are before actually unfolding the strips and counting the seven creases. Since this investigation is intended primarily as a brief, interest-generating activity, you may move quickly on to the discussion section of the text.

8 Geometry

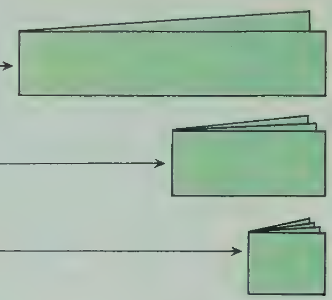
Can you draw parallel lines?

Investigating the Ideas

Fold a strip of paper in half.

Fold it in half again.

Fold it in half a third time.



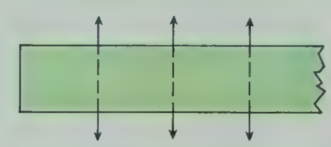
? How many fold lines will you have on your strip? 7

See Investigation.

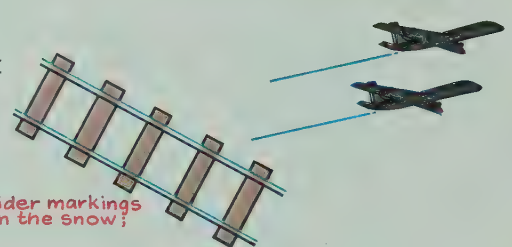
Discussing the Ideas

See Discussion.

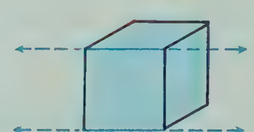
1. The fold lines on your strip suggest **parallel lines**. Can you tell what you think parallel lines are?



2. These pictures suggest parallel lines. Can you think of other things that suggest parallel lines? Lane-divider markings on a highway; ski tracks in the snow; power lines; etc.



3. **Parallel segments** lie on parallel lines. Can you find some objects in your classroom that suggest parallel segments?
Opposite edges of books or wall posters, etc.



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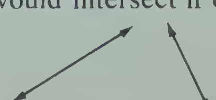
Discussion

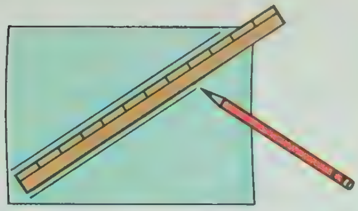
Encourage children to talk about the fold lines they made. Call attention to the illustration for exercise 1 and help the children see that the arrows suggest that they should think of the fold lines as extending on and on. Be sure they grasp the basic idea that parallel lines will always have the same distance between them, that they will not intersect, no matter how far extended. Discuss the meaning of the word *intersect* and draw some lines which would intersect if extended.



Explain that such lines are *not* parallel.

If, for exercises 2 and 3, the children have trouble thinking of other things that suggest parallel lines or parallel segments you might call their attention to such things as the baseboard at the bottom of the wall, parts of window frames, and the borders of the chalkboard. Or you might make a few suggestions to stimulate other ideas, such as lanes painted on a freeway, lines on a sheet of ruled paper, etc. Throughout the discussion, continue to stress the definitive property of parallel lines: that they do not intersect, no matter how far extended.

Using the Ideas



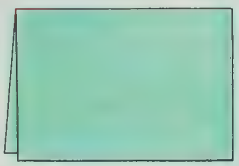
1. You can draw a pair of parallel lines by drawing on each side of your ruler.
   - A Draw two parallel lines using this method.
   - B With your ruler draw two more parallel lines that cross (intersect) the first two lines.

   See Using the Exercises.

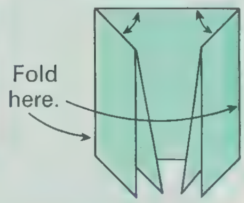
2. Make folds like these and draw along the parallel lines that are formed. Can you fold paper another way to make parallel lines?

   See Using the Exercises.

Crease carefully.

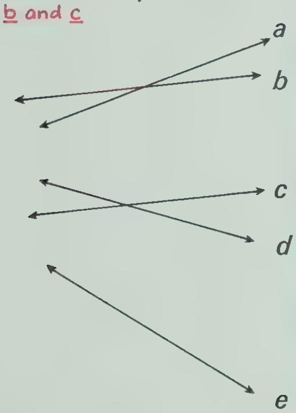


These edges must come together.



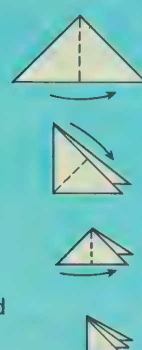
3. Name the pair of lines in the figure that seem to be parallel.

   b and c



think

Suppose you draw a large right triangle with legs the same length and fold it 3 times through the middle. How many small triangles will you be able to count when you unfold it? Guess first. Then try it. 8



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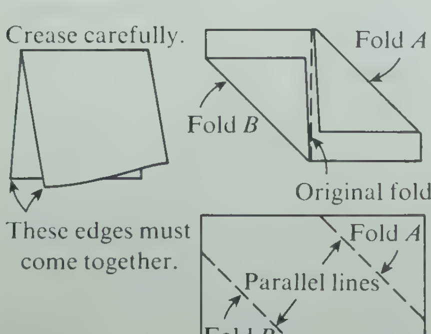
Using the Exercises

This independent activity gives children an opportunity to draw parallel lines by a variety of methods. Here is another way to make parallel lines by folding paper (exercise 2).



It may be necessary for you to give individual guidance to some of the children.

For exercise 3, some of the children might enjoy tracing the lines on a large sheet of paper and then extending them to demonstrate that lines *b* and *c* are the only pair that do not intersect no matter how far they are extended.

Encourage all the children to try the *Think* problem.

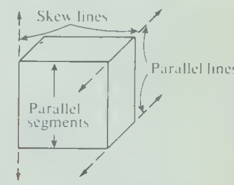
Assignments (page 197)
Minimum: 1–2. Average: 1–3.
Maximum: 1–3.

Mathematics
The central idea of this lesson concerns parallel lines. A formal definition of parallel lines follows:

Two lines are parallel if they are in the same plane and do not intersect.

Note the necessary condition: *two lines lie in the same plane.* In space, it is possible for two lines to not intersect and yet not be parallel lines. Such lines are called *skew lines.*

Parallel segments are two segments which lie on parallel lines. The figure of the cube below will help to clarify these concepts.



Your children are likely to think of lines as the marks which they can draw on their papers rather than as geometric lines which continue indefinitely in both directions. If asked to draw parallel lines, they might draw a pair of line segments which do not intersect on their paper but which are not parallel segments. Lead the children to make such statements as: "Parallel lines do not intersect no matter how far we extend them."

Resources for Active Learning

Franklin Series: *Pencil and Paper Geometry,* "Intersecting Lines," pp. 44–47, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Inquiry in Mathematics via the Geoboard, "Parallel Line Segments," Geo-Cards 27/1-4, Walker. (Available from Fitzhenry & Whiteside)

Mathex: Geometry No. 4, "Parallel Lines," pp. 25–26, Encyclopaedia Britannica Publications Ltd.

Nuffield Project: *Environmental Geometry,* "Drawings and Models," pp. 24–27, Wiley.

Nuffield Project: *Shape and Size* 2, "Which Shapes Fit Together Best?" pp. 25–35; "Parallel Lines," p. 89, Wiley. [Appropriate for the next lesson, also]

PAGES 198–199

Objective

Given 2 parallel lines cut by a transversal, the child will be able to identify pairs of angles that are the same size.

Preparation

Materials

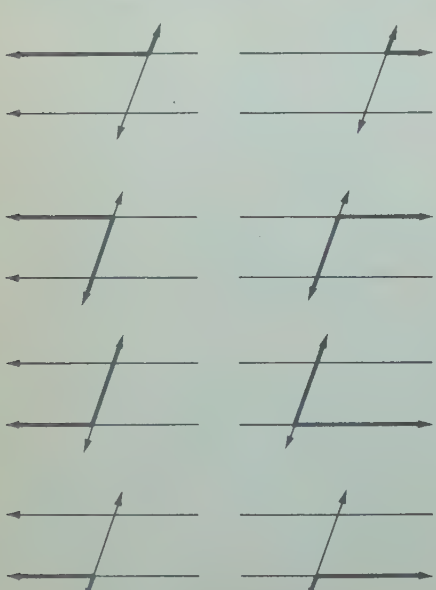
crayons; ruler or straightedge; scissors; unlined paper

Briefly review with the children the meaning of parallel lines. Draw various pairs of lines on the chalkboard and ask children to identify which pairs are parallel. You might also ask for suggestions of things which make us think of parallel lines.

Investigation

Direct the children's attention to the illustrations and accompanying text at the top of page 198. You might ask for volunteers to recall the meaning of the term *angle* (Chapter 4, page 80). When it has been recalled that an angle is two rays with the same endpoint, allow the children time to carry out the investigation according to the instructions in the text (reminding them, if necessary, that they can use the opposite edges of their rulers to draw a pair of parallel lines).

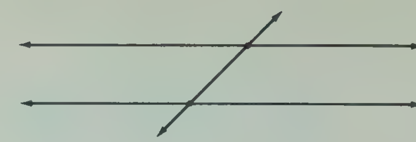
If some children finish the activity quickly, you might wish to suggest that they find and color other angles. The angles they may find are shown below.



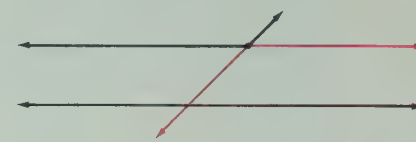
Let's explore angles and parallel lines.

Investigating the Ideas

When a line crosses two **parallel lines**, eight angles are formed.



One angle is shown in red.

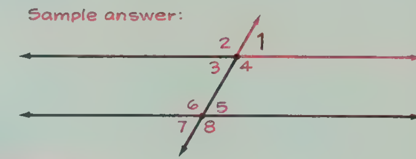


? Can you draw a figure like this and use your crayons to color another angle? See Investigation.

Discussing the Ideas

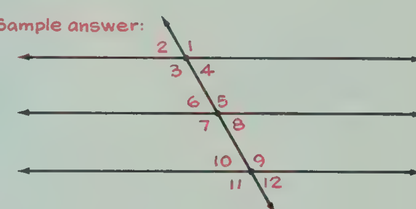
1. In the figure, angle 1 is shown in color. Show how you would draw and number the other seven angles. See Discussion.

Sample answer:

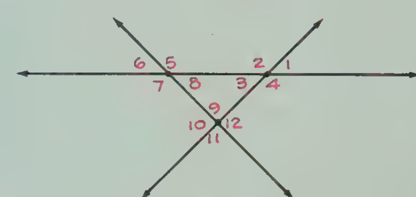


2. The figure shows a line that crosses three parallel lines. How many angles can you find? Draw a picture and number them. 12 See Discussion.

Sample answer:



3. How many angles can you find in this figure? 12 See Discussion.



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Discussion

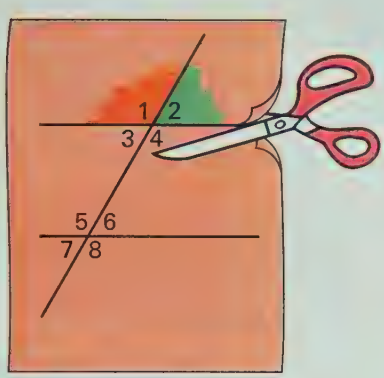
Before discussing these exercises, have prepared on the chalkboard illustrations like the three that appear in the discussion section of the text. Then read through the first exercise with the children and ask for volunteers to mark and number the angles with colored chalk. For exercise 2, allow children to speculate about the probable number of angles before they actually draw the figure and number the angles. Again, for exercise 3, encourage children to guess the number of angles before you select a volunteer to go to the chalkboard and enumerate them. (By this time many children will have perceived that four angles are formed by each intersecting pair of lines; however, there is no need for any explicit statement of this fact.)

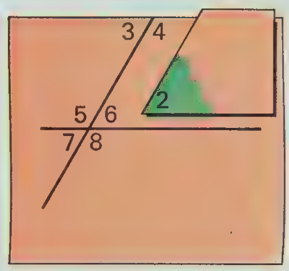
Note that the assignment of specific numbers to specific angles is entirely arbitrary; the angles may be numbered in any order that is convenient.

Using the Ideas

See Using the Exercises.

1. Draw a line that crosses two parallel lines.
Number the eight angles that are formed as shown below.
Color the insides of angles 1 and 2 and cut them out.

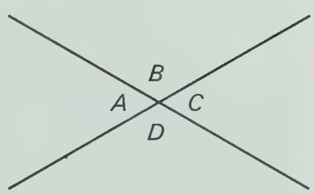




Which of the other angles (3, 4, 5, 6, 7, or 8) are the same size as angle 2? Use angle 2 to help you find out.
Angles 3, 6, and 7

2. Which of the other angles are the same size as angle 1?
Use angle 1 to help you find out.
Angles 4, 5, and 8

3. Draw a pair of lines that cross each other. Letter the angles as in the figure.



A Which angle is the same size as angle *A*? Angle C
B Which angle is the same size as angle *B*? Angle D

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Using the Exercises

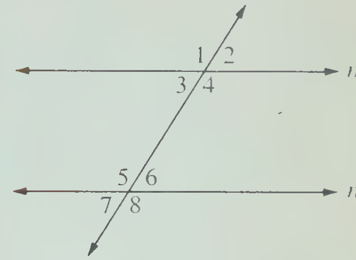
Review with the children the ways they learned to make parallel lines: by using their ruler edges, and by folding sheets of paper in a certain way. If you think it necessary, read the directions with the class. Then give the children an opportunity to make the discoveries in exercises 1, 2, and 3 before you discuss the ideas. For exercise 3, it would be helpful for them to draw a pair of intersecting lines and then cut out and manipulate one angle to find another angle which matches it.

As the children talk about their discoveries, they may speak of the congruent angles as "equal" angles. This matter of terminology should not be belabored but, by your own example, encourage use of the phrase "the same size" in preference to "equal." To help the children understand, intuitively, what we mean when we speak of the size of an angle, you might use two strips fastened with a paper fastener or a commercial chalkboard compass to show angles of different sizes.

Assignments (page 199)
Minimum: 1–2. Average: 1–3.
Maximum: 1–3.

Mathematics

The figure shows parallel lines *m* and *n* cut by a third line called a *transversal,* forming angles 1 through 8.



Angles 1 and 4, 2 and 3, 5 and 8, and 6 and 7 are pairs of *vertical angles.* Angles 1 and 5, 2 and 6, 3 and 7, and 4 and 8 are pairs of *corresponding angles.* Angles 3, 4, 5, and 6 are pairs of *interior angles.* Angles 1, 2, 7, and 8 are *exterior angles.* Angles 3 and 6, and 4 and 5 are *alternate interior angles.* Angles 1 and 8, and 2 and 7 are *alternate exterior angles.* If lines *m* and *n* are parallel, the pairs of angles named above are congruent (the same size). These ideas are introduced to the children informally and without emphasis on the encumbering terminology.

Follow-up

To give children practice in recognizing parallel lines, suggest that they bring in pictures of things which make them think of parallel lines. You might use these pictures to talk about perspective—how the lines we draw to picture parallel lines are not themselves parallel as were the lines drawn in this lesson, but how the objects they represent suggest parallel lines.

Resources for Active Learning

Mathex: Geometry No. 4, "Lines and Angles—Activity 5," p. 29, Encyclopaedia Britannica Publications Ltd.

Nuffield Project: *Shape and Size* 3, ". . . Parallels and Angles," pp. 9–10, Wiley.

Workbook, page 75

PAGES 200–201

Objective

Given a four-sided figure, the child will be able to identify it as a quadrilateral.

Preparation

Materials

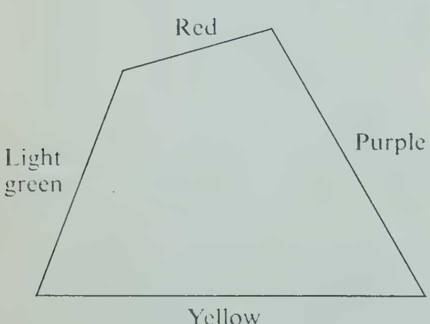
colored strips; ruler or straightedge

The investigation which introduces this lesson can be undertaken without any special preparatory activity.

Investigation

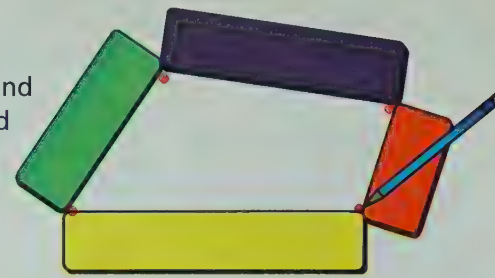
Treat this investigation quite informally, allowing children ample time to try various arrangements of the strips to form different-shaped quadrilaterals. Be on the alert for any signs of frustration that might result from attempts to be overly precise in laying down the strips so that the corners touch exactly. If any children experience real difficulty in performing the manipulations, do not hesitate to give them individual help and encouragement.

Remind the children to record their results by using their rulers to connect the points they have found. It would be helpful for them not only to record the shape of the figure but also to label the segments.



Investigating the Ideas

These strips help you find the corners of a 4-sided figure.



? Can you make a different 4-sided figure by using this set of strips? See Investigation.	Record your figure by marking points at the corners and connecting them.

Discussing the Ideas

A quadrilateral

1. A closed figure like this one is called a quadrilateral
   - A A quadrilateral has __?__ line segments. 4
   - B Can you name some objects that are in the shape of quadrilaterals? Sheets of paper, posters, bulletin boards, etc.
2. How many vertices does a quadrilateral have? 4
3. Can you mark 4 points on the chalkboard so that a quadrilateral is not formed when they are connected? A quadrilateral cannot be formed if more than 2 of the points are on the same straight line.
4. Can you find 4 of your strips that do not form a quadrilateral? See Discussion.

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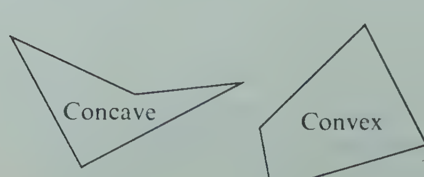
Discussion

Help the children with the word *quadrilateral*. You might break it down for them as quadri- (meaning *four*) and *lateral* (meaning *side*). Let the children discuss the different figures they made using different arrangements of the strips. If feasible, display these and other quadrilaterals on the flannelboard. You might even include a concave polygon, though for the children you should emphasize simply that a quadrilateral is a closed figure with four line segments as sides.

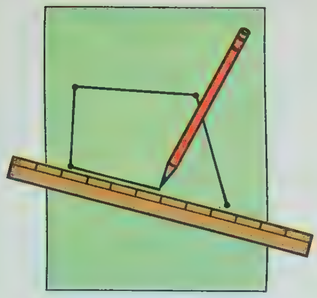


Review the terms *vertex* and *vertices* as you discuss exercise 2. For exercise 3, help the children realize that no three points of the figure may be on the same straight line. For exercise 4, the children will discover through trying various combinations of the strips that any four can be used to form a quadrilateral except when the measure of the longest strip chosen is equal to or greater than the sum of the measures of the other three strips (as in the combination 10, 2, 3, and 4).

Using the Ideas

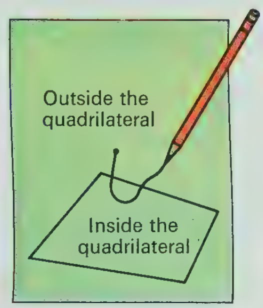
See Using the Exercises.

1. A Mark 4 points on your paper, as in the figure. Be sure that no 3 of them are in a line. Now use your ruler to connect the points like this. When you finish, you will have a quadrilateral.
   B Draw one segment that divides your quadrilateral into two triangles.

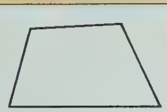
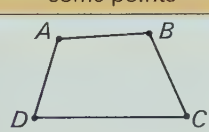


2. Draw a triangle. Draw one segment on this figure that divides the triangle into a smaller triangle and a quadrilateral.

3. Draw a quadrilateral. Mark a point outside the quadrilateral. From this point, draw a path that crosses the quadrilateral 4 times.
   A Where will your pencil point be then, inside or outside? Outside
   B If the path crosses 9 times, where will it end, inside or outside? Inside



★ 4. Study the chart. Then draw and name 5 different quadrilaterals.

We see a quadrilateral	We label some points	We write a name for it	We say
		*ABCD*	"quadrilateral *ABCD*"

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Using the Exercises

If necessary, read the directions with the children, but keep specific directions to a minimum. Some children will need additional help, but see that those who are more capable have a chance to make discoveries alone. Note that in exercise 2 if children draw a segment from a vertex to the opposite side, they will form two triangles instead of a triangle and a quadrilateral. More capable children might extend exercise 3 by trying other numbers of crosses, and eventually they will realize that every even number of crosses will end on the same side as the beginning point.

Although exercise 4 is starred, it would be wise to use the chart as a summary of this lesson. Common quadrilaterals such as square, rectangle, parallelogram, rhombus, and trapezoid should be listed. This can be an excellent introduction to the next lesson.

Assignments (page 201)
Minimum: 1–3. Average: 1–3. Maximum: 1–4.

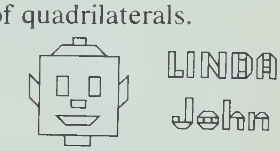
Mathematics

A quadrilateral is any four-sided polygon. It is a closed figure which separates the plane into three sets of points: those inside, or interior to, the quadrilateral; those on the quadrilateral; and those outside, or exterior to, the quadrilateral.

The investigation illustrates that, when the lengths of four sides of a quadrilateral are fixed, many different shapes are possible. If the same experiment were tried for a triangle, using three strips, we would find that all the resulting triangles would be the same size. For this reason, a triangle is sometimes spoken of as a rigid geometric figure whereas quadrilaterals are nonrigid figures.

Follow-up

If the children enjoyed working with quadrilaterals, have them copy or trace some of the figures they recorded during the investigation onto colored paper and mount them on display paper. They might also try to draw or construct interesting objects using only four-sided figures; or they might try to print their names using letters made up of quadrilaterals.



Quadrilateral man

Quadrilateral letters

Resources for Active Learning

Franklin Series: *Learn to Fold . . .*, "A Special Kind of Rectangle," pp. 89–95, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Franklin Series: *Pencil and Paper Geometry*, "Quadrilaterals," pp. 48-51, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Freedom to Learn, "Experimenting with Squares . . . ," p. 152, Addison-Wesley.

Inquiry in Mathematics via the Geoboard, "Polygons," Geo-Cards 10/1, 2, Wiley.

Mathex: Geometry No. 4, "Chinese Tangram – Activity 2," pp. 19–20 (pupil page 35), Encyclopaedia Britannica Publications Ltd.

Objective

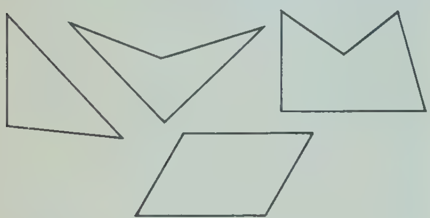
Given one of the special quadrilaterals, the child will be able to identify it as a square, rectangle, parallelogram, rhombus, or trapezoid.

Preparation

Materials

colored strips; ruler; scissors; crayons

To prepare for this lesson, draw figures like the following on the chalkboard and ask the children to point out the quadrilaterals.



Investigation

In the last investigation, the children freely explored a variety of quadrilaterals. In this investigation, children follow guidelines to help them construct specific quadrilaterals. Let the children work as independently as they can; give guidance only to those who definitely need it. Allow children to share ideas, but let each child have a chance to find the strips required for each quadrilateral by himself.

The strips which can be used for the various quadrilaterals are as follows:

A (square)—four 3-strips (light green)

B (rectangle or parallelogram)—two 2-strips (red) and two 3-strips (light green)

C (trapezoid)—various possibilities, including two 2-strips (red), one 3-strip (light green), and one 4-strip (purple); two 3-strips (light green), one 4-strip (purple), and one 5-strip (yellow); etc.

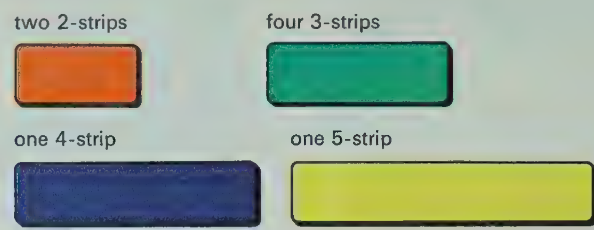
D (quadrilateral)—one of each: 2-strip (red), 3-strip (light green), 4-strip (purple), and 5-strip (yellow)

You might wish to have the children not only list the strips they use for each figure but also record the figures they find by putting dots at the corners and connecting them.

● *Can you name some special quadrilaterals?*

Investigating the Ideas

For this investigation, use these strips.



? Can you make one of the quadrilaterals described below?	List the strips you used.

See Investigation.

A All four sides have the same length.

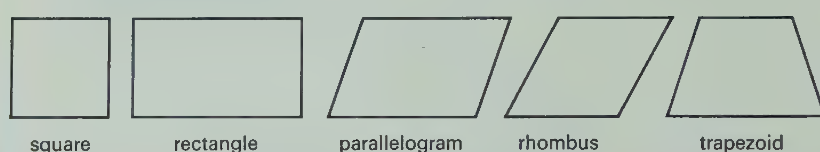
B The two longer sides have the same length and the two shorter sides have the same length.

C Two sides have different lengths and are parallel.

D No two sides have the same length.

Discussing the Ideas

Special Quadrilaterals



1. Which of the special quadrilaterals shown did you make in the Investigation? Answers will vary.

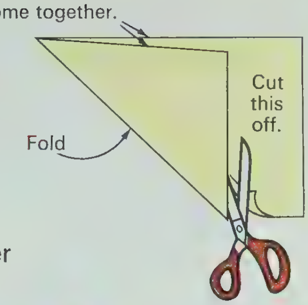
2. In what ways are squares and rectangles alike? In what way are they different? Compare other figures in this way. See Discussion.

Discussion

For an effective discussion of exercise 1, use felt strips on a flannelboard to show the strips which might be used for each quadrilateral, as listed in the preceding section. As you discuss each shape, write the corresponding term on the chalkboard and help children associate it with the figure on the flannelboard as well as the one they made in the investigation. It would be helpful for them to copy the correct term, labelling the quadrilaterals they have drawn on their papers.

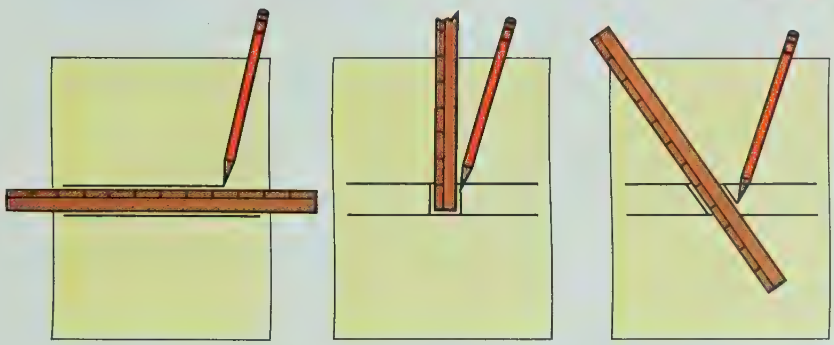
In exercise 2, help the children compare the figures by asking questions such as the following: How are a rhombus and a square alike? (Both have four equal sides.) How do rectangles and parallelograms differ? (Angles of a rectangle must be right angles; angles of a parallelogram need not be right angles.) How is a trapezoid different from a parallelogram? (A parallelogram has 2 pairs of parallel line segments; a trapezoid has only 1 pair.)

Using the Ideas



1. A What kind of figure is suggested by a sheet of tablet paper? Rectangle
   B Use the method suggested in the picture to cut a square from tablet paper.

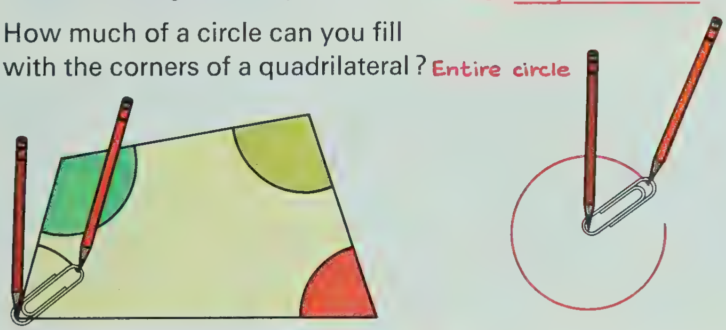
2. Study these pictures. Then use your ruler to draw a square and a rhombus.



Draw a line along each side of your ruler. | Draw like this to form a **square**. | Draw like this to form a **rhombus**.

Can you use your ruler like this to draw a rectangle and a parallelogram? See Using the Exercises.

★ 3. How much of a circle can you fill with the corners of a quadrilateral? Entire circle

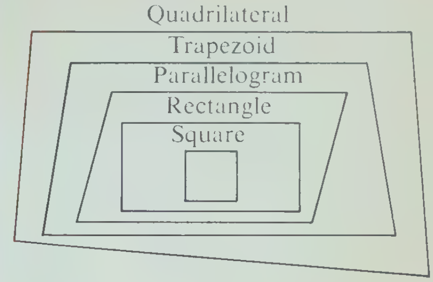


Using the Exercises
Direct the children to work through these exercises without group guidance. In exercise 2 some children may observe that a square is a rectangle and a rhombus is a parallelogram, but encourage them to draw other kinds of rectangles and parallelograms. You might give more specific, verbal directions for exercise 3, instructing the children to mark each corner with a paper clip arc as illustrated, color and cut each out, and try to fit the four corners into a circle drawn with the same paper clip.

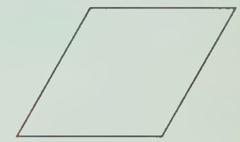
To extend these exercises, challenge the children to try cuts (other than the one suggested in exercise 1) to form other shapes.

Assignments (page 203)
Minimum: 1–2. Average: 1–2.
Maximum: 1–3.

Mathematics
The kinds of quadrilaterals may be pictured as in the diagram.

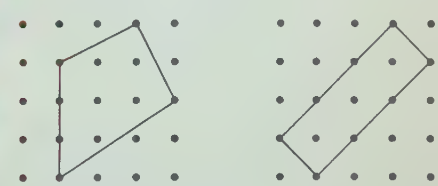


A rhombus is also a kind of quadrilateral: all sides of equal length.



Follow-up
Now that the children have studied many basic geometric shapes, you might have them do some paper folding. Suggest that they experiment and try to make a paper hat, an airplane, or a bird. Directions for making such items may be found in numerous books such as those recommended in the introduction of this chapter. Children may also enjoy origami activities, for which kits are available at many art or five-and-dime stores.

This would also be a good time for the children to try to form different quadrilaterals on a geoboard.



Resources for Active Learning
Developmental Math Cards, F³14, G²17, Addison-Wesley.
Math Activity Cards, "Nail Boards and Four-sided Shapes," A 12, Macmillan.

PAGES 204–205

Objectives

Given a parallelogram, the child will be able to divide it into two congruent triangles.

Given a quadrilateral, the child will be able to construct a parallelogram by connecting the midpoints of the sides of the quadrilateral.

Preparation

Materials

tracing paper; scissors; crayons; construction paper

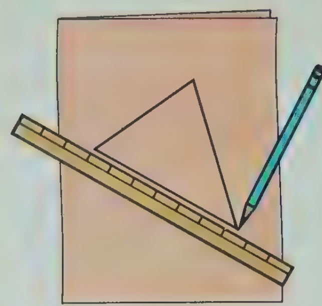
Prepare for this lesson briefly by reviewing the term *parallelogram*. For this purpose have several different-sized parallelograms to display, as well as quadrilaterals that are not parallelograms.

Investigation

If necessary, read the directions with the children, stressing that the two triangles they cut out are to be the same size and shape. Let each child work at his own rate and, if some finish the investigation quickly, suggest that they repeat the activity using different triangles. If some use a right triangle, the parallelogram they construct will be a rectangle. You might like to have the children display their results on a bulletin board, after they have labelled their finished parallelogram.

Let's explore parallelograms.

Investigating the Ideas



Draw a triangle on a piece of folded paper.

Cut the folded paper to get two triangles that are just alike. Color them different colors.

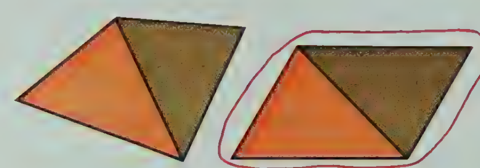
? How many different quadrilaterals can you make by placing sides of your triangles together?

See Investigation.

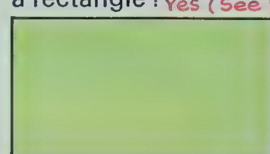
Discussing the Ideas

1. How many parallelograms did you find in the Investigation?

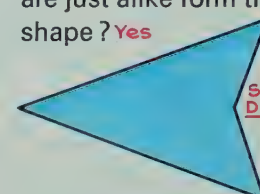
2. Jack made these two shapes with his triangles. Which one is a parallelogram?



3. Can you cut out two triangles that are just alike and form a rectangle? Yes (See Discussion.)



4. Could two triangles that are just alike form this shape? Yes



See Discussion.

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Discussion

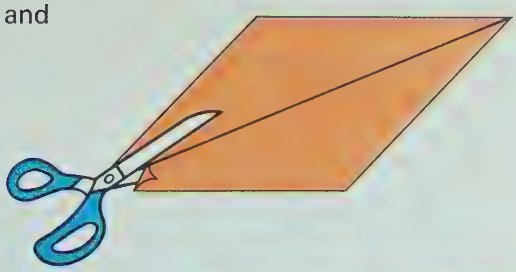
Allow the children an opportunity to verbalize their discovery that a parallelogram can be constructed from two triangles that have the same size and shape. As you discuss their constructions, emphasize that the opposite pairs of sides are parallel.

As you discuss exercise 3, take advantage of the squares and rectangles some children may have constructed during the investigation. Bring out the fact that all squares and rectangles are parallelograms, but not all parallelograms are rectangles and squares; only those parallelograms which have right angles are rectangles or squares.

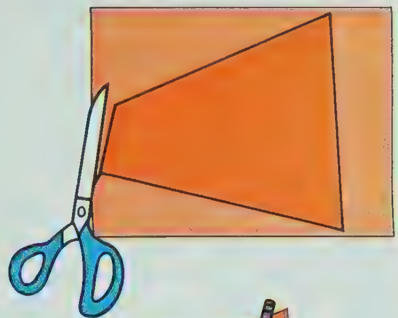
If any children are slow to see the line of symmetry in the figure for exercise 4, you might let them trace and cut out the figure and experiment with various folds to find the two triangles that are the same size.

Using the Ideas

1. Draw a parallelogram and cut it out. Then cut it into two triangles. Do they fit exactly upon each other? Yes



2. Draw a large 4-sided figure on your paper.



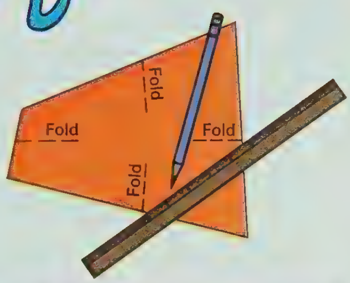
   A Color it and then cut it out.

   B Find the midpoint of each side by folding.

   C Connect the midpoints with segments as in the figure.

   D What kind of a figure do you think you have? A parallelogram

   E Do you think this would work if you started over with a different 4-sided figure? See Discussion.



think

How many pennies does it take to make a stack as high as a penny standing on edge?
5? 6? 7? 8? 9? 10? 11? 13? 15?
Guess. Then check your guess. 13

205

Mathematics

We give below a formal definition of a parallelogram:

> A parallelogram is a quadrilateral having two pairs of parallel sides.

Exercise 1 on page 205 demonstrates an interesting theorem of geometry:

> Either diagonal of a parallelogram divides it into two congruent triangles.

Follow-up

Give each of the children an envelope containing quadrilaterals of different sizes and shapes. Suggest that they color, cut, paste, draw lines, or use any other means they have learned to prove to themselves which of the shapes are parallelograms. Expect some children to try to find matching triangles by folding along the diagonals. Others will probably draw diagonals and cut out triangles so that they can be manipulated.

Resources for Active Learning

Inquiry in Mathematics via the Geoboard, "Congruent Triangles," Geo-Card 22, Walker. (Available from Fitzhenry & Whiteside)

Using the Exercises

Some children may need specific directions on how to draw a parallelogram. You might suggest using the ruler as pictured on page 203. Then direct the children to follow the illustrations in the text, giving help when necessary. For example, you may need to remind the children that to find the midpoint of an edge they must bring the adjacent corners together and mark the place of the resulting centre fold. Give children a chance to try exercise 2 with different-shaped quadrilaterals. You might encourage more capable children to begin with a square and then observe the kind of parallelogram formed by connecting the midpoints (a smaller square). Similarly if they begin with a rhombus, which is not a square, the quadrilateral formed by connecting the midpoints will be a rectangle.

If time permits, all the children would enjoy trying the *Think* problem at the bottom of the page.

Assignments (page 205)
Minimum: 1–2. Average: 1–2.
Maximum: 1–2.

PAGES 206–207

Objective

Given a polygon, the child will be able to draw and identify the diagonals.

Preparation

Materials

scissors; tracing paper

Display models of some of the figures the children have already studied, such as triangles, squares, parallelograms, and quadrilaterals, and help children identify each with its proper geometric name. Introduce the term *diagonal* and show a *diagonal* in several of the figures. A diagonal is a line segment, other than a side of the polygon, which joins one vertex to another vertex.

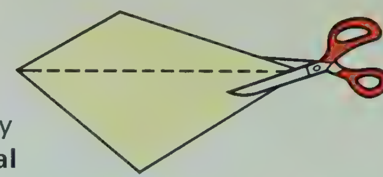
Investigation

Study the illustrated polygons with the children, pronouncing with them the new words, *pentagon*, *hexagon*, and *polygon*. Explain that a polygon is a many-sided figure. Have them count the number of line segments in the pentagon and the hexagon. Review the procedure of cutting from corner to corner as they did with the parallelogram, emphasizing that they are cutting along a *diagonal*. Remind them to record their answers to the investigation question.

Let's explore some polygons.

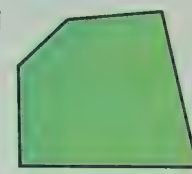
Investigating the Ideas

You can separate any quadrilateral region into two triangular regions by cutting along a **diagonal** (corner to corner).

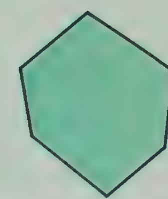


quadrilateral

Draw a **pentagon** (5-sided polygon) and a **hexagon** (6-sided polygon) and cut them out.



pentagon



hexagon

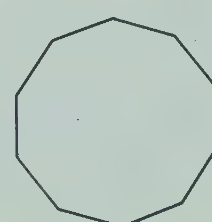
? Can you find how many triangles are formed by cutting along the diagonals from one corner of a pentagon ? 3 of a hexagon ? 4

Discussing the Ideas

1. Can you explain how to complete the table ?

Number of sides of polygon	Number of triangles formed
3	1
4	2
5	3
6	4
7	5

2.



This polygon has ten sides and is called a **decagon**. Into how many triangles can it be separated by cutting along the diagonals from one corner ? 8

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Discussion

Read the discussion questions with the children and together fill in the table. Explain that the corner from which a diagonal is drawn is called a *vertex*, and when we talk about more than one vertex we use the term *vertices*.

For exercise 2, ask children to see whether they can discover a pattern in the table for exercise 1 that suggests the probable answer. Then let them check the answer by tracing and drawing diagonals or cutting along diagonals.

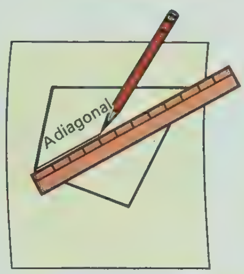
Encourage children who finish quickly to extend the table to polygons of a greater number of sides and to check some of them, such as a 16-sided polygon, to see how many triangles they can cut. (They can construct a 16-sided polygon by folding a piece of paper around one point four times (16 layers) and making one cut across the top, away from the centre.)

If time is available, display a variety of polygons on the chalkboard or overhead projector and ask children to draw in the diagonals.

Using the Ideas

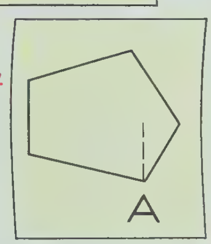
1. **Regular polygons** have all sides and all angles the same. Copy and complete the table for the regular polygons shown.

Name of polygon	Number of sides	Number of angles	Any sides parallel?
Regular triangle (equilateral)	3	3	No
Regular quadrilateral (square)	4	4	Yes, both opposite pairs
Regular pentagon	5	5	No
Regular hexagon	6	6	Yes, 3 opposite pairs



2. Draw a **quadrilateral** on your paper.
   - A Draw all the **diagonals**.
   - B How many diagonals are there? 2
   - C How many are there from each vertex? 1

3. Draw a **pentagon** on your paper. Choose one vertex and label it ***A***.
   - A How many diagonals can you draw from ***A***? 2
   - B Can you draw the same number of diagonals from any vertex? Yes



★ 4. Give the total number of diagonals that can be drawn for each figure.
   - A pentagon 5
   - B hexagon 9
   - C a 10-sided polygon 35

Using the Exercises
Rather than have the children copy the table from the top of page 207, you might prefer to make a table on a duplicating master and distribute a copy to each child.

Exercises 2, 3, and 4 require that children know which figures are represented by the terms quadrilateral, pentagon, and hexagon; if any children become confused, refer them to the chart to see how many sides each has.

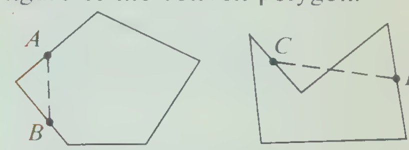
Some capable children may discover a pattern to help them solve exercise 4, or encourage them to construct the polygons to find the correct number of diagonals.

You might wish to extend this exercise to provide data for a table such as the one below.

Number of: Vertices	Number of: Diagonals from each	Total diagonals
4	1	2
5	2	5
6	3	9
7	4	14
8	5	20
9	6	27
10	7	35

Assignments (page 207)*
Minimum: 1–2. Average: 1–3.
Maximum: 1–4.

Mathematics
Polygons may be either convex or concave. This geometry lesson involves only convex polygons like those illustrated at the top of page 206. Convex polygons are polygons such that if any two points of the polygon are connected by a segment all points of that segment are either on the polygon or in its interior. This is illustrated in the figure of the convex polygon.



Convex polygon   Concave polygon

Note that in the figure on the right, however, some points of $\overline{CD}$ are neither on the polygon nor in the interior of the polygon; hence, it is a concave polygon.

Follow-up
Give the children paper on which to construct two polygons that have many sides. Have them draw one polygon, cut it out, and use it to trace a polygon that matches it. Direct them to paste one of the polygons on a piece of construction paper. Then cut the other polygon along diagonals to construct triangles or other figures. Use these cutout triangles to form a design and paste them below the original polygon on the construction paper.

Resources for Active Learning
Developmental Math Cards, E³13, F³20, G³4, Addison-Wesley.
Inquiry in Mathematics via the Geoboard, "Polygons," Geo-Cards 10/4-6; 32/1,2, Walker. (Available from Fitzhenry & Whiteside)
Math Activity Cards, "Nail Boards and Polygons," A13; "Polygons," A 22, Macmillan.
Mathex: Geometry No. 4, "Lines and Angles—Activity 2," p. 27; "Using the Chinese Tangram," pupil pages 36–39, Encyclopaedia Britannica Publications Ltd.
Math Workshop: Games and Enrichment Activities, "Classifying Polygons," pp. 85–86, Encyclopaedia Britannica Educational Corp.

Workbook, page 76

PAGES 208–209

Objective

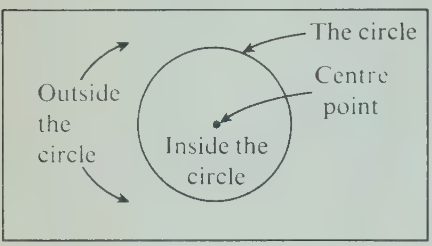
Given a selection of curves, the child will be able to recognize simple closed curves.

Preparation

Materials

cups; paper clips; compasses; string

Draw the diagram below on the chalkboard or on a transparency for the overhead projector.



Point out the points of the diagram, stressing that the *circle* is the black (or chalked) *outline*. Ask the children whether or not they think it would be easy to draw a circle. Explain that in this lesson they will have a chance to find out.

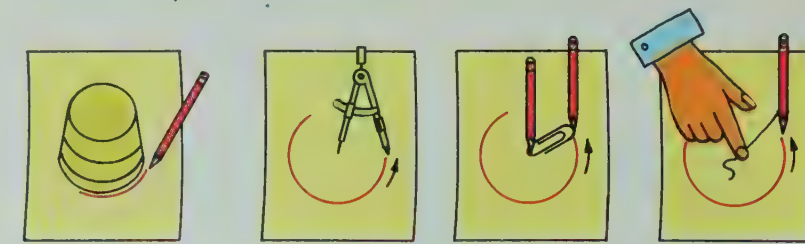
Investigation

The children are presented with four possible ways of drawing a circle. Most children will be able to follow the procedures suggested by the pictures. As you move around the room, watch to see that the children keep the materials in place once they have begun to draw the circle. The cup or can, the point of the compass, the point of the pencil on the centre point, and the finger anchoring the string should remain stationary once the drawing is begun. Help the children to turn the compass properly by holding only the small knob at the top and rotating it, while slightly slanting the stationary centre leg. Give the children sufficient time to try all four methods.

What is a simple closed curve?

Investigating the Ideas

Here are four ways to draw a circle.

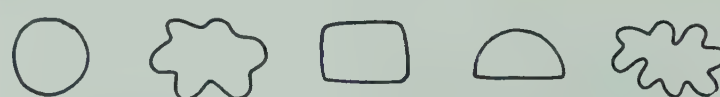


Drawing around a cup or a can | Using a compass | Using a paper clip | Using a string

? Can you use each of these methods to make a circle? Which method was easiest for you? See Investigation.

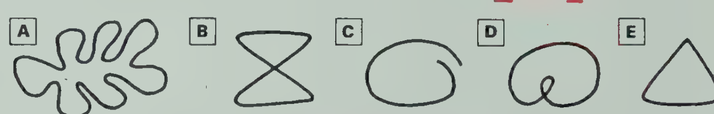
Discussing the Ideas

1. Each of these is a **simple closed curve**.



None of these is a **simple closed curve**.

Which of these are **simple closed curves**? A and E



2. How can you tell whether or not a figure is a **simple closed curve**? See Discussion.

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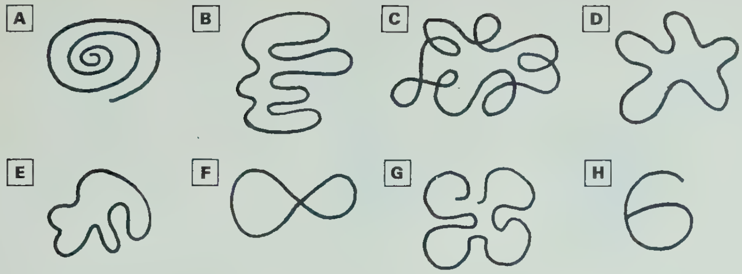
Discussion

Let the children discuss their preferences for a method of drawing a circle. Then point out that the circle has both an *interior* and an *exterior*. An ant travelling beside the path of the circle would never be able to pass from the outside to the inside without crossing the circle. Also, from any point on the circle you may trace a path which will return you to your original point. So the circle is a *closed curve*. The circle is also a *simple curve:* an ant travelling along the path of the circle would never cross a point twice until he returned to his starting point. Explain that a circle is an example of a *simple closed curve*. Encourage children to discuss what is alike in each group of figures in exercise 1. In the first group no curve intersects itself, and there is an interior and exterior to each figure. The figures in the second group either have no interior and exterior, or the path of the curve crosses itself at least once.

In exercise 2 help the children summarize that a closed curve has an interior and an exterior and that a simple curve is a curve that does not cross itself.

Using the Ideas

1.



A Which curves above are closed? B, C, D, E, F

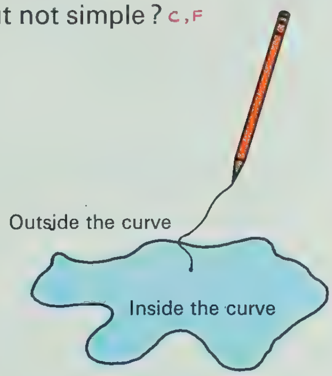
B Which curves above are simple closed curves? B, D, E

C Which curves above are closed but not simple? C, F

2. Draw a simple closed curve.
Mark a point inside the curve.

A Start at the point and draw a path that crosses the curve. Where is your pencil point now, inside or outside? Outside

B If you cross 5 times in all, where are you? 6 times? Inside Outside



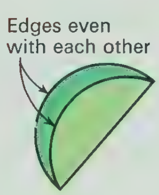
3. A Draw a circle and cut it out.

B Find the **centre** of the circle like this:

▲ Fold the circle as shown here.

▲ Open it and fold it again as shown but in a different place.

C Where is the centre of the circle?
At the point where the fold lines intersect.



Using the Exercises

It would be helpful to let the children study the figures in exercise 1 for a few minutes. Then have them discuss and explain their answers. You might assign exercises 2 and 3 as independent work. However, when you discuss exercise 2, use other examples until the children discover that crossing the path an odd number of times means you are on the side opposite to where you began, and crossing an even number of times brings you back to the side where you began. In exercise 3 the children will discover that the centre of the circle is the point of intersection of the two lines. You may have them make other similar folds to see that the resulting lines (diameters) will always intersect at the centre of the circle.

Assignments (page 209)

Minimum: 1–2. Average: 1–3.
Maximum: 1–3.

Mathematics

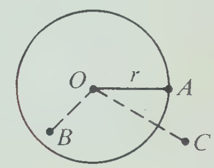
The early geometry lessons in this series are primarily concerned with the geometry of the physical world. Thus the children can call the black outline shown on paper a circle. When they are more mature, they will learn that the black outline is a picture or model of an idea existing in our minds. Such abstractions should be avoided in this early study of geometry.

The following are mathematical definitions for a circle, the interior of a circle, and the exterior of a circle:

A *circle* with centre O and radius r is the set of all points A in a plane such that the length of $\overline{OA}$ is equal to r.

The interior of the circle is the set of all points B such that the length of $\overline{OB}$ is less than r.

The exterior of the circle is the set of all points C such that the length of $\overline{OC}$ is greater than r.



Resources for Active Learning

Franklin Series: *Pencil and Paper Geometry,* "Curves and Points," pp. 57-60, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Mathex: Geometry No. 9, "Open and Closed Curves . . . — Activity 3," pp. 20–22, Encyclopaedia Britannica Publications Ltd.

Math Workshop: Games and Enrichment Activities, "Curves," p. 81; "Polygons," p. 85, Encyclopaedia Britannica Educational Corp.

Nuffield Project: *Environmental Geometry,* "Curves," p. 23, Wiley.

PAGES 210–211

Objective

Given suitable materials, the child will be able to construct a symmetrical figure.

Preparation

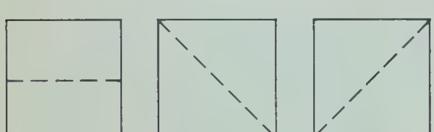
Materials

paper squares suitable for folding (1 per child)

To prepare for this lesson, write the word *symmetrical* on the chalkboard. Display a picture or design which when folded in half forms two matching halves. Explain that such a figure is called a symmetrical figure, and help the children pronounce the word correctly.

Investigation

In this investigation, children explore the symmetry of a square. You might prefer to have squares approximately 15 by 15 centimetres ready to distribute rather than have children cut out their own. Read the directions with the class. Then give them sufficient time to find the other three ways to fold the square. The three possibilities other than the one pictured in the text are indicated by the dashed lines in the figure below.



This investigation could be extended by using other methods to show symmetry. For example, have the children use water paints or ink to draw a simple picture on one half of a piece of drawing paper. While the paint or ink is still wet, instruct the children to fold the halves together, carefully pressing the dry half on top of the wet one. When the halves are again separated the original and the print should form symmetrical halves.

● *What is a symmetrical figure?*

Investigating the Ideas

This square is folded so that one half exactly matches the other half.

? How many different ways can you fold a square so that one half exactly matches the other?

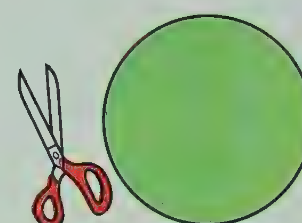
See Investigation.

Discussing the Ideas

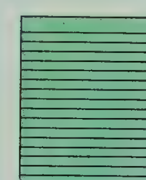
See Discussion.

1. A square is **symmetrical** because you can fold it so that one half exactly matches the other half. Are there other symmetrical figures? Yes

2. Draw a circle and cut it out. Can you fold it so that the halves match? Can you do this in more than one way? Yes; yes



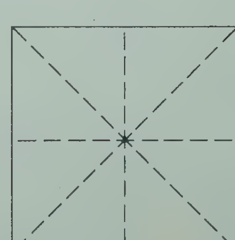
3. A sheet of tablet paper forms a rectangle. Can you fold it to form matching halves? Yes Can you do this in more than one way? Yes



210

Discussion

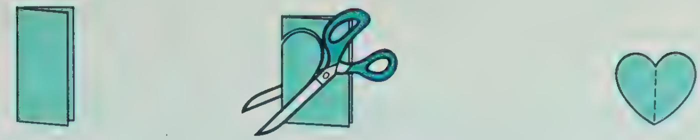
When the children have finished the folding activity, discuss with them the appearance of the halves of the square. They should notice that both halves have folds or creases that match. Explain that the halves are *symmetrical* to each other, no matter which of the four folds is used as the dividing line.



Use discussion exercise 1 to reinforce the children's understanding of the term symmetrical. For exercise 2, remind the children of the methods of drawing a circle investigated on page 208. There are innumerable diameters in a circle, so there are innumerable ways to fold the circle in half. The sheet of tablet paper, in exercise 3, can be folded in symmetric halves both vertically and horizontally.

Using the Ideas

1. Here is a way to make a symmetrical figure.



Fold a piece of paper.

Make a cut that starts and ends on the fold.

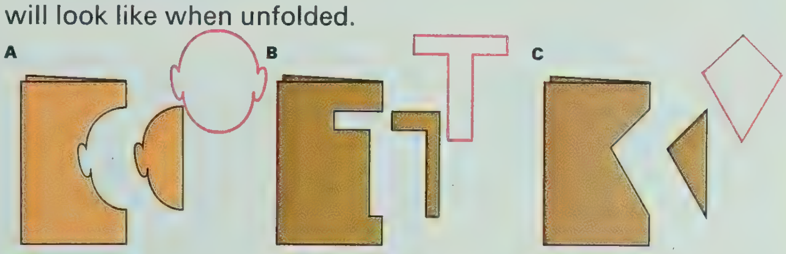
Unfold the piece you cut out. It will be symmetrical.

Make cuts so that the unfolded shape will look like

A a rectangle.	C a triangle.	E a house.	G a rocket.
B a leaf.	D a square.	F a pumpkin.	H a hexagon.

See Using the Exercises.

2. Draw a picture to show what each cut-out piece below will look like when unfolded.



3. Write **S** (symmetrical) or **NS** (not symmetrical) for each figure. If it is symmetrical, think about how you would fold it to make the halves match.

A S B S C NS D S

Using the Exercises

You may prefer to have the children use other methods for constructing symmetrical figures. The ink or paint "print" method suggested in the investigation may be used. Or some children may use the point of a pin or a compass to prick an outline of the figure, and then unfold the paper to see the full symmetrical figure. Whatever method is used, try to keep the children from being discouraged if they draw or cut their half figure incorrectly. Shapes such as the square and the hexagon may be particularly difficult, so have an ample supply of material available and encourage children to make several attempts.

Exercises 2 and 3 give the child practice in visualizing symmetrical figures without actually performing the cutting or pointing. However, some children may need to try the actual folding to answer parts of exercise 3.

Follow-up

This would be an appropriate time to study the patterns in leaves or snowflakes. You might extend the activities of this lesson into a mathematics-art project. Children might display their own symmetrical designs, designs from leaves, or pictures of real-life symmetrical objects. Most leaves give the appearance of being symmetrical but if examined closely are not. However, the outline of a leaf may be used to make a symmetrical figure.

Another method for creating symmetrical designs is to have children fold a piece of drawing paper in half and write their name along the fold either in ink or heavy crayon. Fold the blank half over the name and press. The imprint on the blank side should be dark enough for children to use as a guide to finish the design.

Before folding

After folding

Assignments (page 211)
Minimum: 1. Average: 1–3.
Maximum: 1–3.

Resources for Active Learning

Developmental Math Cards, D³3, F³4, G³5, Addison-Wesley.

Franklin Series: *Learn to Fold . . .*, "Some Same Shapes and Sizes," pp. 31–42, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Franklin Series: *Mirror Magic*, Lyons and Carnahan. [Activities built upon reflection symmetry.] (Available from McGraw-Hill Ryerson)

Mathematics in Modules, SK3, Addison-Wesley.

Mathex: Geometry No. 4, "Reflection," pp. 30–34 (pupil pages 50, 52), Encyclopaedia Britannica Publications Ltd.

Mirror Cards, Guide and Materials, Webster, McGraw-Hill. [This would be a good time to introduce these cards for exploration throughout the year.]

Nuffield Project: *Shape and Size* 2, "Fitting Shapes Together," pp. 36–61, Wiley.

Nuffield Project: *Shape and Size* 3, ". . . Symmetry," pp. 23–26; pictures of students' work on symmetry, pp. 19–22, Wiley.

Duplicator Masters, page 48
Workbook, page 77

PAGES 212–213

Objectives

The child will demonstrate his ability to work with the concepts presented in this chapter.

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

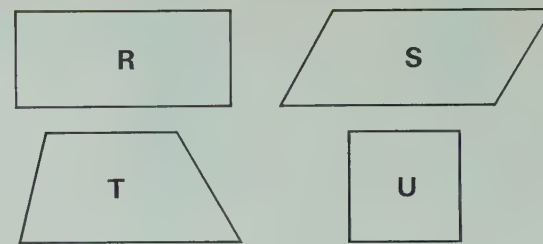
Preparation

The children will benefit from a display of the geometric shapes and models studied in the geometry units. As the children identify each shape, write its name on the chalkboard.

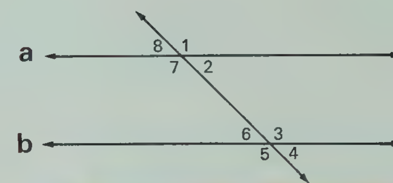
To prepare for page 213, review any concept on the page which has been troublesome for a majority of the children.

Reviewing the Ideas

1. Give the letter of the figure that matches each name.
   A square U
   B parallelogram S
   C rectangle R
   D trapezoid T

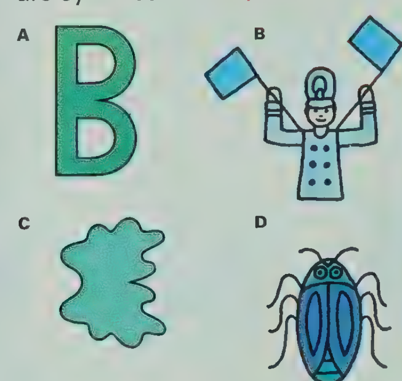


2. Lines **a** and **b** are parallel. Name all of the angles in the figure that are the same size as angle 2.
   Angles 4, 6, and 8

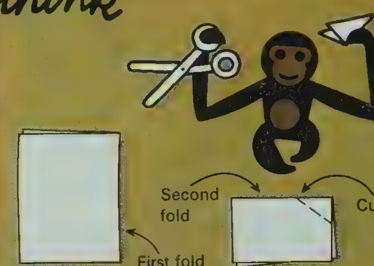


3. Draw a quadrilateral on your paper. Draw the diagonals of the quadrilateral.
   Constructions will vary.

4. Which of these shapes are symmetrical? A, C, D



think



Fold a piece of paper twice and cut off the corner as shown. Guess what the cut-off piece will look like when unfolded and draw a picture. Then unfold it and check your guess. A square or rhombus

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Discussion

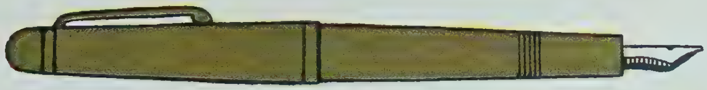
If you choose to use page 212 as a review, work through the exercises with the class. If you choose to use it as evaluation, assign the exercises as independent work.

Keeping in Touch with

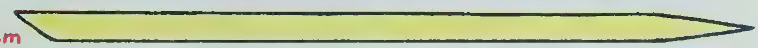
Measurement
Place value
Addition
Subtraction
Fractions

1. Measure each object to the nearest $\frac{1}{2}$ centimetre.

A 11½ cm

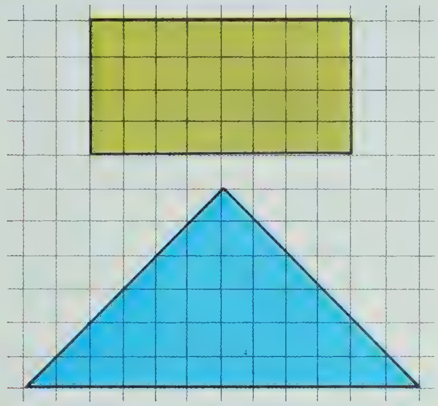


B 12½ cm



2. A Find the area of the rectangular region. 32
B What is the area of half the rectangle? 16
C What is the area of $\frac{1}{4}$ the rectangle? 8

3. A Find the area of the triangular region. 36
B What is the area of half this region? 18



4. Solve the equations.

A $56 = 40 + n$ 16
B $72 = 60 + n$ 12
C $41 = n + 11$ 30
D $90 = 80 + n$ 10
E $84 = n + 14$ 70
F $76 = n + 16$ 60

5. Find the sums and differences.

A	B	C	D	E
24	78	35	73	84
+31	−46	+48	−26	+78
55	32	83	47	162

F	G	H	I	J
121	362	284	375	721
−75	+475	−166	+468	−145
46	837	118	843	576

You are invited to explore

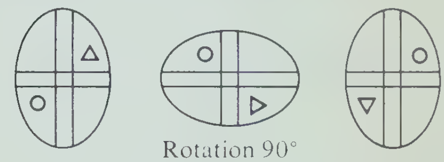
ACTIVITY CARD 10
Page 314

Using the Exercises

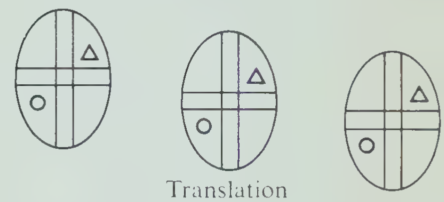
As with the chapter review, you may have the children work through page 213 together or you may assign the exercises as independent work. However, whether you discuss the exercises while the children work through them or when they are finished, go over with the children the process of measuring to the nearest half centimetre. In exercise 3, recall the method of counting half of the unit area for regions like this triangle. Exercise 4 stresses regrouping, and exercise 5 applies the regrouping method in addition and subtraction.

Follow-up

If the children enjoyed working with symmetry, they might also enjoy working with patterns. Potato prints would be appropriate. By turning the potato 90° with every print the child is intuitively introduced to rotation. By positioning the potato differently, the child is introduced to translation.

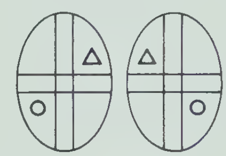


Rotation 90°

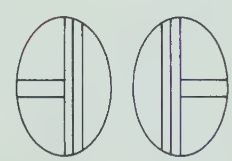


Translation

Only some patterns will show reflection.



No matter how the potato is turned, this reflection will not be produced.



This pattern shows reflection after a rotation of 180°.

Resources for Active Learning

Nuffield Project: *Shape and Size* 2, "Classification of 2-D Shapes," pp. 91–101, Wiley.

Workbook, page 78

CHAPTER 9 | Number Theory

Pages 214–225

General Objectives

To provide experiences with even and odd numbers
To introduce some properties of numeral endings
To explore sums and products involving even and odd numbers
To provide additional experiences with multiples and factors
To introduce prime numbers

The opening pages of this chapter offer intuitive definitions for even and odd numbers and concentrate on work involving these numbers. Then the children are given some experiences involving numeral endings and identification of even or odd numbers.

After the children work with sums and products of even and odd numbers, attention is focussed on multiples and factors, and they are given an opportunity to examine and work with these ideas. Next, prime numbers and some key concepts associated with them are introduced. The last lesson incorporates both a chapter and a cumulative review.

Mathematics

Following the definitions of even, odd, and prime numbers, several examples illustrate each set of numbers defined.

The *even numbers* are the set of all whole numbers x, such that

$$x = 2 \times n$$

where n is a whole number.

For example:

$$\left.\begin{array}{lcl} n=0 & \rightarrow & x=0 \\ n=1 & \rightarrow & x=2 \\ n=2 & \rightarrow & x=4 \\ n=3 & \rightarrow & x=6 \\ \vdots & & \vdots \end{array}\right\} \text{even numbers}$$

The *odd numbers* are the set of all whole numbers x, such that

$$x = (2 \times n) + 1$$

where n is a whole number.

For example:

$$\left.\begin{array}{lcl} n=0 & \rightarrow & x=1 \\ n=1 & \rightarrow & x=3 \\ n=2 & \rightarrow & x=5 \\ n=3 & \rightarrow & x=7 \\ \vdots & & \vdots \end{array}\right\} \text{odd numbers}$$

The set of prime numbers is the set of all whole numbers x, such that

x has exactly 2 different factors.

Thus:

2 is a prime number (factors: 2, 1).
3 is a prime number (factors: 3, 1).
5 is a prime number (factors: 5, 1).
7 is a prime number (factors: 7, 1).
11 is a prime number (factors: 11, 1).

Also:

1 is not prime (has only one factor: 1).
6 is not prime (factors: 1, 2, 3, 6).
91 is not prime (factors: 1, 7, 13, 91).

Teaching the Chapter

Materials

Colored strips
Graph paper
Unit centimetre strips or squares cut from graph paper

Vocabulary

even number	odd number
factor	prime number
multiple	skip counting

Multiple, factor, and *prime* are important words in the study of arithmetic and will be used throughout the remaining books in the *Investigating School Mathematics* series.

Lesson Schedule

This chapter is designed to be covered in one to two weeks. You may wish to continue into prime factorization and factor trees with more capable children.

Evaluation of Progress

In your evaluation of children's work with this material, attempt to judge both understanding and mastery of the fundamental concepts of even numbers, odd numbers, multiples, factors, and primes.

Though you will find that the average and above-average children may be quite excited by the ideas of this chapter, this can be a difficult chapter for the slower children. Attempt to prevent them from becoming discouraged by working through much of the chapter as a class activity and by maintaining a game-like atmosphere. Employ pattern recognition and demonstrations as part of the discussion activities.

Pages 224 and 225 provide chapter and cumulative reviews to help you evaluate children's understanding of the concepts presented in this chapter.

Resources for Active Learning

General Activities

Developmental Math Cards, E19, Addison-Wesley

Freedom to Learn, "Basic Number Facts," pp. 113–116, Addison-Wesley. This reference contains information for using the 100-Square and 200-Chart to find even and odd numbers, number patterns, primes, and factors, and to play addition and subtraction games.

Mathex: Numeration No. 2, "Patterns Using Number Cards," pp. 40–41, Encyclopaedia Britannica Publications Ltd.

Mathex: Numeration No. 7, "Sieve of Eratosthenes," pp. 9–12 (pupil pp. 5–7, Encyclopaedia Britannica Publications Ltd.

Nuffield Project: *Computation and Structure* 2, "The '100' Square," pp. 51–57, Wiley

Manipulative Devices

Cubical Counting Blocks (Milton Bradley; school supplier)

Cuisenaire Rods (Cuisenaire Co.)

Hundred Board and Cylinders (Educational Teaching Aids; Responsive Environments Corp.)

Pattern Blocks (Selective Educational Equipment; Webster, McGraw-Hill)

Sigma Chips (Sigma, Scott Scientific)

Commercial Games

Think-A-Dot (Childcraft; Edmund Scientific)

PAGES 214–215

Objectives

Given an even number, the child will recognize that it can be matched with a train of 2-strips.

Given an odd number, the child will recognize that it can be matched with a train of 2-strips and one 1-strip.

Preparation

Materials

set of colored strips

To introduce this chapter, you might ask volunteers to tell the class what their favorite or "lucky" number is. After all have had a turn, direct them to the text and explain that in this chapter they will learn some interesting things about numbers.

Investigation

In this investigation, the children use the strips to discover that some numbers may be matched exactly with a train of twos.

Read the investigation section in the text with the children. Encourage them to record the strips they find that can be matched with a train of 2-strips. Also encourage them to observe which strip would fit in the extra space when the strip and the train of 2-strips do not match.

9 Number Theory

● *What are odd and even numbers?*

Investigating the Ideas

You can match the 6-strip with a "train" of 2-strips.

? Which of the other strips can be matched with a train of 2-strips? 4-, 8-, and 10-strips

Discussing the Ideas

The numbers whose strips can be matched with a train of 2-strips are called **even numbers**.
Zero is also an even number.
The other numbers are called **odd numbers**.

1. A Name the even numbers less than 50. 0, 2, 4, 6, 8, 10, 12, . . . 48
   B Name the odd numbers between 50 and 100. 51, 53, 55, 57, 59, 61, ... 99

2. Every odd-numbered strip can be matched with a train of 2-strips and how many extra 1-strips? 1

3. Can you think of an easy way to decide whether a number is even or odd?
   Every even number has a numeral that ends in 0, 2, 4, 6, or 8.
   Every odd number has a numeral that ends in 1, 3, 5, 7, or 9.

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Discussion

As the children tell you the numbers they found which they were able to match with the trains of 2-strips, list the first few even numbers on the chalkboard.

Even: 2, 4, 6, 8, 10, 12, . . .

Then relate these numbers to the discussion in the text.

For exercise 1, have volunteers list the even and odd numbers on the chalkboard.

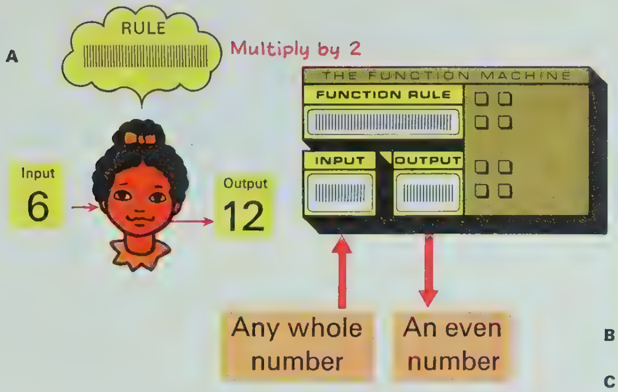
Help children realize that two times a whole number is an even number and that every odd number is one more than an even number.

In discussion exercise 3, you might have to direct attention to the last digit in each numeral. This idea will be developed more fully in the next lesson.

Using the Ideas

1. Give the function rules and the missing numbers.

A

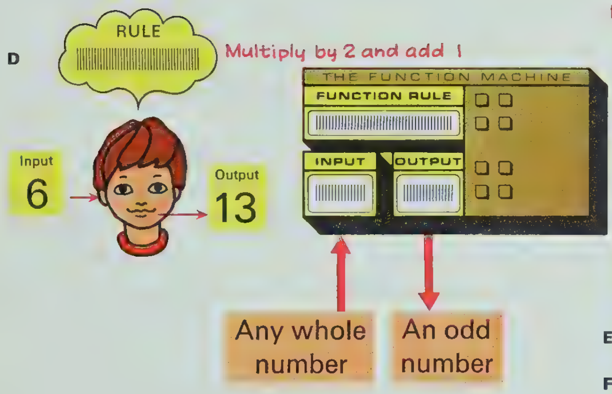


Multiply by 2

Function Rule

	Input	Output	
	1	2	
	2	4	
	3	6	
	4	8	
B	7		14
C	9		18

D



Multiply by 2 and add 1

Function Rule

	Input	Output	
	1	3	
	2	5	
	3	7	
	4	9	
E	7		15
F	9		19

2. Complete a table like this one for these numbers: 34, 35, 47, 48, 60, 61.
See Answers.

Number	Equation	Odd or Even
24	24 = **12** + **12**	Even
25	25 = **12** + **13**	Odd

Using the Exercises

If the children have understood the meaning of the term *even number,* they should realize that the function rule for exercise 1 is "Multiply by 2." Some may choose to think of it as adding the number to itself. However, lead the children to see that when the machine is used, the rule (× 2) is appropriate. Discuss the function rule for part D and how it relates to exercise 2, helping children formulate an expression for generating odd numbers, (× 2) + 1 (multiply by 2 and add 1). Children will benefit from trying to formulate such a rule by sharing ideas and testing rules.

As the faster children finish exercise 2, you might challenge them to continue the input-output table for other, more difficult numbers, such as 54, 82, 38.

Answers, exercise 2, page 215

Number	*Equation*	*Odd or Even*
34	34 = 17 + 17	Even
35	35 = 17 + 18	Odd
47	47 = 23 + 24	Odd
48	48 = 24 + 24	Even
60	60 = 30 + 30	Even
61	61 = 30 + 31	Odd

Assignments (page 215)

Minimum: 1, oral. Average: 1.
Maximum: 1–2.

Mathematics

The development of this lesson is derived from the definition of even and odd numbers. The even numbers are the set of all whole numbers x such that $x = 2 \times n$ where n is a whole number. The odd numbers are the set of all whole numbers x such that $x = (2 \times n) + 1$ where n is a whole number. Note that since $0 = 2 \times 0$, 0 is an even number.

Follow-up

The following activity should help children see the use of even and odd numbers in relation to the study of polygons developed in the previous chapter.

Have children examine various regular polygons. Consider the number of sides each polygon has. Is there a relation between the number of sides a polygon has and whether or not its opposite sides are parallel? What do you notice about the number of sides of any regular polygon which has opposite sides parallel? If you relate these questions to the even and odd numbers treated in this lesson, some children will notice that only regular polygons with an even number of sides have all opposite sides parallel. That is, no regular polygon with an odd number of sides has any pair of sides parallel.

Resources for Active Learning

Inquiry in Mathematics via the Geoboard, "Even Numbers," Geo-Card 7/1, 2; "Odd Numbers," Geo-Card 8/1,2, Walker. (Available from Fitzhenry & Whiteside)

Math Activity Cards, "Order," A43, Macmillan.

Mathex: Numeration No. 2, "Even and Odd Numbers," pp. 37–38, Encyclopaedia Britannica Publications Ltd. [The activities included here are appropriate for the next lesson, too.]

Math Workshop: Games and Enrichment Activities, "Squared Paper Patterns," pp. 29–30, Encyclopaedia Britannica Educational Corp.

Nuffield Project: *Computation and Structure* 3, "Odd and Even," p. 50, Wiley.

PAGES 216–217

Objectives

Given a base-ten numeral, the child will be able to identify the numeral as representing an even or an odd number by noting the last digit of the numeral.

Given sums and products of even and odd numbers, the child will be able to recognize patterns which indicate whether the sum or product will be even or odd.

Preparation

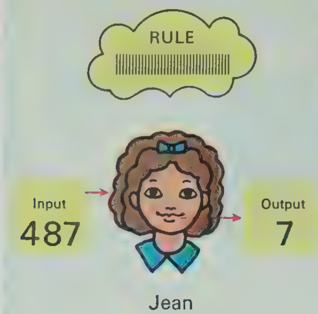
Use a short oral warm-up activity in which you name a number less than 100 and have the class respond "even" or "odd." Alternatively, you might say "even" or "odd" and have children respond with an appropriate number.

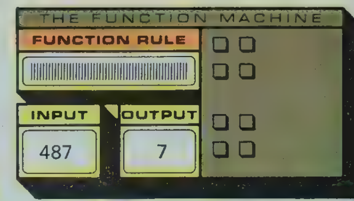
Investigation

The primary purpose of this investigation is to focus attention on the last digit in multi-digit numerals, since that is the digit with which children will be concerned as they look for the patterns suggested in the discussion exercises. Allow time for the children to try to discover Jean's function rule for themselves. You might list other input numbers for those who finish quickly.

● *What are some patterns of odd and even numbers?*

Investigating the Ideas





Function Rule

Input	Output	
19	9	
20	0	
76	6	
A 45	▯	5
B 960	▯	0
C 871	▯	1
D 408	▯	8

? Can you find Jean's function rule and use it to complete the table? See Investigation.

Discussing the Ideas

1. Can you tell whether a number is even or odd if you know only the last digit of the numeral? Yes

2. Study the examples. Then complete the exercises.
   Examples: 57 ends with 7. 57 is an odd number.
   86 ends with 6. 86 is an even number.
   - A 34 ends with ▯. Is 34 an even or an odd number? 4; even
   - B 43 ends with ▯. Is 43 an even or an odd number? 3; odd
   - C 30 ends with ▯. Is 30 an even or an odd number? 0; even
   - D 138 ends with ▯. Is 138 an even or an odd number? 8; even
   - E 469 ends with ▯. Is 469 an even or an odd number? 9; odd

3. Is the number for this 3-digit numeral odd or even? even

4. Answer "even" or "odd."
   - even A Each __?__ number ends with 0, 2, 4, 6, or 8.
   - odd B Each __?__ number ends with 1, 3, 5, 7, or 9.

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Discussion

Have children identify as even or odd the numbers in Jean's table. Help them see the relation between the input numbers and the output numbers: the output number is the ones' digit of the input number. Thus, when the input number is odd, so is the output number; and when the input number is even, so is the output number.

Next, work through the discussion exercises and help the children realize that an even number will always end in 0, 2, 4, 6, or 8 and an odd number in 1, 3, 5, 7, or 9. Have the children use several examples to illustrate the statements that they complete in exercise 4.

Using the Ideas

1. Find the sums or products. Then tell whether "even" or "odd" should go in the blank.

A	6	16	60	78	D	4	2	6	8
	+8	+28	+78	+54		×6	×6	×8	×2
	14	44	138	132		24	12	48	16

The sum of two even numbers is an __?__ number. even

The product of two even numbers is an __?__ number. even

B	5	15	37	65	E	7	3	1	7
	+7	+1	+45	+87		×3	×5	×7	×5
	12	16	82	152		21	15	7	35

The sum of two odd numbers is an __?__ number. even

The product of two odd numbers is an __?__ number. odd

C	6	47	38	57	F	6	2	3	6
	+7	+6	+11	+38		×3	×5	×8	×7
	13	53	49	95		18	10	24	42

The sum of an even and an odd number is an __?__ number. odd

The product of an even and an odd number is an __?__ number. even

2. Answer "even" or "odd."

A The sum of an even number and 1 is an __?__ number. odd
B The sum of an odd number and 1 is an __?__ number. even
C The product of an odd number and 1 is an __?__ number. odd
D No __?__ number is less than 1. odd
E Every __?__ number is greater than 0. odd
F There are two __?__ numbers less than 3. even
G There is only one __?__ number less than 3. odd
H The product of 0 and an odd number is an __?__ number. even
I The sum of an even number and 0 is an __?__ number. even
J The product of an even number and 0 is an __?__ number. even
K The sum of two odd numbers is an __?__ number. even
L The product of two odd numbers is an __?__ number. odd
M The sum of an even and an odd number is an __?__ number. odd
N The product of an even and an odd number is an __?__ number. even

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Using the Exercises

Ask the children to do the additions in exercise 1A and then to attempt to tell what word goes in the blank. Give them time to think about the missing word before someone says the answer. Send several "doubters" to the board to try other examples involving sums of two even numbers.

Continue this type of discussion for parts B and C. Then have the children complete parts D, E, and F by themselves.

Direct the children to take turns completing the statements in exercise 2, and discuss with them any questions which arise.

Assignments (page 217)*
Minimum: 1. Average: 1–2.
Maximum: 1–2.

Follow-up

A patterned worksheet using frames or placeholders for unknowns should give pupils an opportunity to gain some understanding of odd and even numbers. On a duplicating master, reproduce a page containing problems similar to the following, or write the problems on the chalkboard.

> Fill in the missing numbers to make true statements. Repeat the numeral when there are two missing numbers in an equation.
>
> $4 \times 2 = \triangle$
> $(4 \times 2) + \square = 9$
> $(2 \times \triangle) = 10 + 8$
> $(2 \times \square) + 1 = 10 + 9$
> $15 + \triangle = 2 \times 15$
> $15 + 16 = (2 \times \square) + 1$
> $\triangle + \triangle = 2 \times 7$
> $7 + 8 = (2 \times \square) + 1$

Resources for Active Learning

Developmental Math Cards, D11, Addison-Wesley. [An "odd-even" number activity]

Math Activity Cards, "Odd and Even," A44, B42, Macmillan.

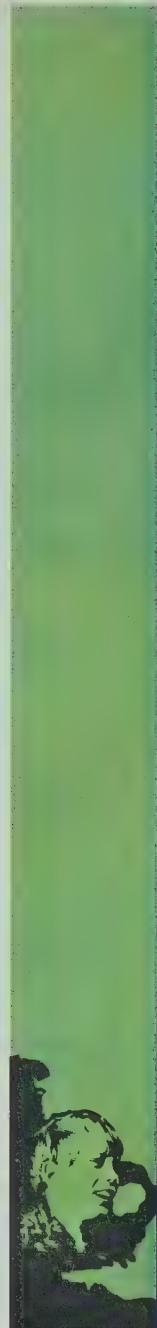
Workbook, page 79

Objective

Given a number from 2 to 10, the child will be able to list the multiples of the number.

Preparation

Unless you prefer to begin immediately with the discussion, you might briefly review basic multiplication facts by saying something like this: "I'm thinking of the product seven times six. What's my number?"



Can you list multiples of a number?

Discussing the Ideas

	×	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
A	2	0	2	4	6	8	10	12	14	16	18	20	22	24	26	
B	3	0	3	6	9	12	15	18	21	24	27	30	33	36	39	
C	4	0	4	8	12	16	20	24	28	32	36	40	44	48	52	

The numbers in row **A**, {0, 2, 4, 6, . . .}, are **multiples** of 2.

The numbers in row **B**, {0, 3, 6, 9, . . .}, are **multiples** of 3.

The numbers in row **C**, {0, 4, 8, 12, . . .}, are **multiples** of 4.

28, 30, 32, . . .

1. Give a multiple of 2 not shown in the table. How does the diagram above suggest another way to describe the even numbers? They are all multiples of 2.

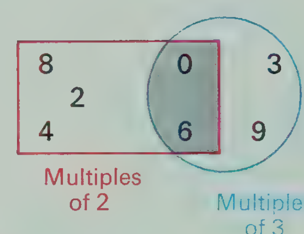
42, 45, 48, . . .

2. Give a multiple of 3 not shown in the table. Use the table to count by threes to 39. See table.

3. Find a number in the table (other than 0) that is a multiple of 3 and a multiple of 4. Find another such number. 12, 24 or 12, 36 or 24, 36

4. Why would 18 be in the list of multiples of 6? Because 18 = 3 × 6

★ 5. The rectangle contains **only** multiples of 2. The circle contains **only** multiples of 3. Can you explain how to find some more numbers for the shaded space? Find numbers that are multiples of both 2 and 3.



218

Discussion

Point out that the multiplication table at the top of the page shows only three rows: the 2 row, the 3 row, and the 4 row. The children should check several products to see that they are right.

Then observe that the numbers in row *A* are the multiples of 2, and that we get them when we multiply by 2. The numbers in row *B* are the multiples of 3, and we get them when we multiply by 3. The numbers in row *C* are the multiples of 4, and we get them when we multiply by 4.

Work through the discussion exercises with the children, referring to the table at the top. Note that in exercise 3 children should expand the lists of multiples of 3 and of 4 in order to find other numbers which appear in both lists. You might present other questions similar to exercise 4, such as, "Why would 28 be in the list of multiples of 7?" and expect a response such as, "Because 4 × 7 = 28."

Starred exercise 5 may be a challenge even for more capable children. You might choose to discuss it with a group of faster children if you do not think the whole class would benefit. Help them see that numbers which may be put in both the rectangle and the circle should be put in the shaded region, such as 0, 6, 12, etc.

Using the Ideas

1. List the multiples
   - A of 5 up to 50.
   - B of 6 up to 60.
   - C of 7 up to 70.
   - D of 8 up to 80.
   - E of 9 up to 90.
   - F of 10 up to 100.

   See Answers.

think

Of the numbers with 2 digits,
I'm the smallest with this fate.
I'm a multiple of six
And a multiple of eight.
WHO AM I? 24

2. Find the missing numbers.
   - A 8 is a multiple of 2.
     8 is also a multiple of ▒. 4
   - B 6 is a multiple of both ▒ and ▒. 2, 3
   - C Since $3 \times 4 = 12$, 12 is a multiple of both 3 and ▒. 4
   - D Since $5 \times 6 = 30$, 30 is a multiple of both ▒ and 6. 5
   - E Since $7 \times 8 = 56$, 56 is a multiple of both ▒ and ▒. 7, 8

3. A In the table on page 218, find a number (other than 0) that is a multiple of 2, 3, and 4.
   B Find another multiple of 2, 3, and 4. $\Big\}$ 12, 24
   C Find a number less than 50 that is a multiple of 2, 4, 5, 8, and 10. 40

★ 4. Write the numbers 1 through 100 in rows as shown. Circle the multiples of 5. What pattern do you see? Mark the multiples of another number. Is there a pattern? Mark multiples of other numbers to show as many patterns as you can. See Using the Exercises.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	. . .								
⋮									

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Using the Exercises

You may choose to work through one or two parts of exercise 1, to make sure children know how to form a list of multiples (by adding the number successively). Exercise 2 relates the concepts of multiple and factor, but the factors of a number will be developed in the following lesson. Just help the children see that, if $3 \times 4 = 12$, then 12 will appear as a multiple in the lists of both factors. Exercise 4 is starred, but the entire class would benefit from discussing it. The children should observe that all multiples of 5 end in 5 or 0. Suggest that they circle multiples of 8 to find a pattern. (Diagonal straight lines will connect them. This is also true for other multiples, most clearly for 9.)

Assignments (page 219)
Minimum: 1. Average: 1–3.
Maximum: 1–4.

Answers, exercise 1, page 219

A 0, 5, 10, 15, 20, 25, 30, 35, 40, 45, 50
B 0, 6, 12, 18, 24, 30, 36, 42, 48, 54, 60
C 0, 7, 14, 21, 28, 35, 42, 49, 56, 63, 70
D 0, 8, 16, 24, 32, 40, 48, 56, 64, 72, 80
E 0, 9, 18, 27, 36, 45, 54, 63, 72, 81, 90
F 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100

Follow-up/"Buzz"

An enjoyable activity to help children become aware of multiples is the "Buzz" game. Choose a set of multiples. Have the class begin counting by ones. Every time a multiple of the set comes up, the child whose turn it is should say "buzz." For example, if multiples of 4 are chosen, the counting will be:

1, 2, 3, buzz, 5, 6, 7, buzz, 9, 10, 11, buzz, ...

To make the game more exciting, choose two sets of multiples and use another word such as "clang" for the new set. If the multiples of 3 and 4 are chosen, the counting will be:

1, 2, clang, buzz, 5, clang, 7, buzz, clang, 10, 11, buzz-clang, 13, . . .

Resources for Active Learning

Mathex: Operations No. 3, "Hundreds Board and Multiples – Activity 1," and "Graphs – Activity 3," pp. 32–33, Encyclopaedia Britannica Publications Ltd.

Math Workshop: Games and Enrichment Activities, "Multiplication and Division Tables," pp. 74–79, Encyclopaedia Britannica Educational Corp.

Nuffield Project: *Computation and Structure* 3, "Dominoes," p. 34; "Tables," pp. 30–38, Wiley.

PAGES 220–221

Objective

Given a number less than 50 that is a multiple of 2, 3, 4, or 5, the child will be able to find the factors of the number.

Preparation

You might use a short warm-up activity such as the "Buzz" game suggested on page 219 or "What's My Rule" for reviewing multiplication facts.

Investigation

Read the directions with the children. Then allow time for them to write equations for the missing factors. If you wish to extend this activity, direct the children to follow the same procedure for equations you write on the chalkboard, such as:

$\_\_\_\_ \times \_\_\_\_ = 24$

$\_\_\_\_ \times \_\_\_\_ = 36$

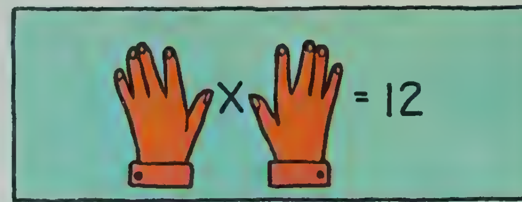
$\_\_\_\_ \times \_\_\_\_ = 30$

$\_\_\_\_ \times \_\_\_\_ = 16$

As children write out the equations, some may need to refer to a multiplication table. It would be helpful to have at least the more difficult part of the multiplication table displayed for any who need it. If the facts are displayed, children's ability to work with multiples and factors will not be hindered by a lack of mastery of the basic multiplication facts.

Can you find the factors of a number?

Investigating the Ideas



? Can you record all the different equations that might be hidden? 1×12=12; 2×6=12; 3×4=12; 12×1=12; 6×2=12; 4×3=12

Discussing the Ideas

1. Study this diagram.

   $\boxed{3} \times \boxed{4} = 12$ Since the product of 4 and 3 is 12, we say **4 is a factor of 12** and **3 is a factor of 12.**

   What are some other factors of 12? 1,2,6,12

2. Kevin is covering up two factors of 20.

   A Can you be certain about what these factors are? No

   B Give a pair of numbers that Kevin might be hiding. 1,20; 2,10; 4,5

   C Write three equations, using different factors that might be on the board. 1×20=20; 2×10=20; 4×5=20

   D List six numbers that are factors of 20. 1,2,4,5,10,20

   E Is 7 a factor of 20? Why? No, because there is no whole number that can be used as a factor with 7 to give the product 20.

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Discussion

After children have had time to compare and discuss the equations they completed for the investigation, direct their attention to the discussion section.

Exercise 1 reviews the meaning of the term factor. To answer the questions in it, have children write the different equations for 12 which they found in the investigation. Accept equations such as

$2 \times 6 = 12$ and $6 \times 2 = 12$

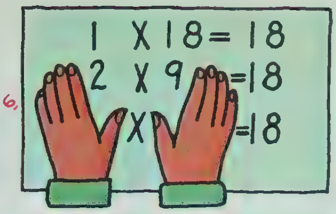
but point out that both equations contain the same factors. Help the children develop this list of factors from the equations: 1, 2, 3, 4, 6, 12.

Continue with exercise 2 similarly. Note in part C that only one of equation pairs such as $2 \times 10 = 20$ and $10 \times 2 = 20$ should be used.

According to the ability and needs of your class, continue a similar development with other numbers such as 24, 36, 30, and 16. For each number, have the children list all the factors.

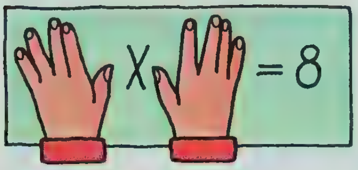
Using the Ideas

1. A Write a third equation to show other factors of 18. 3×6=18
   B List six different factors of 18. 1,2,3,6, 9,18
   C Is 4 a factor of 18? No



2. For each exercise, write as many equations as you need to show all the factors of the product.

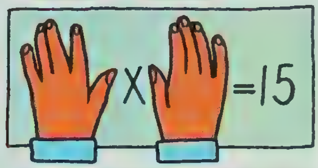
A



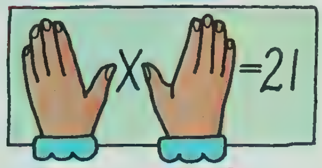
1×8=8 ; 2×4=8

B

1×15=15
3×5=15



C



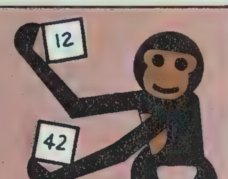
1×21=21 ; 3×7=21

3. List all the factors of each number. Use exercise 2 if you need help.
   A 8 1,2,4,8   B 15 1,3,5,15   C 21 1,3,7,21

4. A Which of the numbers 3, 4, 5 is a factor of 16? 4
   B Which equation can you solve? 2
   $\boxed{1}$ $16 \div 3 = n$
   $\boxed{2}$ $16 \div 4 = n$
   $\boxed{3}$ $16 \div 5 = n$

★ 5. Which of the numbers 2, 3, 4, 5, 6 are factors of:
   A 15? (Answer: 3 and 5)
   2,5 B 10?   F 30? 2,3,5,6
   2,4,5 C 20?   G 31? None
   2,3,4,6 D 24?   H 32? 2,4
   2,4 E 28?   I 60? 2,3,4,5,6

think



I'm proud to be a factor of
Both twelve and forty-two.
No larger number makes this claim.
You need no other clue.

WHO AM I? 6

221

Follow-up

As an additional activity, ask more capable children to try to find three factors whose product is a given number. List numbers such as 24, 36, 40, 48, 54, and 60 on the chalkboard, and instruct the children to find as many different sets of three factors for each of the given numbers as they can. Avoid using 1 as a factor. For example:

$$40 = 2 \times 4 \times 5 \qquad 24 = 2 \times 3 \times 4$$
$$= 2 \times 2 \times 10 \qquad = 2 \times 2 \times 6$$
$$48 = 2 \times 3 \times 8 \qquad 60 = 2 \times 5 \times 6$$
$$= 2 \times 4 \times 6 \qquad = 4 \times 5 \times 3$$
$$= 2 \times 2 \times 12 \qquad = 2 \times 2 \times 15$$

Resources for Active Learning

Developmental Math Cards, G#12, Addison-Wesley. [A factoring game]

Discovery, Section I, Activity 20, p. 23, Encyclopaedia Britannica Educational Corp. [A "composite number" problem]

Mathex: Operations No. 3, "Patterns—Activity 2," "Array Building—Activity 4," "Building Blocks—Activity 5," "Factor Decks—Game 2," pp. 32–35, Encyclopaedia Britannica Publications Ltd.

Math Workshop: Games and Enrichment Activities, "Factor Sets," pp. 64–65, Encyclopaedia Britannica Educational Corp.

Nuffield Project: *Computation and Structure* 3, "Dominoes," p. 34; "Factors and Primes," pp. 67–69, Wiley.

Workbook, page 80

Using the Exercises

You may assign exercises 1 and 2 as independent work, or you may work through them with the class. In exercise 4, the children should see that the answer is dependent on the multiplication fact, $4 \times 4 = 16$. Exercise 5 and the *Think* problem are intended as enrichment material, but the whole class would benefit from a discussion of the solutions.

Assignments (page 221)

Minimum: 1–2. Average: 1–4.
Maximum: 1–5.

PAGES 222–223

Objective

Given a number which has only 2 factors (itself and 1), the child will be able to identify it as a prime number.

Preparation

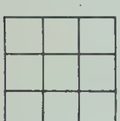
Materials

graph paper; set of squares (centimetre squares or squares cut from graph paper; 20 per child)

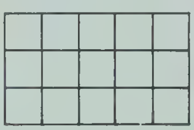
Unless you wish to use a short oral game such as the "Buzz" game, begin immediately with the investigation.

Investigation

In this investigation, children study the fact that any number with more than 2 factors can be represented by a rectangular array of squares or dots. The numbers which cannot be represented in this manner (prime numbers) have only 2 factors (1 and the number itself). Remind the children to record their findings. You might suggest to a few very capable children that they write a multiplication equation for each rectangle they make.



$3 \times 3 = 9$



$3 \times 5 = 15$

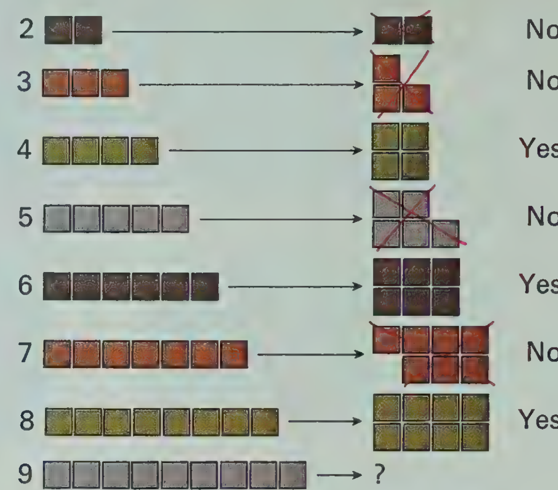
As the children work on constructing the rectangles, ask questions such as:

"Have you found an even number which you cannot represent as a rectangle?" *(No)*

"Are there some numbers for which you could make 2 rectangles?" *(12, 16, 18, 20)*

Investigating the Ideas

The figures show which numbers from 2 through 8 can be shown as rectangles using sets of squares. No single strings of squares are allowed!



?

Can you use sets of squares to find which numbers from 9 through 20 will form rectangles?

Record your findings on graph paper.

9, 10, 12, 14, 15, 16, 18, 20 (See Investigation.)

Discussing the Ideas

1. The numbers greater than 1 that do not make "rectangles" are called **prime numbers**.
   - A Which numbers from 2 through 8 are prime numbers? 2, 3, 5, 7
   - B What prime numbers did you find from 9 through 20? 11, 13, 17, 19
2. Which numbers between 20 and 30 do you think are prime numbers? 23, 29
3. The number 39 is the product of 3 and 13. How could you use this fact to convince someone that 39 is not a prime number?
   39 will make the "rectangle" 3×13; 39 has more than two factors.

222

Discussion

Direct the children to write the numbers which can be made into rectangles in one list and those which cannot in another list. Use these numbers as you discuss exercise 1.

Prime: 2, 3, 5, 7, 11, 13, 17, 19, 23
Not prime: 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24

For several numbers, show the rectangle of squares on the flannelboard and write corresponding multiplication equations pointing out the factors. For example, on the flannelboard and chalkboard show the following rectangle of squares and the corresponding equation.

F F
$3 \times 5 = 15$

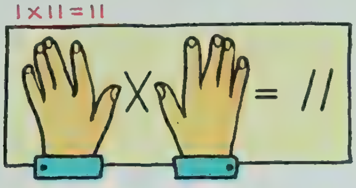
Then say, "We know that one more equation for 15 may be written, $1 \times 15 = 15$. This gives us four factors of 15 – 1, 3, 5, 15 – so 15 is not a prime number."

After you work through other examples for exercise 2, children should be able to see in exercise 3 that 39 may be arranged in 13 rows of 3 (or 3 rows of 13). This is another way of saying that 39 has the factors 1, 3, 13, and 39 and is not a prime number.

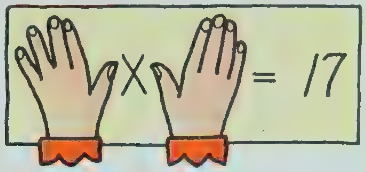
Using the Ideas

1. The product in each exercise is a prime number. Write as many equations as you need to show all the factors of the product.

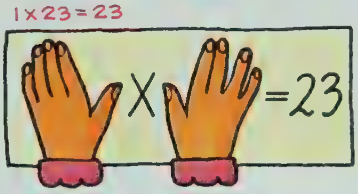
A 1×11=11



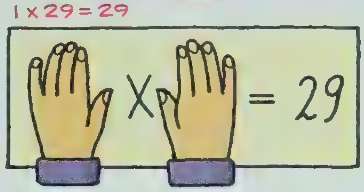
B 1×17=17



C 1×23=23



D 1×29=29



2. A How many factors did each prime number in exercise 1 have? 2
   B Can you find a prime number with more than two factors? No

3. Write an equation for each exercise.
   Example: I know 20 is **not prime** because $4 \times 5 = 20$.
   A I know 32 is **not prime** because ▯ × ▯ = 32. 2,16 4,8
   B I know 33 is **not prime** because ▯ × ▯ = 33. 3,11

4. List the prime numbers between 40 and 50.
   41, 43, 47

think

Now I'm a strange prime number.
No others are like me.
All my cousins are so odd,
As odd, as odd can be.

WHO AM I? 2

More practice, page A-27, Set 37

Using the Exercises

Use exercises 1 and 2 as a basis for futher discussion. Help the children realize that in each case the only equation is that in which 1 and the number itself are the factors. This idea can be presented by discussing the possibility of arranging 11, 17, 23, or 29 squares in rectangular form. If necessary, have children try to form rectangles using squares on the flannelboard. Emphasize in exercise 2 that numbers which can be expressed with only one multiplication equation (or in other words, which have only two factors, one and the number itself) are *prime numbers*.

You may expand exercise 3 by writing similar equations on the chalkboard.

12 is not a prime,
because _____ × _____ = 12.
40 is not a prime,
because _____ × _____ = 40.

In exercise 4, help children realize that no even number greater than 2 is prime; for example $2 \times 22 = 44$, $2 \times 23 = 46$. The *Think* problem is enrichment, but it points out that 2 is the only *even* prime number.

Assignments (page 223)
Minimum: 1–4, oral. Average: 1–4.
Maximum: 1–4.

Mathematics

A whole number is a prime number if it has exactly two different factors (one and the number itself). Thus:

- 2 is a prime number (factors: 2, 1).
- 3 is a prime number (factors: 3, 1).
- 5 is a prime number (factors: 5, 1).
- 7 is a prime number (factors: 7, 1).
- 11 is a prime number (factors: 11, 1).

The set of prime numbers is an endless or infinite set.

Note:

- 1 is not prime (has only one factor: 1).
- 6 is not prime (factors: 1, 2, 3, 6).
- 91 is not prime (factors: 1, 7, 13, 91).

Numbers larger than 1 which are not prime are called *composite* numbers.

Resources for Active Learning

Discovery, Section I, Activity 7, p. 9; Activities 17, 18, pp. 20–21, Encyclopaedia Britannica Educational Corp. [All about "prime and composite numbers"]

Math Activity Cards, "Factors and Primes," B43, Macmillan.

Mathex: Numeration No. 7, "Prime Numbers," pp. 6–9, Encyclopaedia Britannica Publications Ltd.

Math Workshop: Games and Enrichment Activities, "Prime Numbers," pp. 21–22; "Products, Factors . . .," pp. 65–69, Encyclopaedia Britannica Educational Corp.

Workbook, page 81

PAGES 224–225

Objectives

The child will demonstrate his ability to work with the concepts presented in this chapter.

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

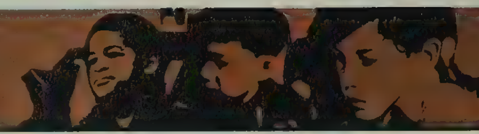
If you use each of these pages as a separate lesson, plan an oral warm-up activity for each. For example, to prepare for page 224 you might pattern your activity as: "I'm thinking of a number which has 1, 2, 4, and 8 as its factors. What's my number, and is it prime or not?" Capable children will see that the last factor is the number, so the challenge is really to determine whether or not the number is prime.

For page 225 you might review basic facts with mental chain games such as the following:

"Start with 7 . . . Add 8 . . . Subtract 9 . . . What's my number?" (6)

"Start with 14 . . . Subtract 4 . . . Add 2 . . . Subtract 7 . . . What's my number?" (5)

Reviewing the Ideas



1. Which numbers are even and which are odd?

A 6 even B 9 odd C 5 odd D 12 even E 32 even F 31 odd G 146 even H 283 odd

2. Find the missing numbers.

- A 14 is a multiple of 2 because $2 \times 7 = n$. 14
- B 12 is a multiple of 3 because $n \times 4 = 12$. 3
- C Since $4 \times 6 = 24$, we call ▒ a multiple of 4. 24
- D Since $n \times 12 = 36$, we call 36 a multiple of 3. 3
- E Since $5 \times 6 = 30$, we know that ▒ is a multiple of 5. 30
- F 45 is a multiple of 5 since $5 \times 9 = n$. 45

3. Find the missing numbers.

- A ▒ and ▒ are factors of 18 because $3 \times 6 = 18$. 3, 6
- B 2 and 9 are factors of ▒ because $2 \times n = 18$. 18, 9
- C Since $4 \times 5 = 20$, ▒ and ▒ are factors of 20. 4, 5
- D Since $2 \times 3 \times 4 = 24$, 2, 3, and 4 are factors of ▒. 24
- E Since $21 \div 3 = 7$, ▒ and ▒ are factors of 21. 3, 7

4. List the factors of the following numbers.

A 6 1,2,3,6 B 5 1,5 C 9 1,3,9 D 10 1,2,5,10 E 11 1,11 F 12 1,2,3,4,6,12 G 20 1,2,4,5,10,20 H 29 1,29

★ **5.** Give the digits.

- A If a number is a multiple of 2, then it ends in one of the digits ▒, ▒, ▒, ▒, or ▒. 0, 2, 4, 6, 8
- B If a number is a multiple of 5, then it ends in one of the digits ▒ or ▒. 0, 5
- C The multiples of 3 may end in any one of the digits 0 to 9. Show this by listing the first ten multiples of 3. 0,3,6,9,12,15,18,21,24,27
- D What other numbers between 1 and 10 have multiples that may end in any one of the digits 0 to 9? 7, 9

224

Discussion

You may prefer to assign page 224 as independent work and discuss it while you check the children's work, or you may choose to work through the exercises together. Direct the children to write only the answers. Although exercise 5 is intended for the more capable children, you might invite all to study the exercise before you give the answers. You might have to list the multiples of 4, 6, 7, 8, 9 to convince children that multiples of 4, 6, 8 will always end in an even digit and that 7 and 9 are the numbers which have multiples that end in any one of the digits 0 to 9.

Keeping in Touch with Addition, Subtraction, Measurement, Place value, Inequalities

1. Solve the equations.
   - A $84 = 80 + n$ 4
   - B $384 = 300 + 80 + n$ 4
   - C $58 = n + 8$ 50
   - D $458 = n + 50 + 8$ 400
   - E $763 = 700 + n + 3$ 60

2. Find the sums and differences.

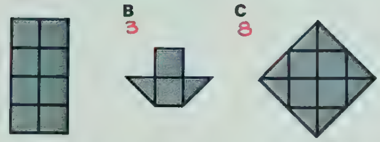
A $24 + 35$ = 59	B $78 - 26$ = 52	C $42 + 30$ = 72	D $79 - 20$ = 59	E $65 + 18$ = 83	F $94 - 56$ = 38
G $38 + 27$ = 65	H $54 - 19$ = 35	I $76 + 88$ = 164	J $134 - 75$ = 59	K $84 + 16$ = 100	L $100 - 23$ = 77

3. Solve the equations.
   - A $7 + n = 15$ 8
   - B $16 - 9 = n$ 7
   - C $13 - 7 = n$ 6
   - D $n + 5 = 12$ 7
   - E $10 - 4 = n$ 6
   - F $6 + n = 14$ 8

4. Give the sign $>$, $<$, or $=$ for each ●.
   - A 482 ● 472 $>$
   - B 6286 ● 6296 $<$
   - C $50 + 8$ ● $50 - 8$ $>$
   - D $70 - 2$ ● $70 - 3$ $>$
   - E $70 + 4$ ● $70 + 5$ $<$
   - F $80 + 0$ ● $80 - 0$ $=$

5. Find the area of each region.

A 8 B 3 C 8



6. Find the volume of each region.

A 4 B 4 C 10

You are invited to explore

ACTIVITY CARD 11
Page 314

Follow-up

Below are sample worksheets you might use to reinforce the concepts developed in this chapter.

Complete the equations. Check *Yes* if the product is a prime number. Check *No* if the product is not a prime number.

	Yes	*No*
$\_\_ \times 5 = 35$	___	___
$\_\_ \times 11 = 11$	___	___
$\_\_ \times 9 = 36$	___	___
$\_\_ \times 1 = 43$	___	___
$\_\_ \times 8 = 64$	___	___
$\_\_ \times 13 = 13$	___	___
$3 \times \_\_ = 24$	___	___
$19 \times \_\_ = 19$	___	___
$5 \times \_\_ = 25$	___	___
$7 \times \_\_ = 42$	___	___

In each row below are portions of series of multiples. In the column at the right are numbers which are factors of the multiples in each row. Match each row with the number which is a factor of each multiple in that row. The first one is done for you.

14, 16, 18, 20, 22	3
35, 42, 49, 56, 63	4
24, 28, 32, 36, 40	2
20, 25, 30, 35, 40	6
16, 24, 32, 40, 48	7
21, 24, 27, 30, 33	8
12, 18, 24, 30, 36	5

Workbook, page 82

Using the Exercises

You will probably want to assign the exercises on page 225 as independent work. Remind the children that these exercises are samples of the many kinds of problems they have learned to solve so far in Book 3. You might want to precede the assignment with a review of expanded-value notation. If so, dictate 2- to 5-digit numerals, and ask the children to write them in place-value notation. For example, they should write 2786 as $2000 + 700 + 80 + 6$. Also review inequalities with these numbers. To expand the review of area, give children sheets of graph paper and ask them to design regions which have areas of 5, 8, and 9 units.

If you use these pages to help evaluate the children's achievement thus far, assign worksheets or activities to help reinforce concepts or skills. Keeping a file of extra worksheets such as these will often prove useful after a review has determined the areas of need.

Pages 226–255

General Objectives

To introduce the multiplication algorithm
To provide appropriate practice with word problems
To maintain understanding of multiplication concepts

The first objective of this chapter is to ensure that the children understand the multiplication algorithm. To accomplish this, it is necessary for them to develop an understanding of multiplication by 10, by 100, and by multiples of 10. In addition, they will need to use the distributive principle and become familiar with estimation.

Consider the following problem and the steps that are needed to arrive at the multiplication algorithm using 1-digit multipliers.

$$\begin{array}{r} 463 \\ \underline{\times 5} \end{array}$$

Step 1: $5 \times 3 = 15$ (The children should be able to do this step.)

Step 2a: $5 \times 60 = ?$ (This step must be developed.)
$5 \times 60 = 5 \times (6 \times 10)$
$= (5 \times 6) \times 10 = 30 \times 10$

Step 2b: $30 \times 10 = ?$ (This step must be developed.)
$30 \times 10 = (3 \times 10) \times 10$
$= 3 \times (10 \times 10) = 3 \times 100$
$= 300$

Step 3: $5 \times 400 = ?$ (This step must be developed as in step 2.)

Step 4: $15 + 300 + 2000 = 2315$ (The children should be able to compute this sum.)

Steps 2 and 3, based on the grouping principle, must be broken down so that the children can multiply factors which are multiples of ten. These steps should then be combined with the distributive principle to find products such as 6×432. Much of the early part of the chapter emphasizes the background necessary to present short-cuts.

Toward the end of the chapter, the children are given practice in using the multiplication algorithm and in solving word problems that require multiplication.

Mathematics

Using the distributive principle to find products such as 4×23 is the main mathematical concept presented in this chapter. The following example illustrates this procedure.

$$\begin{aligned} 4 \times 23 &= 4 \times (20 + 3) \\ &= (4 \times 20) + (4 \times 3) \\ &= 80 + 12 \\ &= 92 \end{aligned}$$

This illustrates why we chose the descriptive words *break apart* when we described the distributive principle in the last chapter. The children should recognize that when they multiply two numbers such as 4×23, they should break the 23 apart to get 20 and 3, and multiply 4×20 and 4×3. Then they should add the two products to get the final answer.

Teaching the Chapter

Materials

Crayons
Felt squares, 30 by 30 centimetres, marked into 100 units
Felt strips, 3 by 30 centimetres, marked into 10 units
Felt units, 3 by 3 centimetres
Flannelboard
Graph paper
Overhead projector
Scissors

Vocabulary

calendar
estimate
horizontal notation
map
multiplication-addition (distributive) principle
vertical notation

Since the main ideas of this chapter rely on concepts and principles developed earlier, you will probably use set materials very sparingly for most classes and perhaps eliminate them altogether for some classes.

Lesson Schedule

Plan to spend four to four-and-a-half weeks on this chapter. Vary the schedule according to the interests and abilities of your class. Allow time at the beginning of the chapter for the children to review multiplication facts and to work toward mastery and efficiency in using them.

Evaluation of Progress

Evaluating the children's work in a chapter that has the development of an algorithm as its chief objective is, for the most part, simple. You should have no problem determining whether or not the children have mastered the algorithm. However, the fact that children can *use* the algorithm successfully does not guarantee that they *understand* it. As we have recommended before, your evaluation of the children's understanding of multiplication concepts should be based on day-to-day observation.

Deficiencies in accurate recall of the multiplication facts can hinder understanding of the algorithm. By this time, children should not have to worry about multiplication facts; they should be concentrating on the more important purpose of co-ordinating familiar ideas to attain a new goal. Some children may need to refer to a multiplication table in order to concentrate on the algorithm. Be sure to provide games and review activ-

ities as you develop multiplication for multiples of 10.

Problems that review this chapter may be found on pages 252 and 253. Use these pages to help detect current weaknesses or to evaluate progress. Or, use the review as a guide in designing your own chapter test.

Concepts and skills presented earlier are reviewed on pages 254 and 255.

Resources for Active Learning

General Activities

[Refer to the introductory section of Chapter 6 for games and activities that may be used to review basic multiplication facts and principles. Do not hesitate to reuse the favorites. Some others are listed below.]

Developmental Math Cards, $F^{\underline{1}}19$, $F^{\underline{4}}13$, $G^{\underline{4}}12$, $G^{\underline{1}}7$, $G^{\underline{4}}2$, Addison-Wesley

Franklin Series: *From Fingers to Computers*, "Finger Computation," pp. 5–17, Lyons and Carnahan. A "handy" method for computation. (Available from McGraw-Hill Ryerson)

Franklin Series: *Patterns and Puzzles*, "Operations," p. 93, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Mathex: Operations and Problem Solving No. 8, "Finger Multiplication," pp. 18–19; "Properties of Operations," pp. 25–26, Encyclopaedia Britannica Publications Ltd.

Nuffield Project: *Computation and Structure* 3, "Multiplication," pp. 24–38; "Distributive Property," pp. 39–47; "Money, Weights, . . . ," pp. 56–60, Wiley

Manipulative Devices

Dienes Multibase Arithmetic Blocks (Herder and Herder)

SEE Calculator (Selective Educational Equipment)

Commercial Games

Quinto (Hammett; Selective Educational Equipment)

TUF (Creative Publications; Cuisenaire Co.; TUF)

PAGES 226–227

Objective

Given a pair of factors to multiply, one of which is 10 or 100 and the other a single-digit numeral, the child will be able to find the product by annexing the appropriate number of zeros.

Preparation

To prepare for this lesson, you might ask the children to name some numbers which they think are particularly useful numbers. If no one mentions the numbers 10 or 100, bring them up yourself and explain that after this lesson they will probably add 10 and 100 to their list of useful or interesting numbers, especially in multiplication.

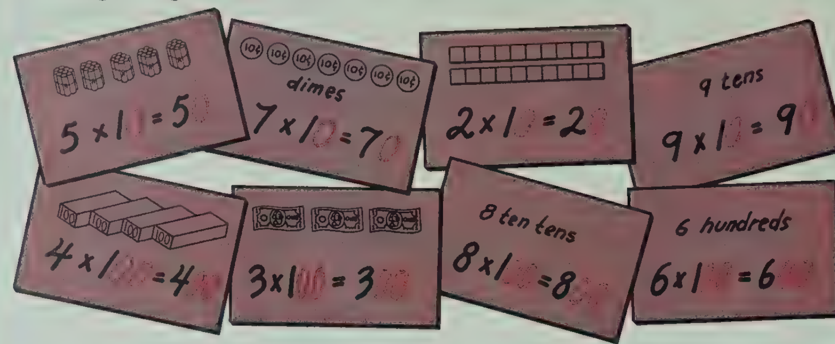
Investigation

Direct the children to study the equations illustrated at the top of page 226 by themselves. If any children do not feel "comfortable" about the equations, remind them that 5×10 means 5 tens or $10 + 10 + 10 + 10 + 10$. Have them perform the addition or use skip counting to arrive at the product. If necessary, point out the red zeros to help the slower children discover the rule by focussing attention on the zeros. However, a formal statement of the rule need not be developed until the discussion period.

10 Multiplying

● *Is there an easy rule for multiplying by 10 and 100?*

Investigating the Ideas



? Can you give a rule for multiplying by 10? by 100?
See Investigation.

Discussing the Ideas

1. A How many sets of 10 are in figure A? 6
   B Write the numeral for 6 tens. 60
   C Solve: $6 \times 10 = n$ 60

A

2. A How many sets of 100 are in figure B? 3
   B Write the numeral for 3 hundreds. 300
   C Solve: $3 \times 100 = n$ 300

B

3. A Explain your rule for multiplying a 1-digit number by 10.
   B Explain your rule for multiplying a 1-digit number by 100.
   See Discussion.

Discussion

Encourage the children to verbalize their discovery and let them state it in their own way. As you discuss the equations in the investigation, refer to the illustration with the cards showing 5 bundles of ten or 7 dimes, etc. Then develop the discussion exercises. If you feel it is necessary, provide set demonstrations using a given number of tens and hundreds. For example, use the felt 10-strips to show 6 tens, and on the chalkboard write $6 \times 10 = 60$; or use the hundred-squares to show 6 hundreds, and write $6 \times 100 = 600$. Not all children will need such a set demonstration. Assess the needs of your class; you might want simply to give them more examples of multiplying single-digit numerals by 10 or by 100. Help the children summarize the lesson in exercise 3: (A) To multiply a 1-digit number by 10, you write a zero on the right side of the number. To multiply a 1-digit number by 100, you write two zeros on the right side of the number.

Using the Ideas

1. A How many tens? 8
   B Write the numeral for this number of tens. 80
   C Solve: $8 \times 10 = n$ 80

2. A How many hundreds? 2
   B Write the numeral for this number of hundreds. 200
   C Solve: $2 \times 100 = n$ 200

3. Find the products.

A 5×10 50	E 8×100 800	I 9×100 900	M 3×100 300
B 7×100 700	F 9×10 90	J 7×10 70	N 6×10 60
C 2×10 20	G 4×10 40	K 8×10 80	O 1×100 100
D 4×100 400	H 5×100 500	L 2×100 200	P 6×100 600

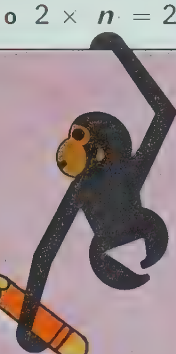
4. Solve the equations.

A $6 \times 10 = n$ 60	F $n \times 10 = 50$ 5	K $4 \times n = 40$ 10
B $3 \times 10 = n$ 30	G $n \times 100 = 300$ 3	L $6 \times n = 600$ 100
C $4 \times 100 = n$ 400	H $n \times 10 = 40$ 4	M $7 \times n = 70$ 10
D $7 \times 10 = n$ 70	I $n \times 100 = 600$ 6	N $3 \times n = 300$ 100
E $9 \times 100 = n$ 900	J $n \times 100 = 800$ 8	O $2 \times n = 200$ 100

think

Study the pattern.
Then copy the equations, giving the missing numbers.

$1 \times 9 = 10 - 1$
$2 \times 9 = 20 - 2$
$3 \times 9 = 30 - 3$
$4 \times 9 = 40 - 4$
50, 5 $5 \times 9 = \square - \square$
80, 8 $8 \times 9 = \square - \square$
130, 13 $13 \times 9 = \square - \square$



Using the Exercises

Assign these exercises as independent work. Check the papers carefully, for knowing how to multiply by 10 and 100 is essential if a child is to learn the multiplication algorithm. You may challenge capable children to try to find the following pattern, which is similar to the one in the *Think* problem.

$1 \times 8 = 10 - 2$
$2 \times 8 = 20 - 4$
$3 \times 8 = 30 - 6$
$4 \times 8 = 40 - 8$
$7 \times 8 = 70 - 14$
$9 \times 8 = 90 - 18$

Assignments (page 227)
Minimum: 1–3. Average: 1–4.
Maximum: 1–4.

Follow-up

Since this chapter develops the multiplication algorithm, the children must have the basic multiplication facts accessible. This would be a good time to decide with the class what facts they know for quick recall and what facts they need to have displayed. You might give a short quiz to help you, and them, decide which facts to display. Then encourage some of the children to help you prepare a bulletin board to display the troublesome facts.

The following tables might be used for a timed test.

×	3	2	8	5	4
6					
7					
5					
9					

×	7	4	3	2	6
6					
9					
8					
5					

×	6	9	3	2	4
4					
3					
5					
8					

Resources for Active Learning

See the introductory section of this chapter for ideas for games to review basic facts, especially multiplication.

PAGES 228–229

Objective

Given a pair of factors one of which is 10 or 100 and the other a 2-digit numeral, the child will be able to find the product by annexing the appropriate number of zeros.

Preparation

Materials

colored strips

To prepare for this lesson, review the interpretation of 2- and 3-digit numbers as groups of ten. For example, ask: "How many tens in eighty? in one hundred ten? in one hundred fifty? in two hundred forty?" Children should respond, "Eight tens, eleven tens," and so on.

Investigation

It would be appropriate for children to work on this investigation in small groups. Encourage them to discuss among themselves the answers to the first two questions. As they do, move around the room to make sure that they correctly interpret the train as 13 when the 1-strip is the unit and as 130 when the 1-strip is thought of as 10.

As the children work into the challenge question, keep the amount of help you give minimal. If they ask for guidance, encourage them first to talk it over with each other. However, if a child seems confused after this, give hints to help him set up a train of strips for 24 and then discuss what this would represent if each single unit was thought of as 10.

For those who finish the challenge question quickly, suggest that they also show strips for 150 or 220 or 200.

● *Does the "10 rule" work for products like 23 × 10?*

Investigating the Ideas

If your white strip (□) is 1, how long is this train? 13

If your white strip were 10, how long would this train be? 130

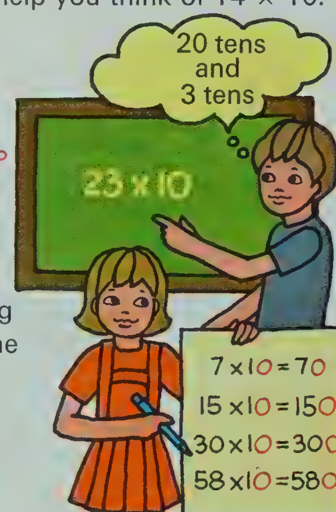
? Can you show 240 with your strips if you think of your white strip as 10?

See Investigation and Discussion.

Discussing the Ideas

1. Explain how you can use strips to help you think of 14 × 10. See Discussion.

2. A Explain how Jack is thinking about 23 × 10. See Discussion.
   B Write the numeral for 20 tens. 200
   C Write the numeral for 3 tens. 30
   D Solve: 23 × 10 230



3. Jill knows a shortcut for multiplying by ten. She made a chart to help the other children discover her rule.
   A What is Jill's rule for multiplying by 10?
   B What is a simple rule for multiplying by 100?
   See Discussion.

228

Discussion

Have several children explain how they showed 240 with their strips. Although there are a variety of ways to show 24 (2 orange strips and 1 purple, 3 blue strips and 1 dark green, 4 yellow strips and 1 purple), the important thing to stress is that any of these combinations may be thought of as representing 240 if we think of the unit strip as 10. This is expressed as 24 × 10 = 240. Use this discussion and exercise 1 to help children see the relation between 24, 24 × 10, and 240; and between 14, 14 × 10, and 140. Point out that the product of any number and 10 is simply that number with a zero annexed to it.

As you discuss exercise 2, show the children how use of the multiplication-addition (distributive) principle can help them solve problems like 23 × 10. They can think of 23 as 20 + 3 and then multiply in parts.

$$\begin{aligned} 23 \times 10 &= (20 + 3) \times 10 \\ &= (20 \times 10) + (3 \times 10) \\ &= 200 + 30 \\ &= 230 \end{aligned}$$

When you have used other examples and are sure the children can follow these steps, discuss exercise 3 and the rule the children discussed in exercise 1: to multiply by 10, annex one zero. Do not

Using the Ideas

1. Find the value in cents for each set of dimes.

A 140¢


B 230¢


2. Give the value in cents for each coin collection.

A 5 dimes 50¢	C 20 dimes 200¢	E 27 dimes 270¢	G 52 dimes 520¢	I 56 dimes 560¢
B 10 dimes 100¢	D 15 dimes 150¢	F 40 dimes 400¢	H 91 dimes 910¢	J 48 dimes 480¢

3. Solve the equations.

A $(40 \times 10) + (3 \times 10) = n$ 430
$43 \times 10 = n$ 430

B $(70 \times 10) + (2 \times 10) = n$ 720
$72 \times 10 = n$ 720

C $(30 \times 10) + (1 \times 10) = n$ 310
$31 \times 10 = n$ 310

D $(60 \times 10) + (5 \times 10) = n$ 650
$65 \times 10 = n$ 650

E $(20 \times 10) + (9 \times 10) = n$ 290
$29 \times 10 = n$ 290

F $(50 \times 10) + (6 \times 10) = n$ 560
$56 \times 10 = n$ 560

4. Find the products.

A $10 \times 10 = n$ 100
B $27 \times 10 = n$ 270
C $10 \times 43 = n$ 430
D $50 \times 10 = n$ 500
E $10 \times 19 = n$ 190
F $96 \times 10 = n$ 960
G $10 \times 73 = n$ 730
H $10 \times 65 = n$ 650
I $34 \times 10 = n$ 340
J $80 \times 10 = n$ 800
K $8 \times 100 = n$ 800
L $52 \times 100 = n$ 5200

think

Now I am a number
You rarely can beat.
When I am a factor
I surely am neat.
Use the other factor.
Make zero the tail.
You'll see the product.
You really can't fail.

WHO AM I? 10

More practice, page A-28, Set 38

expect the children to verbalize this rule in precisely these terms; let them express the shortcut in their own words. Then display a variety of equations which follow the pattern of $7 \times 100 = 700$ and $18 \times 100 = 1800$, until the children are able to generalize the shortcut rule for multiplying by 100.

Using the Exercises

The first exercise relates multiplying by 10 to working with dimes. The children find that 5 dimes correspond to 5 tens; and since each dime is worth 10 cents, 5 dimes are worth 50 cents.

The *Think* problem is built around the observations made in discussion exercise 3 on page 228. Most of the children will be able to solve it, and all should understand it when the correct answer is given.

Assignments (page 229)

Minimum: 1–2, 4. Average: 1–4.
Maximum: 1–4.

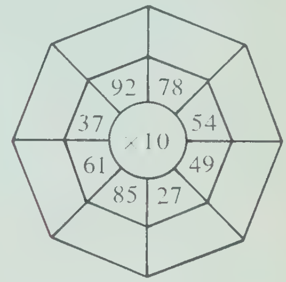
Mathematics

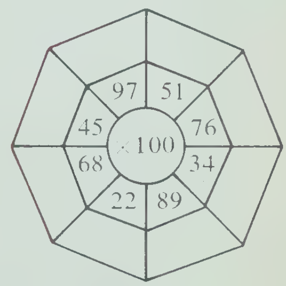
The distributive principle is used to develop the algorithm for finding products such as 72×10. The following example shows how this principle is used.

$$\begin{aligned} 72 \times 10 &= (70 + 2) \times 10 \\ &= (70 \times 10) + (2 \times 10) \\ &= 700 + 20 \\ &= 720 \end{aligned}$$

Follow-up/Practagons

Duplicate practagons adapted to further work on multiplying by 10 and 100. Leave some of the practagons blank so that the children can make up problems of their own.





Workbook, page 83

PAGES 230–231

Objective

Given a pair of factors, one of which is a multiple of 10 and the other a 1-digit numeral, the child will be able to find the product.

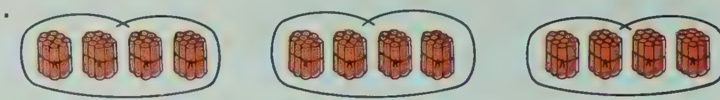
Preparation

You might prefer to begin immediately with the text, but if you choose to have an oral warm-up activity, review multiplying 1- and 2-digit numerals by 10 and 100.

Let's explore products like 3×40 and 3×400.

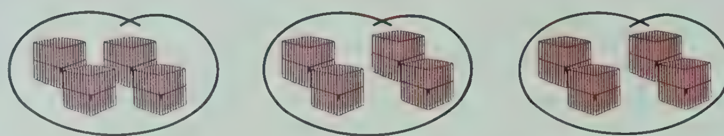
Discussing the Ideas

1.



- A How many tens in each ring? 4
- B How many tens in all? 12
- C How many sticks in all? 120
- D 3×4 tens is how many tens? 12
- E Solve: $3 \times 40 = n$ 120

2.



- A How many hundreds in each ring? 4
- B How many hundreds in all? 12
- C How many sticks in all? 1200
- D 3×4 hundreds is how many hundreds? 12
- E Solve: $3 \times 400 = n$ 1200

Fran

See Discussion.

3.
- A Explain how Fran is thinking about the product 5×30.
- B Write the numeral for 15 tens. 150
- C Solve: $5 \times 30 = n$ 150
- D How would Fran think about 2×70? 4×60? 7×30?

Discussion

To encourage each child to think independently for this section, read each question aloud while the class reads silently with you. Do not solicit an oral response after each question; encourage the children to write the answer to each question on their own. Work through exercises 1 and 2 in this manner. Then discuss each question, asking volunteers for their answers. Encourage the children to think of 3×40 as 3×4 tens and of 3×400 as 3×4 hundreds. As you work into exercise 3, point out the use of the associative principle for multiplication by examples like:

$$5 \times 30 = 5 \times (3 \times 10) = (5 \times 3) \times 10 = 15 \times 10 = 150$$

Do the same with a 3-digit factor.

$$7 \times 500 = 7 \times (5 \times 100) = (7 \times 5) \times 100 = 35 \times 100 = 3500$$

Work through as many examples as you think necessary. Throughout your discussion, stress thinking of products as multiples of 10 or 100.

Using the Ideas

1. Give the missing numbers.

4 × 20	5 × 30	6 × 40
A 4 × ___ tens 2	B 5 × ___ tens 3	C 6 × ___ tens 4
___ tens in all 8	___ tens in all 15	___ tens in all 24
4 × 20 = n 80	5 × 30 = n 150	6 × 40 = n 240

2. Solve the equations.

A (4 × 2) × 10 = n 80
4 × (2 × 10) = n 80
4 × 20 = n 80

B (5 × 3) × 10 = n 150
5 × (3 × 10) = n 150
5 × 30 = n 150

C (6 × 4) × 10 = n 240
6 × (4 × 10) = n 240
6 × 40 = n 240

D (7 × 3) × 10 = n 210
7 × (3 × 10) = n 210
7 × 30 = n 210

3. Find the products.

A 4 × 6 24	C 3 × 7 21	E 3 × 9 27	G 7 × 4 28
4 × 60 240	3 × 70 210	3 × 90 270	7 × 40 280
4 × 600 2400	3 × 700 2100	3 × 900 2700	7 × 400 2800
B 6 × 3 18	D 5 × 7 35	F 6 × 6 36	H 5 × 8 40
6 × 30 180	5 × 70 350	6 × 60 360	5 × 80 400
6 × 300 1800	5 × 700 3500	6 × 600 3600	5 × 800 4000

4. Find the products.

A 3 × 70 210	K 8 × 40 320
B 3 × 700 2100	L 3 × 80 240
C 4 × 30 120	M 2 × 90 180
D 4 × 300 1200	N 9 × 30 270
E 2 × 80 160	O 5 × 50 250
F 2 × 800 1600	P 8 × 400 3200
G 6 × 10 60	Q 2 × 900 1800
H 6 × 100 600	R 9 × 300 2700
I 7 × 20 140	S 5 × 500 2500
J 7 × 200 1400	T 4 × 800 3200

think

If it's products you are after,
I will lend a helping hand.
Just use me over and over,
On the answer you will land.

WHO AM I? Addition

More practice, page A-29, Set 39

Mathematics

The ideas of this lesson depend upon the associative principle to find products such as 3 × 40.

$$\begin{aligned} 3 \times 40 &= 3 \times (4 \times 10) \\ &= (3 \times 4) \times 10 \\ &= 12 \times 10 \\ &= 120 \end{aligned}$$

The use of the associative principle in the second step justifies our use of the language "3 times 4 tens" in multiplying 3 × 40. We could use similar reasoning in finding a product such as 3 × 400.

Follow-up

Children usually enjoy meeting a challenge. You might give them the following problems and suggest that, if they think they have found the rules for multiplying by 10 or 100, they should try to find these products.

Find the products.

1. 275 × 10
2. 10 × 654
3. 789 × 10
4. 695 × 10
5. 10 × 790
6. 7285 × 10
7. 7006 × 10
8. 10 × 8307
9. 6520 × 10
10. 100 × 8300
11. 68 293 × 100
12. 100 × 74 657
13. 90 000 × 100
14. 35 000 × 100
15. 100 × 60 200
16. 100 × 100 000
17. 756 298 × 100
18. 100 × 654 200

Duplicator Masters, page 49
Workbook, page 84
Skill Masters, page 49

Using the Exercises

Exercise 1 could be used as a basis for discussion. Then the remaining exercises might be assigned as independent work. If necessary, give individual help with exercise 4. For example, walk around the room as the children work, and quietly ask a child to think his way through one of the problems. He might say, for instance: "3 × 70 is the same as 3 × 7 tens; and this is the same as 21 tens or 210."

You may choose to use the *Think* problem as enrichment. Even though all children may not be able to solve it, when they have heard the answer, most will understand that repeated addition can help them find a product.

Assignments (page 231)

Minimum: 3–4. Average: 1–4.
Maximum: 1–4.

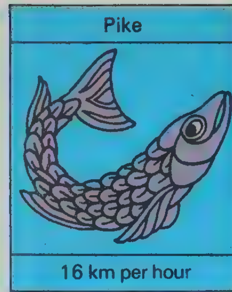
PAGES 232–233

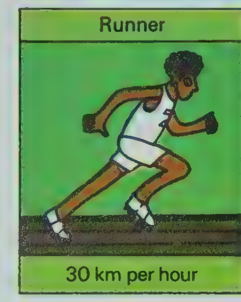
Objective

Given a simple multiplication word problem containing a multiple of ten as one of the factors, the child will be able to find the product.

Preparation

Use a short oral game to review multiplying 1- and 2-digit numerals by 10 and 100. You might say, for instance, "I'm thinking of the product that is 5 × 6 tens. What's my number?"





Story Problems

1. A helicopter can fly about 3 times as fast as a homing pigeon. How fast can the helicopter fly? 300 km/h

2. Racing bikes can be made to go 2 times as fast as a deer. How fast can racing bikes go? 160 km/h

3. A jet speedboat goes 9 times as fast as a sailboat. How fast does a jet speedboat go? 450 km/h

4. What object can go 10 times as fast as a pike can swim? Pitched Baseball

5. A racing car runs for 10 hours. A jet flies for 7 hours. Which goes farther? How much farther? The jet goes 600 kilometres farther

6. A propeller plane can fly 9 times as fast as a deer can run. How fast can a propeller plane fly? 720 km/h

7. A golden eagle can fly about 6 times as fast as a runner can run. How fast can the eagle fly? 180 km/h

8. A bullet goes how many times as fast as a pitched baseball? 10

Discussion

Unless you have a very capable class, you will probably prefer to work through most of these problems together. Use the problem-solving guidelines suggested previously, so that your approach will exemplify a consistent method of attacking problems.

(1) "What do I know?" Here refer the children to the pictures, whose captions give the approximate top speeds for each item.

(2) "What must I find?" In many of the problems the child is asked to find the speed of an unlisted item by comparing it to the speed of some item on the chart. A few problems (e.g., 5, 8, and 9) ask for a comparison of the speeds of items on the chart.

(3) "What must I do?" Point out the phrase "times as fast as" and help children see that it implies multiplication.

(4) "Does my answer make sense?" After the children solve each problem, direct them to reread the problem and try to put the answer in context to see if it makes sense.

As you work through the problems, remember that the preceding guidelines are only *guidelines*. Help the children use the common sense approach. For example, you might

9. If a cheetah could run 6 times as fast as usual, could it catch a racing car? Yes
How fast would the cheetah be running? 660 km/h

10. A man flies in a jet plane from Halifax, Nova Scotia, to Edmonton, Alberta. It takes him 4 hours for the trip. About how far is it between the two cities? 4000 km

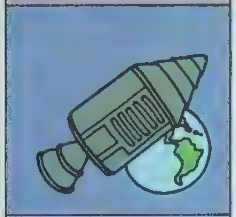
11. A boy rode for 2 hours on a monorail train. How far did he ride? 260 km

12. A homing pigeon was taken 500 kilometres from home. About how many hours would it take the pigeon to fly back home? 5 hours

13. A racing car can go how many times as fast as a deer? 8

14. How much faster than its usual rate would a vulture have to fly to pass a jet plane?
More than 860 km/h faster

15. A satellite circles the earth for 10 hours. How far does it travel? 270 000 km

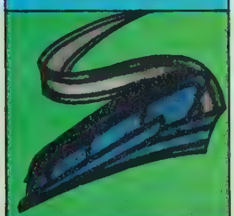
Satellite

27 000 km per hour

Bullet


1600 km per hour

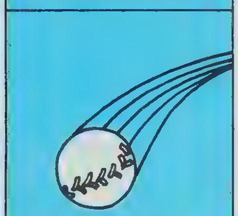
Jet plane

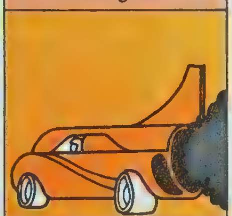
1000 km per hour

Monorail train

130 km per hour

Vulture

140 km per hour

Pitched baseball

160 km per hour

Racing car

640 km per hour

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think through problem 3 as follows: "In order to find out how fast a jet speedboat goes, I must know how fast a sailboat goes. From the chart I see that a sailboat goes 50 kilometres per hour. The speedboat goes 9 times as fast or 9 × 50 kilometres per hour or 450 kilometres per hour."

Problem 8 is a little different: "From the chart I know that a bullet goes 1600 kilometres per hour and a pitched baseball goes 160 kilometres per hour. To see how many times as fast the bullet goes in relation to the baseball, I think: What number × 160 = 1600? Since 16 × 10 = 160, I know that 160 × 10 = 1600, so the bullet goes 10 times as fast."

Exercises 12 through 15 are designed principally for faster children.

Assignments (page 233)
Minimum: 1–11, oral. Average: 1–11. Maximum: 1–15.

Follow-up
For continued practice with multiples of 10 and 100, create some function tables like the following. Direct the children to fill in the blanks.

Function Rule: ×10

Input	Output
9	
12	
	370
	940
88	
	760
45	

Function Rule: (blank)

Input	Output
7	700
15	
	2100
9	900
56	
	7800
	9400

Function Rule: ×30

Input	Output
8	
40	
	1500
	270
30	
70	
	2700

Resources for Active Learning
Mathex: Operations and Problem Solving, No. 8, "Function Machine – Activity 3," p. 18, Encyclopaedia Britannica Publications Ltd.

PAGES 234–235

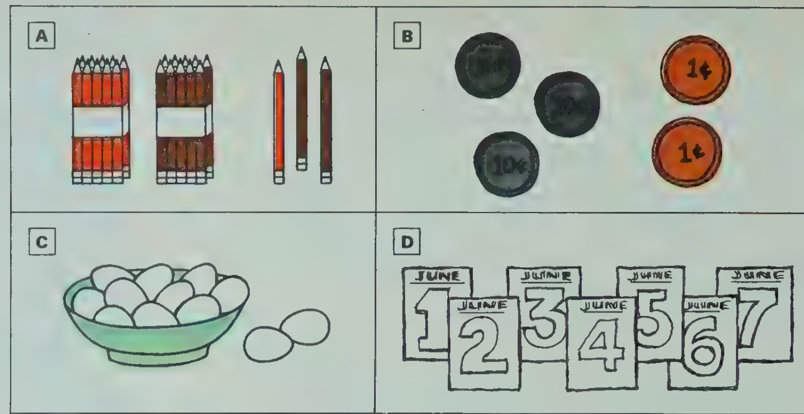
Objective

Given a picture of some objects grouped by tens and ones, the child will be able to solve some simple stories based on the pictures by reasoning and mental multiplication and addition.

Preparation

Continue to use short oral drills to reinforce the children's facility with basic multiplication facts. Occasionally review division by saying something such as: "The product is 28. One factor is 4. What is the other factor?"

Solving Multiplication-Addition Problems



1. A How many pencils do you see in figure A ? 23
   B If John has 2 times as many pencils, how many does he have ? 46
   C Suppose Rose has 3 times as many pencils as in A. How many pencils does she have ? 69

2. A What is the value in cents of the money pictured in figure B ? 32¢
   B If Tom has twice that much money, how much does he have ? 64¢
   C Suppose a toy cost 3 times as much as the amount pictured in B. How much does the toy cost ? 96¢

3. A One dozen is 10 and 2. (See figure C.) How many are in a dozen ? 12
   B How many are in 3 dozen ? 36
   C How many are in 4 dozen ? 48
   D How many are in 5 dozen ? 60

4. A Look at figure D and tell how many days are in a week. 7
   B How many days are in 10 weeks ? 70
   C How many days are in 50 weeks ? 350
   D How many days are in 52 weeks ? 364

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Discussion

These problems are intended to prepare children for further applications of the multiplication-addition principle. Consequently they should not be solved algorithmically but by logical reasoning, and simple mental multiplication and addition. Work through the first problem with your class in a manner similar to the following.

In figure A, there are 23 pencils. If John has 2 times as many pencils, we can double the 2 tens, which makes 4 tens or 40, and we can double the 3 single pencils, which makes 6. So we know that John has 46 pencils. If Rose has 3 times as many as 23 pencils, we can think 3 times the 2 tens gives us 6 tens or 60 and 3 times the 3 single pencils gives us 9. So 3 times 23 is 69.

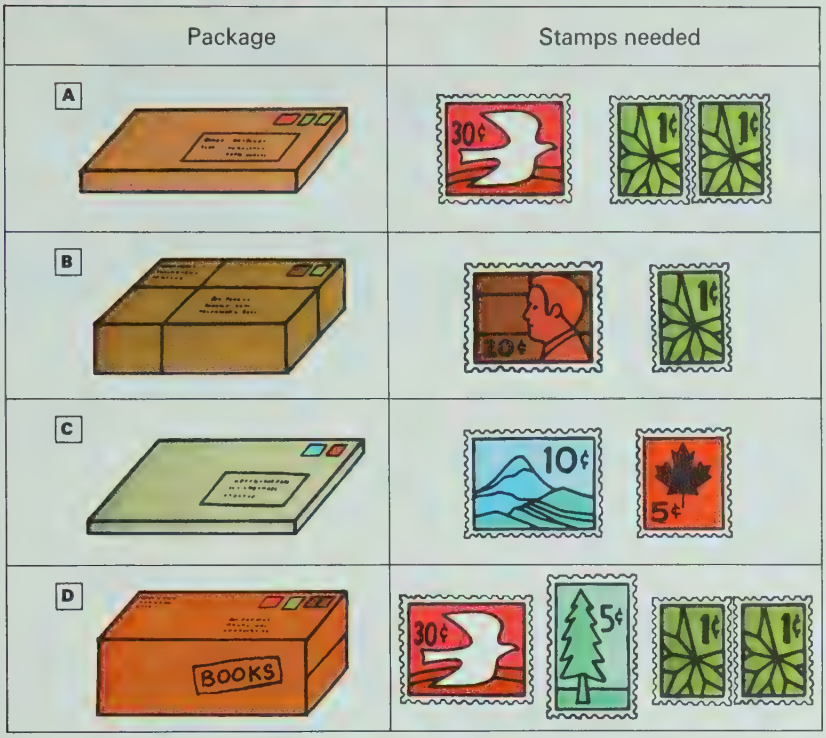
Treat the other problems similarly. In this manner, children are informally breaking apart the factor and using the multiplication-addition principle.

Assignments (page 234)

Minimum: 1–4, oral. Average: 1–4. Maximum: 1–4.

Solving Story Problems

MAILING PACKAGES



1. A How much did it cost to mail package A ? 32 ¢
   B How much would it cost to mail 3 packages like this one ? 96 ¢

2. A How much did it cost to mail package B ? 21 ¢
   B How much would it cost to mail 4 packages like this one ? 84 ¢

3. A How much did it cost to mail package C ? 15 ¢
   B How much would it cost for 3 such packages ? 45 ¢

4. A How much did it cost to mail package D ? 37 ¢
   ★ B How much would it cost to mail 5 such packages ? 185 ¢ or $1.85

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Follow-up/Story Problems
Give the children paste, scissors, newsprint, and a bundle of newspapers. Suggest that they design four to six story problems using information, advertisements, and illustrations from the newspapers. Sunday supplements, sale circulars, and midweek grocery advertisements are usually a good source of material for an activity like this. The children can then trade papers and solve someone else's problems. Suggest that they discuss any unclear problem with the person who wrote it, and that they co-operate in making good problems.

Assignments (page 235)
Minimum: 1–4A, oral. Average: 1–4A. Maximum: 1–4.

PAGES 236–237

Objective

Given two single-digit factors such as 6 × 8, the child will demonstrate his understanding of the multiplication-addition principle by "breaking apart" the second factor to get (6 × 5) + (6 × 3).

Preparation

Materials

graph paper (no smaller than 1-cm grid); crayons; scissors

Because of the nature of the investigation, no preparatory activity is needed. However, if you prefer, you might continue to review multiplication facts and multiplication in which one factor is a multiple of 10.

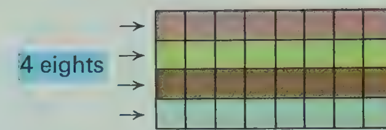
Investigation

In this investigation, children study the breaking-apart concept by actually cutting apart rectangles which represent various products. After they have worked with the 4-by-8 rectangle, suggest other rectangles that they might also break apart, such as 3 by 5, 4 by 6, and 7 by 3. As you move around the room, emphasize that when a factor is broken apart, both parts are still multiplied by the other factor.

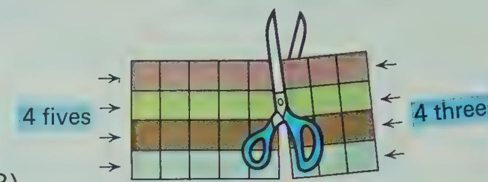
Let's explore the multiplication-addition principle again.

Investigating the Ideas

Cut a 4-by-8 rectangle from graph paper. Color each row of 8 a different color.



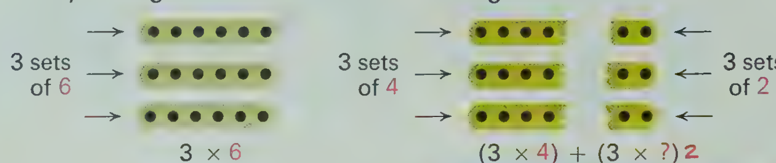
Another way to think about 4 eights is shown by the cut. Now you have 4 fives and 4 threes.
4 × 8 = (4 × 5) + (4 × 3)



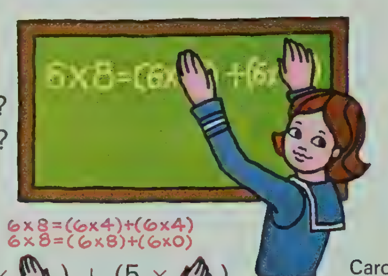
? See Investigation. Can you show how to think about 4 eights in a different way by cutting another 4-by-8 rectangle? (4×4)+(4×4) (4×2)+(4×6) (4×7)+(4×1)

Discussing the Ideas

1. Study the figure below. Give the missing number.



2. Carol's hands are covering two numbers.



Yes **A** Could the numbers be 6 and 2?

No **B** Could the numbers be 3 and 2?

C Write 5 different equations to show what pairs of numbers Carol might be hiding. See Answers, T.E. page 237. Samples: 6×8=(6×4)+(6×4) 6×8=(6×8)+(6×0)

D Repeat part **C** for 5 × 9 = (5 ×) + (5 ×). Samples: 5×9=(5×8)+(5×1) 5×9=(5×5)+(5×4)

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Discussion

Lead into the discussion exercises by discussing the investigation with the rectangles. Exhibit appropriate equations to relate the cutting of the rectangle to the written symbol. Then develop discussion exercises 1 and 2. Finally, extend the principle to a 2-digit factor, in preparation for the exercises on page 237.

On the flannelboard, overhead projector, or chalkboard, show an array having 7 rows and 12 columns. Emphasize that this array shows 7 × 12. Then put a piece of yarn or a ragged line between column 5 and column 6, and ask the children to tell you the products that correspond to the two smaller portions of the array. They should be able to respond 7 × 5 and 7 × 7. Write the equation 7 × 12 = (7 × 5) + (7 × 7) on the chalkboard and continue moving the divider to make other separations, including 7 × 10 and 7 × 2. Write the corresponding equations, and ask the children to find the partial products and to complete each equation to see which partition gives them the easiest way to find 7 × 12.

Using the Ideas

1. Give the missing word.

A 4 fives and 4 __?__ twos
4×7

B 7 threes and 7 __?__ twos
7×5

C 5 fives and 5 __?__ twos
5×7

D 7 sixes and 7 __?__ twos
7×8

E 3 fives and 3 __?__ fours
3×9

F 6 fours and 6 __?__ threes
6×7

G 4 tens and 4 __?__ twos
4×12

H 2 twenties and 2 __?__ sixes
2×26

2. Find the missing number.

A $8 \times 6 = (8 \times 3) + (8 \times n)$ 3
B $4 \times 7 = (4 \times 5) + (4 \times n)$ 2
C $3 \times 8 = (3 \times 4) + (3 \times n)$ 4
D $7 \times 7 = (7 \times 2) + (7 \times n)$ 5
E $9 \times 3 = (9 \times n) + (9 \times 2)$ 1
2 F $6 \times 12 = (6 \times 10) + (6 \times n)$
8 G $5 \times 18 = (5 \times 10) + (5 \times n)$
20 H $3 \times 23 = (3 \times n) + (3 \times 3)$

think

Subtract 24 from me,
Or take me from 42.
Your answer is the same,
Whichever one you do.

WHO AM I? 33

237

Using the Exercises

Use portions of exercise 1 as a basis for discussion. Then assign the rest of exercise 1 and exercise 2 for the class to work independently. However, if any problems arise, you may wish to do a few parts of exercise 2 orally. For example, say "8 sixes are 8 threes and 8 threes," or "6 twelves are 6 tens and 6 twos." Help the children see that the best way to break apart such factors as those in 2F, 2G, and 2H is to break the right-hand factor into an expression of tens and ones.

To solve the *Think* problem, the children must realize that the answer has to be larger than 24, because 24 is subtracted from it; and it has to be less than 42, because it is subtracted from 42. If the children list the numbers 25 through 41, they should discover by trial and error that 33, midway between 24 and 42, is the solution because $33 - 24 = 9$ and $42 - 33 = 9$.

Assignments (page 237)
Minimum: 1, oral. Average: 1–2.
Maximum: 1–2.

Mathematics
A formal statement of the distributive (multiplication-addition) principle follows.

If a, b, and c are any whole numbers, $a \times (b + c) = (a \times b) + (a \times c)$.

The principle helps the children multiply in parts.

$$4 \times 37 = 4 \times (30 + 7)$$
$$= (4 \times 30) + (4 \times 7)$$

Follow-up/"Use Your Head"
Encourage the more capable children to find the products by thinking about the multiplication-addition principle.

1. Since $5 \times 70 = 350$, we know that $5 \times 71 = n$.
2. Since $4 \times 60 = 240$, we know that $4 \times 62 = n$.
3. Since $3 \times 80 = 240$, we know that $3 \times 83 = n$.
4. Since $6 \times 50 = 300$, we know that $6 \times 53 = n$.
5. Since $4 \times 30 = 120$, we know that $4 \times 33 = n$.
6. Since $5 \times 50 = 250$, we know that $5 \times 52 = n$.

Treat these exercises as a game, and encourage children to think through the answers for themselves. Avoid giving them a rule for finding the products.

Resources for Active Learning
Developmental Math Cards, E[1]19, Addison-Wesley.

Answers, discussion exercises 2C and D, page 236

2C. $6 \times 8 = (6 \times 0) + (6 \times 8)$;
$6 \times 8 = (6 \times 1) + (6 \times 7)$;
$6 \times 8 = (6 \times 2) + (6 \times 6)$;
$6 \times 8 = (6 \times 3) + (6 \times 5)$;
$6 \times 8 = (6 \times 4) + (6 \times 4)$

2D. $5 \times 9 = (5 \times 0) + (5 \times 9)$;
$5 \times 9 = (5 \times 1) + (5 \times 8)$;
$5 \times 9 = (5 \times 2) + (5 \times 7)$;
$5 \times 9 = (5 \times 3) + (5 \times 6)$;
$5 \times 9 = (5 \times 4) + (5 \times 5)$

Workbook, page 85

PAGES 238–239

Objective

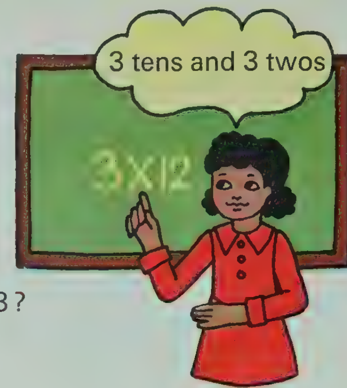
Given a 2-digit factor and a 1-digit factor, the child will be able to find the product by applying the multiplication-addition principle.

Preparation

To prepare for this lesson, use an oral warm-up activity with examples patterned after exercise 2 on page 237. For instance, ask the children to complete expressions such as "3×9 can be thought of as 3×5 plus . . . ," and expect a child to respond "3×4."

How can you use the multiplication-addition principle?

Discussing the Ideas



1. Explain how Sue is thinking about the product 3×12. See Discussion.
2. Write the numeral for 3 tens. 30
3. Write the numeral for 3 twos. 6
4. Give the product for 3×12. 36
5. How would Sue think about 2×13? See Discussion.
6. Find the product for 2×13. 26
7. Give the missing numbers.

   A 4 tens and 4 _?_ twos — 4×12

   B $(4 \times 10) + (4 \times n)$ 2 — 4×12

8. Solve the equations.

   A $4 \times 12 = (4 \times 10) + (4 \times n)$ 2
   B $2 \times 13 = (2 \times 10) + (2 \times n)$ 3
   C $3 \times 11 = (3 \times 10) + (3 \times n)$ 1
   D $5 \times 11 = (5 \times 10) + (5 \times n)$ 1
   E $3 \times 23 = (3 \times n) + (3 \times 3)$ 20
   F $5 \times 14 = (5 \times n) + (5 \times 4)$ 10
   G $4 \times 24 = (4 \times n) + (4 \times 4)$ 20
   H $3 \times 27 = (3 \times n) + (3 \times 7)$ 20

9. A What is 4×10? 40
   B What is 4×5? 20
   C What is $(4 \times 10) + (4 \times 5)$? 60
   D What is 4×15? 60

10. A What is 3×10? 30
    B What is 3×4? 12
    C What is $(3 \times 10) + (3 \times 4)$? 42
    D What is 3×14? 42

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Discussion

One of the main purposes of this lesson is to help children break apart a 2-digit factor into tens and ones. As you discuss the exercises on page 238, relate the first four to the illustration. Draw from the children the idea that Sue is thinking about breaking 12 into $10 + 2$ and multiplying each part by 3. The product in discussion exercise 4 can be found by adding together the two partial products that answered discussion exercises 2 and 3. Write the steps on the chalkboard as you discuss them.

$$3 \times 12 = (3 \times 10) + (3 \times 2) = 30 + 6 = 36$$

Develop exercises 5 and 6 similarly, thinking "2×1 ten and 2 threes," and writing

$$2 \times 13 = (2 \times 10) + (2 \times 3) = 20 + 6 = 26$$

Work through the balance of the exercises, emphasizing those in which the 2-digit factor has more than one ten.

Using the Ideas

Find the products and sums.

1. A $3 \times 10 = n$ 30
B $3 \times 2 = n$ 6
C $(3 \times 10) + (3 \times 2) = n$ 36
D $3 \times 12 = n$ 36

2. A $4 \times 10 = n$ 40
B $4 \times 2 = n$ 8
C $(4 \times 10) + (4 \times 2) = n$ 48
D $4 \times 12 = n$ 48

3. A $2 \times 10 = n$ 20
B $2 \times 3 = n$ 6
C $(2 \times 10) + (2 \times 3) = n$ 26
D $2 \times 13 = n$ 26

4. A $2 \times 30 = n$ 60
B $2 \times 4 = n$ 8
C $(2 \times 30) + (2 \times 4) = n$ 68
D $2 \times 34 = n$ 68

5. A $2 \times 20 = n$ 40
B $2 \times 3 = n$ 6
C $(2 \times 20) + (2 \times 3) = n$ 46
D $2 \times 23 = n$ 46

6. A $3 \times 20 = n$ 60
B $3 \times 3 = n$ 9
C $(3 \times 20) + (3 \times 3) = n$ 69
D $3 \times 23 = n$ 69

7. A $6 \times 10 = n$ 60
B $6 \times 3 = n$ 18
C $(6 \times 10) + (6 \times 3) = n$ 78
D $6 \times 13 = n$ 78

8. A $4 \times 20 = n$ 80
B $4 \times 4 = n$ 16
C $(4 \times 20) + (4 \times 4) = n$ 96
D $4 \times 24 = n$ 96

think

Make 8 of these — Make 5 of these

This is a game for two. The object is to cover the 10-unit strip exactly with the 1-unit and 2-unit pieces. Start at the left and take turns placing either a 1-unit or a 2-unit side by side until the 10-unit is exactly covered. The last one to put down a strip wins the game.

Start									

Try this game using an 11-unit strip. Try it with a 12-unit strip. See Solution.

More practice, page A-29, Set 40

Using the Exercises
You might choose to work through the first exercise on page 239 with the children. The exercises are so constructed that a careful analysis of the papers will indicate which areas need more concentration.

Part A of each exercise requires multiplying by a multiple of ten; part B involves basic multiplication facts; part C requires finding the sum of parts A and B; and part D requires the recognition and application of the multiplication-addition principle to see that the number for n in part D is the same as for n in part C.

Most children will enjoy the game suggested in the *Think* problem. Do not discuss it until they have had ample opportunity to play the game and discover their own strategies.

Assignments (page 239)
Minimum: 1–4. Average: 1–8.
Maximum: 1–8.

Follow-up/Reconstruction Problems
To provide another form of review of basic multiplication facts, give the children a worksheet similar to the sample below and ask them to reconstruct the problems. Remind them that each box represents a missing digit.

Find the missing numbers.

$\square$	9	7	$\square$	7
$\times 5$	$\times \square$	$\times \square$	$\times 8$	$\times 5$
40	36	21	48	$\square\square$
6	4	8	7	$\square$
$\times \square$	$\times 3$	$\times \square$	$\times \square\square$	$\times 6$
42	$\square\square$	56	7 0	54
8	$\square$	$\square$	7	9
$\times \square$	$\times 9$	$\times 8$	$\times \square$	$\times 7$
64	45	32	49	$\square\square$

Solution to *Think*, page 239
Some children may discover the winning tactics by accident, while others may keep track of moves to see which ones enable them to win. By working a winning strategy a second time, they may be able to determine that they can win if they are the player who covers the seventh position. After more play, or through analysis of their moves, children may even find that they can always cover the seventh space and thus win, if they can control the units to cover the fourth space. Some may even realize that if their opponents start with 2, or they themselves start with 1, and if playing errors are avoided, they can always win.

Workbook, page 86

PAGES 240–241

Objective

Given a 1-digit factor and a 2-digit factor written in vertical notation, the child will be able to find the product by applying the multiplication-addition principle.

Preparation

You may choose to begin immediately with the investigation. However, if you prefer, continue to review multiplication facts and breaking apart 2-digit factors.

Investigation

Read the material at the top of page 240 with the class. Help the children interpret the table. Impress upon them that the left column is formed by breaking apart the 2-digit factor, and that the blank space, the product of 54 and 3, is the sum of the first two products. Then allow the children sufficient time to make their own tables.

To extend the investigation, you might suggest other products (4 × 54, 5 × 76, etc.), and duplicate blank tables to be filled in by the children.

Blank table

Multiply by

Input	Output

Filled-in table

4 × 29

Multiply by 4

Input	Output
9	36
20	80
29	116*

*Add 36 + 80 to get this number.

Point out that the third line of the output column may be solved by adding the first two lines and that this is an application of the multiplication-addition principle.

● *How can you find products like 3 × 54?*

Investigating the Ideas

You know that
3 × 54 = (3 × 50) + (3 × 4).

Let's use a function-machine table to help us find this product. What is the missing number in the table? 162

3 × 54

Multiply by 3

input	output
4	12
50	150
54	▥

? Can you make tables of your own to find these products?
1. 2 × 34 (68) 2. 4 × 36 (144) 3. 3 × 65 (195)

See Investigation.

Discussing the Ideas

Let's look at a shorter way to write the work.

3 × 48

Multiply by 3

8	24
40	120
48	144

See Discussion.

Step 1	Step 2	Step 3
48 × 3 24	48 3 24 120	48 × 3 24 120 144
3 × 8 = 24	3 × 40 = 120	24 + 120 = 144

1. Which part of the table is like step 1?
2. Which is like step 2?
3. Which is like step 3?
4. Now try the same method with these.
   A 3 × 64 — 64, ×3, 12, 180, 192
   B 2 × 76 — 76, ×2, 12, 140, 152

Discussion

Have several volunteers put their completed tables on the chalkboard. Use some of these to show how the multiplication-addition principle is used. Then develop the discussion in the text.

During the discussion of questions 1, 2, and 3, relate the steps in the table to the steps in the illustration at the right. Then have the children work through exercise 4, and afterward, let volunteers explain and show at the chalkboard how they did parts A and B. It would be helpful to relate the method of this lesson with the horizontal equation studied previously.

$$\begin{array}{r} 64 \\ \times 3 \\ \hline 12 \\ 180 \\ \hline 192 \end{array} \qquad \begin{aligned} 3 \times 64 &= (3 \times 60) + (3 \times 4) \\ &= 180 + 12 \\ &= 192 \end{aligned}$$

Work through other examples step by step, and then let the children try a few on their own but under your supervision and guidance.

Using the Ideas

Find the products.

1. 14 ×5 = 70	2. 38 ×2 = 76	3. 24 ×3 = 72	4. 19 ×4 = 76	5. 47 ×4 = 188	6. 34 ×2 = 68
7. 67 ×3 = 201	8. 35 ×2 = 70	9. 36 ×5 = 180	10. 58 ×3 = 174	11. 75 ×4 = 300	12. 17 ×4 = 68
13. 18 ×3 = 54	14. 22 ×4 = 88	15. 94 ×3 = 282	16. 26 ×3 = 78	17. 29 ×5 = 145	18. 18 ×2 = 36
19. 31 ×8 = 248	20. 73 ×4 = 292	21. 19 ×3 = 57	22. 42 ×7 = 294	23. 27 ×3 = 81	24. 32 ×8 = 256

Short Stories

1. 12 dogs, 4 legs each. How many legs in all? 48

2. 14 ants, 6 legs per ant. How many legs? 84

3. 12 spiders, 8 legs per spider. How many legs? 96

4. 13 octopuses, 8 arms each. How many arms? 104

5. 14 crayfish, 10 legs each. How many legs? 140

6. 36 crickets, 6 legs each. How many legs? 216

7. 42 fish, no legs per fish. How many legs in all? 0

8. 4 centipedes, 36 legs each. How many legs? 144

More practice, page A-30, Set 41

Using the Exercises
Select several of the exercises for the children to do on their own. It is not necessary that all the children do all the exercises. Capable students do not need much drill on this intermediate step, and slower students will find it too long and cumbersome. Only a few of the children should want or need to continue using this method after the shortcut algorithm is introduced.

In the short story exercises, you may wish to have children write the equation for each problem and use the vertical notation to find the product.

Assignments (page 241)
Minimum: Odd-numbered problems. Average: All. Maximum: All.

Mathematics
Point out during this lesson that, although the vertical notation is different, the multiplication ideas are the same. This is simply a different and more convenient way for the children to write down their method of finding a solution.

Note that the distributive (multiplication-addition) principle is being used in exactly the same way as it is for working problems in horizontal notation. Here, the only difference is in how the work is organized on paper. This is emphasized in the discussion on page 240, where the related equation for each step is shown beneath it.

Follow-up/"Roll a Problem"
A game for small groups or a pair of children can be made by gluing the numerals 0, 1, 2, 3, 4, and 5 on the faces of a 2-cm cube; the numerals 6, 7, 8, 9, 10, and 11 on the faces of a second cube; and two addition signs, two multiplication signs, one subtraction sign, and one division sign on a third cube. Ask the children to take turns shaking the three cubes in a container (such as an empty juice can) dumping them out, and performing the operation indicated, if possible. A child should be free to use the numbers in any order, but he must be able to get a whole-number answer using the numbers and operation which show on the upturned faces of the cubes. Give him one point each time he successfully completes the operation, and allow him to continue throwing the cubes until he is unable to get a whole-number answer, cannot perform the indicated operation, or makes an error in calculations. For example, if a child throws the symbols 11, 2, and ÷, he will be unable to give a whole number for the operation and his turn will end. If he throws the symbols 10, 2, and ÷ and then answers "Five," he has earned a point and may continue to play. Decide the best rule for ending the game—at 21 points, when one person is 10 points ahead, or at a given time.

Workbook, pages 87, 88

Objective

Given a 2-digit factor and a 1-digit factor, the child will be able to find their product by using the multiplication shortcut.

Preparation

To prepare for this lesson, devise mental chain games to review basic multiplication and addition facts. Here are a few examples.

"Start with 7 . . . Multiply by 8 . . . Subtract 6 . . . Multiply by 10 . . . What's my number?" (500)

"Start with 6 . . . Add 12 . . . Add 6 more . . . Think of that sum as a product and 3 as a factor . . . What number would be the missing factor?" (8)

"Start with 21 . . . Subtract 7 . . . Multiply by 10 . . . What's my number?" (140)

Let's look at a shortcut for finding products.

Discussing the Ideas

See Discussion.

1. Explain each step of the long method.
2. Explain how step 1 of the shortcut is like step 1 of the long method.
3. Explain how step 2 of the shortcut puts together step 2 and step 3 of the long method.
4. Explain each step in this exercise.

$$\begin{array}{r} {\scriptstyle 2} \\ 18 \\ \times\ 3 \\ \hline 54 \end{array}$$

5. Now try the shortcut with this one. 4 × 23 92
6. Carl used the long method, but he made a mistake. Can you explain what he did wrong and find the correct product? He multiplied 6 x 2 instead of 6 x 20. Correct product: 144
7. Beth Ann used the shortcut, but she also made a mistake. Can you explain what she did wrong? She forgot to add the 2 from the first partial product (6 x 4).

Long Method

Step 1	Step 2	Step 3
$\begin{array}{r} 46 \\ \times 3 \\ \hline 18 \end{array}$	$\begin{array}{r} 46 \\ \times 3 \\ \hline 18 \\ 120 \end{array}$	$\begin{array}{r} 46 \\ \times 3 \\ \hline 18 \\ 120 \\ \hline 138 \end{array}$
3×6=18	3×40=120	18+120=138

Shortcut

Step 1	Step 2
$\begin{array}{r} {\scriptstyle 1} \\ 46 \\ \times 3 \\ \hline 8 \end{array}$	$\begin{array}{r} {\scriptstyle 1} \\ 46 \\ \times 3 \\ \hline 138 \end{array}$
3×6=18	3×40=120 120+10=130

Carl

$$\begin{array}{r} 24 \\ \times 6 \\ \hline 24 \\ 12 \\ \hline 36 \end{array}$$

Beth Ann

$$\begin{array}{r} 24 \\ \times 6 \\ \hline 124 \end{array}$$

Discussion

Have the children work a few problems using the method introduced in the previous lesson, such as:

$$\begin{array}{r} 75 \\ \times 6 \\ \hline \end{array} \qquad \begin{array}{r} 75 \\ \times 6 \\ \hline 30 \\ 420 \\ \hline 450 \end{array}$$

Ask volunteers to explain to the class the three steps for working each problem. Then continue discussion of the exercises in the text related to the methods illustrated in the boxes at the right.

While discussing the shortcut, lead the children to think of the 1 ten in 18 as follows:

46	3 × 6 is eighteen.
×3	write down the 8.
8	*Remember* the 1 ten as you write it above the 4.

Continuing:

1	
46	3 × 40 is 120.
×3	Add 1 ten from 18.
138	Write down 13 tens.

(After children become accustomed to this algorithm, you will want to encourage them to discontinue writing the "reminder" numeral. But for now it serves as an important part of the shortcut.)

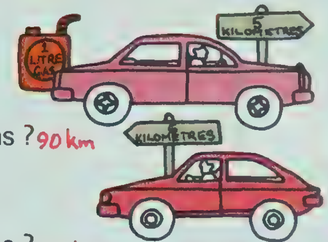
Development of skill in using the shortcut algorithm requires much classroom time and many activities, including discussions, boardwork, practice sheets, and competition. Though we devote only one lesson to the algorithm itself, this lesson follows careful developmental work and serves as an introduction to the reinforcement and practice that must continue in class.

Using the Ideas

In exercises 1 through 18, find the product.

1. 24 × 4 = 96	**2.** 15 × 3 = 45	**3.** 38 × 2 = 76	**4.** 24 × 6 = 144	**5.** 19 × 3 = 57	**6.** 47 × 3 = 141
7. 39 × 2 = 78	**8.** 21 × 4 = 84	**9.** 30 × 2 = 60	**10.** 63 × 4 = 252	**11.** 37 × 2 = 74	**12.** 37 × 3 = 111
13. 26 × 3 = 78	**14.** 54 × 5 = 270	**15.** 67 × 5 = 335	**16.** 75 × 6 = 450	**17.** 54 × 4 = 216	**18.** 68 × 4 = 272

Solving Story Problems



1. Mr. McCoy can drive his car about 6 kilometres on one litre of gas. How far can he drive on 15 litres of gas ? 90 km
2. Mr. Ito's car goes only 5 kilometres on each litre of gas. How far can he drive on 17 litres of gas ? 85 km
3. Mr. Sims has a small car and can drive 8 kilometres on 1 litre of gas. How far can he drive on 14 litres of gas ? 112 km
4. Mr. Le Blanc can drive 56 kilometres on 7 litres of gas. Mr. Brown can drive 7 kilometres on 1 litre of gas. Who can drive farther
   - A on 8 litres of gas ? Mr. LeBlanc
   - B on 4 litres of gas ? Mr. LeBlanc
   - C on 2 litres of gas ? Mr. LeBlanc
   - D on 1 litre of gas ? Mr. LeBlanc

think

Together, a doll and a dress cost $11. The doll cost $10 more than the dress. How much was the doll ? $10.50

More practice, page A-31, Set 42

243

Using the Exercises

Assign selected exercises from those at the top of page 243 and, after they have been completed, use them as a basis for discussion as needed by the class.

Unless the children need help in reading the story problems, assign them as independent work for the whole class.

Allow some time to discuss the exercises after the children finish. During this discussion period, it might be worthwhile to review the steps involved in the multiplication algorithm.

The *Think* problem is a variation of a classical trick problem. At first glance, many children will say the doll was $10 and the dress $1, but these answers do not meet the conditions of the problem. Most children will benefit from a lively discussion of the solution after they have been given ample time to think about the problem.

Assignments (page 243)

Minimum: Odd-numbered problems; story problems, oral.
Average: 1–18; story problems.
Maximum: 1–18; story problems.

Follow-up/Chain Game

Children need continued practice in adding, subtracting, multiplying, and dividing, particularly with multiples of 10 and 100. The following chain games may provide a stimulating form of drill.

Start — End

4	×10		−20		×3		÷3	20

Start — End

10	×70		÷7		−20		÷20	4

Start — End

100	÷10		×6		÷3		÷10	2

Resources for Active Learning

Franklin Series: *Patterns and Puzzles*, "Without a Word," p. 46, Lyons and Carnahan. [A type of chain game] (Available from McGraw-Hill Ryerson)

Duplicator Masters, page 50
Workbook, page 89
Skill Masters, page 50

PAGES 244–245

Objective

Given problems that require finding products of a 2-digit and a 1-digit factor, the child will be able to find the product by using the intermediate or shortcut algorithm developed in the previous lessons.

Preparation

To prepare for this lesson, conduct a brief oral review of the basic multiplication facts. Then select a couple of problems, such as 5×30 and 56×4, and have the class work through them with you, first treating them both as equations in which the multiplication-addition principle is used and then using the vertical notation and shortcut algorithm.

Building Multiplication Skills

1. Find the products.

A 3×6 18	G 3×7 21	M 6×4 24	S 5×6 30
B 7×4 28	H 8×3 24	N 4×8 32	T 5×5 25
C 3×9 27	I 4×9 36	O 9×5 45	U 9×6 54
D 5×8 40	J 6×8 48	P 5×7 35	V 6×7 42
E 9×8 72	K 9×9 81	Q 7×9 63	W 5×4 20
F 7×7 49	L 8×7 56	R 6×6 36	X 8×8 64

2. Find the products.

A 6×10 60	F 6×50 300	K 9×90 810	P 70×4 280
B 10×9 90	G 10×34 340	L 10×84 840	Q 3×90 270
C 40×4 160	H 80×10 800	M 7×30 210	R 2×60 120
D 15×10 150	I 7×20 140	N 80×6 480	S 70×10 700
E 5×30 150	J 10×68 680	O 5×90 450	T 85×10 850

3. Find the products.

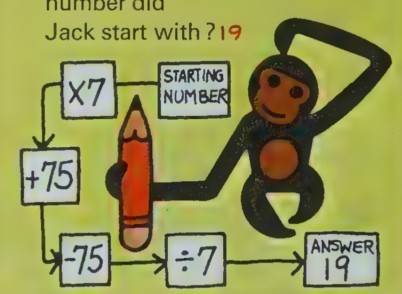
A 32×3 96	B 43×2 86	C 44×2 88
D 17×2 34	E 28×3 84	F 19×4 76

4. Find the products.

A 47×4 188	B 56×5 280	C 87×2 174
D 27×3 81	E 56×4 224	F 65×6 390
G 73×6 438	H 65×5 325	I 76×3 228
J 74×8 592	K 67×7 469	L 89×4 356

think

Jack started with a number and multiplied it by 7. Then he added 75, subtracted 75 and divided by 7. Jack's answer was 19. What number did Jack start with? 19



Discussion

If necessary, use parts of exercises 3 and 4 as a basis for discussion. However, most of the material on page 244 should be worked independently by the children. For exercises 1 and 2, direct the children to write down only the answers. When they have finished all the exercises, check their work and review any troublesome areas.

Assignments (page 244)

Minimum: 1–4F. Average: 1–4.
Maximum: 1–4.

Solving Story Problems TIME

1. There are 60 seconds in 1 minute. How many seconds are in 5 minutes? 300

2. There are 60 minutes in 1 hour. How many minutes are in 9 hours? 540

3. There are 24 hours in 1 day, and 7 days in a week. How many hours are in a week? 168

4. There are 12 months in 1 year. How many months are in 8 years? 96

5. There are about 52 weeks in 1 year. About how many weeks are in 6 years? 312

S	M	T	W	T	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

6. In each year, there are 7 months that have 31 days each. How many days in all are in these 7 months? 217

7. In each year, there are 4 months that have 30 days each. How many days in all are in these 4 months? 120

★ 8. How many days are in a year when February has 28 days? (Use your answers to exercises 6 and 7.) 365

★ 9. How many days are in a year when February has 29 days? 366

★ 10. How many seconds are in one hour? 3600

★ 11. How many seconds are in one day? 86 400

think

The large blocks weigh the same and the small blocks weigh the same. Each large block weighs twice as much as a small block. All the blocks together weigh 35 kilograms. How much does a large block weigh? 10 kg

Using the Exercises

Before assigning page 245, direct the children to the Table of Measure for Time on page A44. Read through the given equivalents, discussing any that are of particular interest to the children. Then return to page 245; you might want to work through two or three of the problems together. If children have benefitted from use of the problem-solving guidelines, use them again here. For example, you might think through problem 1 as follows:

What do I know? (There are 60 seconds in 1 minute.)

What must I find? (5 minutes is 5 times as many as 1 minute, so I must find the number that is 5 times as many as 60.)

What do I do? (To find the number that is 5×60, multiply: $5 \times 60 = 300$.) Does my answer make sense? (If there are 60 seconds in 1 minute, it is sensible that there are 300 seconds in 5 minutes.)

Developing each problem in this manner is not essential, but working through one or two will help children to think through the others with understanding.

Assignments (page 245)

Minimum: 1–7, oral. Average: 1–7.
Maximum: 1–11.

Follow-up

All of the children can benefit from some concentrated study of the calendar. Duplicate a worksheet for them that shows each month of the year. Display a current calendar to help the children accurately fill in the dates for each month. This might be a good opportunity to help them learn the seven months that have 31 days, the four months that have 30, and the one that commonly has 28. You might even present some background information on the difficulty men have had developing a calendar. (*See* Ruth Brindze's *The Story of Our Calendar*, or Jeanne Bendick's *First Book of Time*, in the Books to Explore section of this text, on pages A38 and A39, respectively.)

You might then have the children use their calendar to answer questions such as the following:

1. What day is 20 days from May 17?
2. What day is 25 days from August 16?
3. Circle your birthday.
4. What day is 4 weeks from your birthday?
5. What day is 1 month from your birthday?
6. Are the answers to questions 4 and 5 the same?

*7. Could the answer to question 6 be different from the one you gave?

*8. In what month are there usually exactly 4 weeks?

Resources for Active Learning

Developmental Math Cards, G²15, Addison-Wesley. [A good activity to develop graphing skill; it could be used with the next chapter, too.]

Nuffield Project: *Computation and Structure* 2, "Time," pp. 82–90, Wiley.

PAGES 246–247

Objective

The child will have an opportunity to demonstrate his ability to work with the concepts presented thus far in the chapter and to practice finding products when one factor is a 2-digit numeral.

Preparation

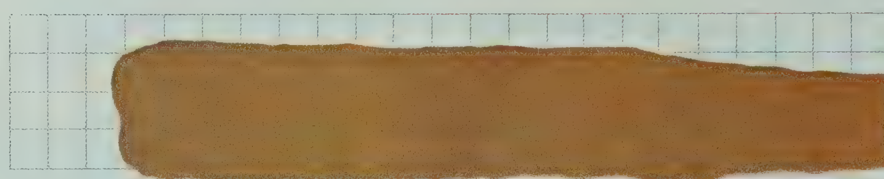
To prepare for this lesson, use a short oral activity of your choice. For example, review basic multiplication facts by using this pattern: "I'm thinking of the product, 8 × 7. What's my number?" Then progress to products in which one factor is a multiple of 10 and the other a single-digit factor. Finally, give products of 2-digit factors and 10; and of 2-digit factors and 100.

Improving Multiplication Skills

1. Find the products.

A 37 × 4 = 148	B 56 × 2 = 112	C 95 × 3 = 285	D 43 × 4 = 172	E 82 × 5 = 410	F 96 × 4 = 384
G 54 × 3 = 162	H 38 × 6 = 228	I 69 × 5 = 345	J 57 × 2 = 114	K 70 × 3 = 210	L 58 × 2 = 116
M 49 × 5 = 245	N 68 × 4 = 272	O 65 × 4 = 260	P 85 × 4 = 340	Q 66 × 2 = 132	R 77 × 3 = 231
S 72 × 6 = 432	T 48 × 5 = 240	U 75 × 4 = 300	V 57 × 6 = 342	W 79 × 2 = 158	X 59 × 3 = 177

2. Find the area of this rectangle. 92



3. How many rooms in each?

A 7 stories high. 25 rooms on each floor. 175

B 9 stories high. 23 rooms on each floor. 207

think

Jim had 1 minute to decide which of these allowances he wanted.

▶$1.00 per week

or

▶Each week he gets 1¢ the first day, 2¢ the second, 4¢ the third, and so on for 7 days.

Which would you take? 1¢ the 1st day, 2¢ the 2nd, etc. Give yourself 1 minute to decide. Then figure it out.

246

Discussion

The exercises on page 246 may be used as a basis for discussion if you think your class would benefit. You may have volunteers explain how they solve an exercise, or you yourself might present an explanation of the shortcut method for one or two examples. It would be helpful to discuss exercise 2 and encourage children to explain how they would find the area. Their reasoning might be: "There are 4 rows and 27 in each row, so there are 4 × 27 area units." For this and for exercise 3, encourage children to use the vertical notation.

Of the two possible choices given in the *Think* problem, most children will probably choose the dollar-per-week allowance. However, when the problem is worked, they should all see why the second choice would give them a greater allowance. That is, $1 + 2 + 4 + 8 + 16 + 32 + 64 =$ $1.27.

Assignments (page 246)

Minimum: 1A–1L. Average: 1–3. Maximum: 1–3.

Solving Story Problems

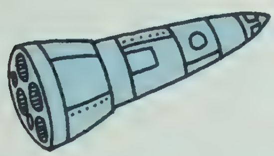
A MOON TRIP

Peter thinks he would like to be an astronaut and go to the moon. He read some books to learn more about the moon. Peter wrote this paper to show his teacher.

Moon Facts

1. The United States Astronauts, Neil A. Armstrong and Edwin E. Aldrin, Jr., were the first men to land on the moon, July 20, 1969.
2. People weigh 6 times as much on earth as they do on the moon.
3. The moon goes around the earth once in about 28 days.
4. The moon is about 384 000 kilometres from the earth
5. Scientists have found no air or water on the moon.

Peter L.



1. Peter figured he would weigh only 6 kilograms on the moon. How much does Peter weigh on earth? 36 kg
2. Peter's father would weigh 13 kilograms on the moon. How much does he weigh on earth? 78 kg
3. About how long does it take the moon to go around the earth 4 times? 112 days
4. Suppose you could fly straight to the moon and back. How far would you travel? 768 000 km
5. Peter drinks water 4 times a day. Each time he drinks about one fourth of a litre. How much water would he have to take for a 36-day moon trip? 36 litres

More practice, page A-32, Set 43

Using the Exercises

Depending on the needs of your class, you may prefer to assign both of these pages as independent study. Story problem pages, such as page 247, are provided not only for their value in improving the children's skill in solving word problems and in using arithmetic concepts, but also for the information and stimulation they add to the arithmetic program. To realize full value from these lessons, allow the children to discuss the various aspects of these exercises freely and in detail.

After the children study the illustrations and the moon facts, give them an opportunity to discuss the ideas and to work the exercises.

If the children have difficulty with exercise 5, help them see that Peter drinks about 1 litre per day and hence needs 36 litres for the trip.

Assignments (page 247)

Minimum: 1–5, oral. Average: 1–5. Maximum: 1–5.

Follow-up/"What's Wrong?"

To encourage children to decide what kind of problem is presented in a story exercise before trying to solve it, you can let children work in groups to discuss problems for which a wrong answer is given. Direct the children to explain the mistake and to think of a proper equation and solution to the problem.

You might take problems with typical errors in their answers from the children's own work, or you could create some like the samples below. Be sure to emphasize that the given answers (in parentheses) are *incorrect*, and that the children are to try to find the correct answers.

1. Steve had 10 guinea pigs. He gave away all but 3. How many are left? (7)
2. Kim weighed 25 kilograms while standing on one foot on the school scales. How much does she weigh when standing on the scales with both feet? (50 kilograms)
3. Pete had 10 more marbles than Joe. Joe had 15 marbles. How many marbles do the boys have altogether? (25 marbles)
4. An automobile averaged 80 kilometres per hour for 8 hours. How far was the car driven? (10 kilometres)

Solution to *Think*, page 246

Each Week		Day	Allowance
\$1	or	1	1¢
		2	2¢
		3	4¢
		4	8¢
		5	16¢
		6	32¢
		7	64¢
			Total: 127¢ or \$1.27

Workbook, page 90

PAGES 248–249

Objective

Given problems in which one factor is a 3-digit numeral and the other is a 1-digit numeral, the child will be able to find the product.

Preparation

Materials

blank tagboard cards (3 per child)

Use a short drill to review multiplication by multiples of 10, such as 4×30 and 4×300. If necessary, follow this with a brief review of multiplication involving a 2-digit factor that is not a multiple of 10 and a 1-digit factor. This lesson involves exactly the same ideas with larger numbers. (*Note:* For children who have had difficulty with multiplication, treat this as optional material. It may be omitted altogether if necessary.)

Investigation

In this investigation, the child is given an opportunity to discover how to multiply a 3-digit numeral using partial products and vertical notation by applying his knowledge of basic multiplication facts and his understanding of how to multiply 2- and 3-digit numerals which are multiples of 10 or 100. Have the children work in groups to try to find the products for A, B, and C. As they work, move about the room making sure children see that the cards show the breakdown of 3×326 into a basic-fact problem, a problem involving a multiple of 10, and a problem involving a multiple of 100. Encourage children to work on their own or with their group to make the cards showing the partial products for 4×243.

$$\begin{array}{r} 200 \\ \times 4 \\ \hline 800 \end{array} \qquad \begin{array}{r} 40 \\ \times 4 \\ \hline 160 \end{array} \qquad \begin{array}{r} 3 \\ \times 4 \\ \hline 12 \end{array}$$

Write additional exercises on the chalkboard for fast workers.

Remind children having difficulty to think of place value and of the multiplication-addition principle.

$$326 = 300 + 20 + 6$$
$$3 \times 326 = (3 \times 300) + (3 \times 20) + (3 \times 6)$$

Let's explore larger products.

Investigating the Ideas

Use the products on the three cards to help you give the three products below.

$$\begin{array}{r} 300 \\ \times 3 \\ \hline 900 \end{array} \qquad \begin{array}{r} 20 \\ \times 3 \\ \hline 60 \end{array} \qquad \begin{array}{r} 6 \\ \times 3 \\ \hline 18 \end{array}$$

A $\begin{array}{r} 26 \\ \times 3 \\ \hline 78 \end{array}$ B $\begin{array}{r} 320 \\ \times 3 \\ \hline 960 \end{array}$ C $\begin{array}{r} 326 \\ \times 3 \\ \hline 978 \end{array}$

? Can you make some cards like these that would help a classmate find this product? See Investigation.

$$\begin{array}{r} 243 \\ \times 4 \\ \hline 972 \end{array}$$

Discussing the Ideas

1. Explain each step in the example below. See Discussion.

Step 1	Step 2	Step 3	Step 4
$\begin{array}{r} 237 \\ \times 4 \\ \hline 28 \end{array}$	$\begin{array}{r} 237 \\ \times 4 \\ \hline 28 \\ 120 \end{array}$	$\begin{array}{r} 237 \\ \times 4 \\ \hline 28 \\ 120 \\ 800 \end{array}$	$\begin{array}{r} 237 \\ \times 4 \\ \hline 28 \\ 120 \\ 800 \\ \hline 948 \end{array}$
$4 \times 7 = 28$	$4 \times 30 = 120$	$4 \times 200 = 800$	$28 + 120 + 800 = 948$

2. You know the first product. Find each of the other products.

$\begin{array}{r} 237 \\ \times 4 \\ \hline 948 \end{array}$ A $\begin{array}{r} 2000 \\ \times 4 \\ \hline 8000 \end{array}$ B $\begin{array}{r} 2237 \\ \times 4 \\ \hline 8948 \end{array}$ C $\begin{array}{r} 3237 \\ \times 4 \\ \hline 12,948 \end{array}$

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Discussion

Have volunteers explain how to use the cards they made to find the product 4×243 and any other products they used for the investigation. Then discuss the steps as presented in exercise 1. Point out that the shading shows the relationship between the steps in horizontal and vertical notation. Use other examples as needed.

The following shows another way of relating equations to the vertical form. Notation like this should help the children develop understanding that will be needed for the more difficult multiplication which comes later.

$$\begin{array}{rl} 237 & \\ \times 4 & \\ \hline 28 & 4 \times 7 = 28 \\ 120 & 4 \times 30 = 120 \\ 800 & 4 \times 200 = 800 \\ \hline 948 & 4 \times 237 = 948 \end{array}$$

Exercise 2 may be treated as enrichment, as may the shortcut method for exercise 1. If you choose to develop the shortcut for exercise 1, these steps are suggested:

Step 1	*Step 2*	*Step 3*
$\begin{array}{r} {\scriptstyle 2} \\ 237 \\ \times 4 \\ \hline 8 \end{array}$	$\begin{array}{r} {\scriptstyle 12} \\ 237 \\ \times 4 \\ \hline 48 \end{array}$	$\begin{array}{r} {\scriptstyle 12} \\ 237 \\ \times 4 \\ \hline 948 \end{array}$

Using the Ideas

Find the products.

1. 231 × 3 = 693	**2.** 213 × 4 = 852	**3.** 116 × 5 = 580	**4.** 326 × 2 = 652	**5.** 207 × 3 = 621
6. 128 × 4 = 512	**7.** 382 × 2 = 764	**8.** 144 × 3 = 432	**9.** 143 × 6 = 858	**10.** 162 × 5 = 810
11. 211 × 9 = 1899	**12.** 264 × 3 = 792	**13.** 225 × 4 = 900	**14.** 243 × 6 = 1458	**15.** 415 × 5 = 2075
16. 1345 × 4 = 5380	**17.** 2534 × 3 = 7602	**18.** 1023 × 8 = 8184	**19.** 1203 × 9 = 10,827	**20.** 1620 × 6 = 9720

Solving Story Problems

1. Miss Wright took her pupils to visit the airport. While they were there, 3 jet planes took off for Europe. Each plane carried 125 people. How many people is this in all? 375
2. During the same visit, 4 planes arrived from Europe and Australia. Each plane carried 310 people. How many people is this in all? 1240
3. The large jet airliners have 4 engines. If each engine weighs about 3200 kilograms, what is their total weight? 12800 kg
4. Suppose a large jet plane flies at the rate of 960 kilometres each hour. How far would it fly in 4 hours? 3840 km
5. A pilot told the children that some supersonic planes fly 3 times as fast as jets fly now. How fast is this? (Use the speed in exercise 4.) 2880 km/h

More practice, page A-33, Set 44

Your explanation might be something like this: Step 1: 4 times 7 is 28; write down the 8; remember the 2 (as you write it down above the 3). Step 2: 4 times 3 is 12, plus 2 is 14 (tens); write down the 4; remember the 1. Step 3: 4 times 2 is 8, plus 1 is 9 (hundreds); write down the 9."

Using the Exercises
Assign exercises according to the children's capabilities. If the reading ability of the children allows, have children work in groups of 3 or 4 to solve the story problems. Encourage interested children to read about aviation, its history, and current progress; and perhaps suggest that they make up problems of their own using facts that they find.

Assignments (page 249)
Minimum: 1–10; story problems, oral.
Average: 1–20; story problems.
Maximum: 1–20; story problems.

Duplicator Masters, pages 51, 52
Workbook, page 91
Skill Masters, page 51

PAGES 250–251

Objective

Given two factors such as 4 and 49, the child will be able to estimate the product by rounding the larger factor to the nearest ten or nearest hundred.

Preparation

You might begin immediately with the investigation, or if you prefer, use a short oral warm-up to review multiplying by multiples of 10.

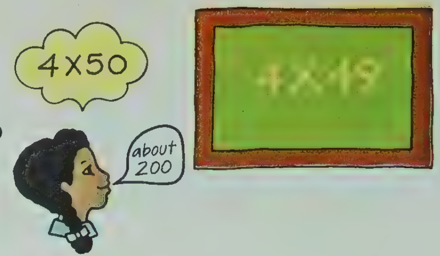
Investigation

Read the top of the page with the children. Make sure they understand the meaning of *estimate*. Have the children form small groups or work as partners. Remind them that each child in the group should write down his estimate for the product before computing it. If some children have difficulty making estimates, suggest that thinking of multiples of 10 might help them. Encourage them to discuss with each other the reasons for their choice of estimate.

● *Can you estimate products like 4 × 49?*

Investigating the Ideas

Sara is estimating the product 4 × 49. When we estimate an answer to a problem, we try to find a number that is "close" to the correct answer.



Work with one or more classmates. Choose one number from Set A and another from Set B. Each of you write down your estimate of the product.

Set A: 3, 5, 2, 4, 6

Set B: 48, 61, 402, 201, 293, 19, 99

? Can you find the difference between your estimate and the actual product? See Investigation.

Discussing the Ideas

1. Explain how Sara found an estimate for 4 × 49.
Instead of 49, she thought of the closest multiple of 10, 50.
2. Find the product in the red cloud. Explain why this is a "good estimate" of the product below.

A	B	C	D
4 × 50 (200)	3 × 40 (120)	12 × 100 (1200)	6 × 200 (1200)
4 × 51	3 × 39	12 × 99	6 × 204

3. Give an estimate for each product.
Explain how you found your estimate.

A 3 × 69 210 B 5 × 38 200 C 7 × 21 140 D 6 × 98 600 E 7 × 297 2100

250

Discussion

Allow the children to share the results of the investigation. You might, for instance, have volunteers write on the chalkboard a pair of factors, the estimates their group gave for this pair, and the multiplication used to find the product. Point out the estimates which used multiples of 10 or 100, and then lead children in a discussion of the exercises in the text. For exercise 3, ask the children first to tell you what multiples to choose and then to try to give the estimated product by mental computation.

Some of the very able children may be capable of finding correct products mentally. Do not discourage this, for a child who can do so can surely estimate also. Indeed, such a child can make estimates for far more difficult problems than are found here.

Using the Ideas

Estimate each product.

1. 3 × 82 240	**4.** 25 × 99 2500	**7.** 5 × 199 1000	**10.** 4 × 299 1200
2. 4 × 69 280	**5.** 6 × 61 360	**8.** 6 × 58 360	**11.** 5 × 302 1500
3. 8 × 99 800	**6.** 3 × 91 270	**9.** 2 × 197 400	**12.** 8 × 493 4000

Which of the three numbers is a "good estimate"?

13. An airplane flew 920 kilometres each hour for 4 hours. About how many kilometres did it fly? A 370 B 3700 C 1800

14. An orchard has 8 rows of trees with 49 trees in each row. About how many trees are in this orchard?
A 100 B 500 C 400

15. Each of 6 books has 312 pages. About how many pages are there in all? A 2000 B 600 C 3000

16. If there are 365 days in each of 3 years, about how many days is this in all? A 100 B 1000 C 10 000

17. A car has 460 moving parts. About how many moving parts are in 2 cars? A 90 B 900 C 9000

★**18.** There are 52 weeks in a year. About how many weeks are in $2\frac{1}{2}$ years?
A 100 B 200 C 125

★**19.** There are 60 minutes in an hour.
About how many minutes are in $8\frac{1}{4}$ hours?
A 450 B 500 C 550

★**20.** There are 24 hours in a day.
About how many hours are in $5\frac{1}{2}$ days?
A 110 B 130 C 150

251

Using the Exercises

Assign these exercises as independent work, making sure the children understand that for the word problems they are to choose the answer they think best. Though exercises 18, 19, and 20 are starred, encourage everyone to try them. When the children have finished, ask volunteers to explain the reasoning they used to arrive at their answers. Having the children discuss the reasons for their estimates is an essential part of this lesson.

Assignments (page 251)*
Minimum: 1–15. Average: 1–20.
Maximum: 1–20.

Follow-up

For capable children, write pairs of factors on the chalkboard or on duplicated worksheets for distribution. Ask the children to estimate the products and tell whether the products are more or less than 100, or 200, or 500.

Estimate the products.

1. Is your estimate greater than or less than 100?

2 × 49	19 × 4
3 × 78	37 × 2
12 × 5	12 × 8

2. Is your estimate greater than or less than 200?

4 × 26	3 × 91
2 × 98	4 × 44
5 × 75	7 × 36

Resources for Active Learning

Nuffield Project: *Computation and Structure* 3, "Approximations," p. 47, Wiley.

Workbook, page 92

PAGES 252–253

Objective

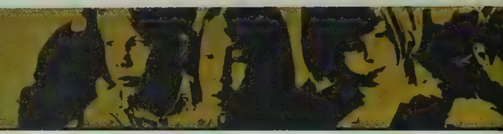
The child will demonstrate his ability to work with the concepts presented in this chapter.

Preparation

Spend a short time reviewing those topics which have caused the most difficulty and which were the main points of this chapter. For example, write a multiplication exercise on the chalkboard and have volunteers show different methods of writing out its solution.

$$\begin{array}{rl} 85 & \\ \times 6 & \\ \hline 30 & 6 \times 5 \\ 480 & 6 \times 80 \\ \hline 510 & 6 \times 85 \end{array} \qquad \begin{array}{r} {\scriptstyle 3} \\ 85 \\ \times 6 \\ \hline 510 \end{array}$$

Reviewing the Ideas



1. Solve the equations.
   - A $4 \times 28 = (4 \times 20) + (4 \times n)$ 8
   - B $2 \times 54 = (2 \times 50) + (2 \times n)$ 4
   - C $3 \times 65 = (3 \times n) + (3 \times 5)$ 60
   - D $7 \times 18 = (7 \times n) + (7 \times 8)$ 10
   - E $6 \times 51 = (6 \times 50) + (6 \times n)$ 1
   - F $8 \times 46 = (8 \times n) + (8 \times 6)$ 40

2. Find the products.

A 100×18 1800	D 2×10 20	G 100×24 2400	J 75×10 750
B 78×10 780	E 10×136 1360	H 75×100 7500	K 10×23 230
C 6×100 600	F 37×10 370	I 20×10 200	L 4×100 400

3. Find the products.

A 3×20 60	D 6×30 180	G 7×20 140	J 3×70 210
B 3×40 120	E 2×80 160	H 4×30 120	K 2×90 180
C 5×50 250	F 9×20 180	I 5×60 300	L 4×60 240

4. Find the products and sums.

A
$3 \times 20 = n$ 60
$3 \times 4 = n$ 12
$(3 \times 20) + (3 \times 4) = n$ 72
$3 \times 24 = n$ 72

B
$4 \times 30 = n$ 120
$4 \times 6 = n$ 24
$(4 \times 30) + (4 \times 6) = n$ 144
$4 \times 36 = n$ 144

5. Find the products.

A $\begin{array}{r} 23 \\ \times 2 \\ \hline 46 \end{array}$	B $\begin{array}{r} 14 \\ \times 2 \\ \hline 28 \end{array}$	C $\begin{array}{r} 32 \\ \times 3 \\ \hline 96 \end{array}$
D $\begin{array}{r} 12 \\ \times 4 \\ \hline 48 \end{array}$	E $\begin{array}{r} 11 \\ \times 5 \\ \hline 55 \end{array}$	F $\begin{array}{r} 43 \\ \times 2 \\ \hline 86 \end{array}$
G $\begin{array}{r} 13 \\ \times 3 \\ \hline 39 \end{array}$	H $\begin{array}{r} 22 \\ \times 4 \\ \hline 88 \end{array}$	I $\begin{array}{r} 33 \\ \times 2 \\ \hline 66 \end{array}$
J $\begin{array}{r} 24 \\ \times 2 \\ \hline 48 \end{array}$	K $\begin{array}{r} 11 \\ \times 6 \\ \hline 66 \end{array}$	L $\begin{array}{r} 12 \\ \times 3 \\ \hline 36 \end{array}$

6. Find the products.

A $\begin{array}{r} 37 \\ \times 4 \\ \hline 148 \end{array}$	B $\begin{array}{r} 46 \\ \times 3 \\ \hline 138 \end{array}$	C $\begin{array}{r} 25 \\ \times 2 \\ \hline 50 \end{array}$
D $\begin{array}{r} 39 \\ \times 5 \\ \hline 195 \end{array}$	E $\begin{array}{r} 43 \\ \times 6 \\ \hline 258 \end{array}$	F $\begin{array}{r} 62 \\ \times 4 \\ \hline 248 \end{array}$
G $\begin{array}{r} 92 \\ \times 3 \\ \hline 276 \end{array}$	H $\begin{array}{r} 49 \\ \times 4 \\ \hline 196 \end{array}$	I $\begin{array}{r} 65 \\ \times 2 \\ \hline 130 \end{array}$
J $\begin{array}{r} 238 \\ \times 3 \\ \hline 714 \end{array}$	K $\begin{array}{r} 1292 \\ \times 4 \\ \hline 5168 \end{array}$	L $\begin{array}{r} 2158 \\ \times 5 \\ \hline 10970 \end{array}$

252

Discussion

Whether you use these pages as evaluation or as review, give the children an opportunity to present and discuss many of the exercises. Note that starred exercise 7 on page 253 is primarily for the faster children, though others may try it if time permits. After the children have completed the short story problems, allow time for a thorough discussion of them.

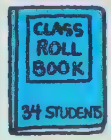
★ **7.** Complete each sentence.

A Since 4 × 27 = 108, we know that 5 × 27 = 108 + *n* . 27
B Since 6 × 78 = 468, we know that 7 × 78 = 468 + *n* . 78
C Since 6 × 78 = 468, we know that 5 × 78 = 468 − *n* . 78
D Since 9 × 57 = 513, we know that 8 × 57 = 513 − *n* . 57
E Since 7 × 83 = 581, we know that 8 × 83 = 581 + *n* . 83
F Since 8 × 47 = 376, we know that 7 × 47 = 376 − *n* . 47
G Since 4 × 348 = 1392, we know that 5 × 348 = 1392 + *n* . 348
H Since 9 × 176 = 1584, we know that 8 × 176 = 1584 − *n* . 176
I Since 27 × 38 = 1026, we know that 28 × 38 = 1026 + *n* . 38
J Since 85 × 67 = 5695, we know that 84 × 67 = 5695 − *n* . 67

Short Stories

1 13 rows of chairs.
6 chairs in each row.
How many chairs? 78

2 3 classes of children.
34 children in each class.
How many children? 102

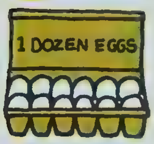


3 427 pages in each book. 7 books. How many pages? 2989

4 36 rooms on each floor.
9 floors. How many rooms? 324

5 10 pencils in each bundle.
68 bundles.
How many pencils? 680

6 7 dozen eggs. How many eggs? 84



7 4 golf balls in a box.
83 boxes.
How many golf balls? 332

8 6 wheels on a truck.
12 trucks.
How many wheels? 72

9 100 centimetres in a metre. How many centimetres in 36 metres? 3600

10 11 players on each team.
How many players on 7 teams? 77

Follow-up/"Find the Mistakes"
To encourage the children to check their own work for errors, give them a worksheet like the sample shown. Use errors detected in their papers to guide your selection of incorrect answers.

Find the errors and circle them with a crayon. Then try to work the problems correctly.

		1	0	4
69	86	92	45	680
×5	×6	×7	×8	×6
45	36	174	324	4086
30	488			
75	524			

Resources for Active Learning
Developmental Math Cards, F⁵21, F¹6, Addison-Wesley. [An opportunity for children to work with a variety of number phrases]

Workbook, pages 93, 94

PAGES 254–255

Objective

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

Spend a short time reviewing any topics with which the children may still have difficulty. Since considerable time has been spent with the processes of multiplication in this chapter, it might be helpful to concentrate most of your effort on the processes for adding and subtracting 2- and 3-digit numbers. One way you might do this would be to put some reconstruction problems (like those in exercise 4) on the chalkboard, and let volunteers complete them, explaining the reasoning behind their solution.

Keeping in Touch with Addition, Subtraction, Multiplication, Division, Fractions

1. Find the sums and differences.

A 34 +51 = 85	B 76 −24 = 52	C 67 +12 = 79	D 87 −32 = 55	E 15 +84 = 99	F 95 −13 = 82
G 67 +89 = 156	H 132 −57 = 75	I 76 +98 = 174	J 161 −82 = 79	K 85 +67 = 152	L 153 −95 = 58

2. Find the missing factors.

A $6 \times n = 42$ 7	D $6 \times n = 48$ 8	G $3 \times n = 27$ 9	J $9 \times n = 0$ 0
B $n \times 5 = 15$ 3	E $n \times 4 = 36$ 9	H $6 \times n = 36$ 6	K $7 \times n = 49$ 7
C $8 \times n = 32$ 4	F $n \times 5 = 35$ 7	I $n \times 7 = 42$ 6	L $n \times 3 = 9$ 3

3. Find the quotients.

A Since $8 \times 9 = 72$, we know that $72 \div 9 = n$. 8
B Since $8 \times 9 = 72$, we know that $72 \div 8 = n$. 9
C Since $6 \times 8 = 48$, we know that $48 \div 6 = n$. 8
D Since $10 \times 27 = 270$, we know that $270 \div 27 = n$. 10

4. There are 12 children playing kickball.

A One half of them are girls. How many of the children are girls? 6

B One fourth of the children wear glasses. How many of the children wear glasses? 3

★ **5.** Copy the problems and give the missing digits.

A 59 −□□ 31 = 28	B 39 +6□ 7 = □□6 (10)	C 83 −3□ 7 = 4 □6	D 23 ×□ 2 = 46	E 4 □9□ 2 +3□8 6 = 860	F □□ (42) ×6 = 252

Discussion

Unless you prefer to present these exercises as a review, have the children work through them independently. Starred exercise 5 is a reconstruction problem in which the missing digits must be found. As you discuss the solutions to these problems, have the children explain how they found the digits. Since a subsequent chapter covers division, you should devote particular attention to exercise 3 during the discussion period. If necessary, provide additional exercises of this type.

Exercises 4A and 4B review simple fractional concepts which children were exposed to in Chapter 1. Some child may know that one half of 12 is 6 and that one fourth of 12 is 3. Or they may think, "$2 \times 6 = 12$, so one half of 12 is 6" and "$4 \times 3 = 12$, so one fourth of 12 is 3."

When the children finish, check the exercises and discuss any questions they may have.

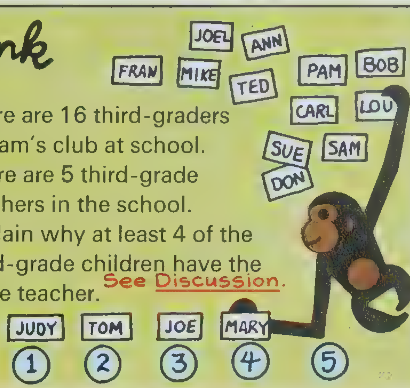
The *Think* problem on page 255 will be a challenge to most children. There are several logical ways to show that at least four children have the same teacher. At one extreme, everyone could have the same teacher; and in that case, the teacher certainly has four pupils.

6. Find the quotients.

A	12 ÷ 4 3	L	15 ÷ 3 5
B	20 ÷ 5 4	M	14 ÷ 7 2
C	12 ÷ 3 4	N	20 ÷ 4 5
D	18 ÷ 6 3	O	12 ÷ 6 2
E	10 ÷ 2 5	P	25 ÷ 5 5
F	15 ÷ 5 3	Q	24 ÷ 8 3
G	16 ÷ 4 4	R	21 ÷ 3 7
H	18 ÷ 3 6	S	28 ÷ 4 7
I	12 ÷ 2 6	T	36 ÷ 9 4
J	30 ÷ 5 6	U	30 ÷ 6 5
K	24 ÷ 6 4	V	35 ÷ 5 7

think

There are 16 third-graders in Pam's club at school. There are 5 third-grade teachers in the school. Explain why at least 4 of the third-grade children have the same teacher. See Discussion.



7. A group of 52 girls and 35 boys visited the bakery. They left at 9 o'clock in the morning and returned at 2 o'clock in the afternoon.
   - A How many children went on the trip? 87
   - B How many more girls went than boys? 17
   - C How long did their trip last? 5 hr
   - D Only 23 girls rode the bus. How many did not ride the bus? 29
   - E The boys visited one room in groups of 7. How many groups of 7 boys were there? 5
   - F Each of the girls got 7 souvenir pencils. How many pencils did they get in all? 364
   - G Each of the boys got 7 souvenir pencils. How many pencils did they get in all? 245
   - H How many more pencils did the girls get than the boys? 119
   - I How many pencils did the children receive in all? 609

You are invited to explore

ACTIVITY CARD 12
Page 315

At the other extreme, if the 16 children are assigned three to a teacher, there will be one child left over. So again, at least one teacher has four children. You might make large and small blocks available to the children who want to work on this problem, with the suggestion that they choose large blocks to represent each of the third-grade teachers and small blocks for the 16 children in Pam's club. Accept any plausible solution, such as "If you divide all the children evenly among the five teachers, they each get three children with one left over; and one teacher will have to have him in addition to her other three."

Follow-up

If you have collected types of errors common to the children in your class, work them into a review sheet. Instruct the children to circle the mistakes in the given answers, and challenge them to write the correct answers if they can. For example:

Find the mistakes and correct them if you can.

72	964	156	49	780
+ 38	+ 523	− 78	× 2	× 6
1(0)10	1(3)87	82	818	4686
(110)	(1487)			

If the children need reinforcement in the basic multiplication facts, duplicate a multiplication table similar to the one shown for them to complete. Using this for a timed test should help the children evaluate their efficiency as well as their accuracy with these facts.

×	7	2	5	8	3	6	0	9	4	1
2										
9										
3										
0										
8										
4										
7										
6										
1										
5										

CHAPTER 11 | Geometry and Graphing

Pages 256–275

General Objectives

To introduce the use of number pairs (co-ordinates) for determining locations in the plane
To introduce the fundamentals of graphing points in the plane
To provide background for future work with graphing concepts
To introduce symmetrical figures on a graph
To provide background for understanding translations of a figure on a graph
To develop the concept of graphing sets of number pairs
To introduce negative numbers and to represent positive and negative numbers on a number line and in the co-ordinate plane
To provide activities involving the use of negative numbers
To provide experiences with vertical bar graphs

One of the main objectives of this chapter is to enable the child to relate number pairs (co-ordinates) with points on a graph. The first few lessons develop the "over and up" pattern for locating a point on the graph. Then the child is given additional experiences in matching number pairs with points and, at the same time, is introduced to symmetry on a graph and translations of a figure. The function machine is used to develop the concept of number pairs on a graph. Children are then introduced to the construction and use of bar graphs. Finally, negative numbers are introduced by association with opposite directions on the co-ordinate axes.

Mathematics

In a certain sense, geometry and arithmetic are brought together in this chapter on geometry and graphing. This is done by establishing a correspondence between the points of a geometric figure and pairs of numbers. We use what is known as a Cartesian co-ordinate system, named after the famous mathematician René Descartes, to establish this correspondence.

To set up a co-ordinate system, we begin by choosing two perpendicular number lines that intersect at zero. We use the number lines or "axes" to locate points in the plane by associating the points with number pairs called the co-ordinates of the points. In beginning work, we use only whole-number points. Later, the whole plane is used as the child learns about negative numbers. Negative integers are introduced through the idea of "opposites." By considering various concepts from the world of the child, where the idea of "opposite" has physical meaning, integers can be easily understood. One of the most effective methods for developing an understanding of positive and negative numbers is through the use of the number line. In this way, the child has a visual image of the concept, part of which is already familiar to him. Following work with the number line, various physical situations can be made very clear. For example, the thermometer is a kind of number line, and the idea of scores above and below zero in certain games can be thought of in terms of the number line.

The concept of a translation, which is introduced in this chapter, can best be thought of by the children in terms of movement. As with symmetry, no attempt at mathematical formality should be made. The ideas are simple when explored on an intuitive level, as they are presented in the text.

Teaching the Chapter

Materials

City maps (simplified)
Colored strips
Crayons
Demonstration grid, 1 m by 1 m
Graph paper: 1 cm grid (*Duplicator Masters*, page 70)
Overhead projector
Paper clips
10-by-10 unit grid on a transparency
Transparent tape

Vocabulary

axes	graph
axis	line of symmetry
bar graph	negative numbers
co-ordinate axes	symmetric
co-ordinates	symmetry

The investigations in this chapter give the children actual experience in using the Cartesian co-ordinate system. In this chapter particularly, the investigations contain the main thrust of the teaching content. Children will need specific demonstrations of how to read a graph to make the concepts they explore definite. However, the lessons on symmetry and translation need not be discussed at great length. They provide an informal introduction to these geometric concepts by providing the child with a meaningful way to practice the technique of graphing which he has just learned. Negative numbers are also developed by expansion of the co-ordinate system, and development of the concepts here would be aided by use of a demonstration grid. In one of the last lessons, the children use their strips to broaden their understanding of "graph," and they learn to read and construct the commonly used bar graph. Finally, a chapter review and a cumulative review help you to see how well the children have understood the concepts presented in this chapter and those treated throughout the text.

Lesson Schedule

Plan to spend a minimum of two weeks on this chapter. Most classes

would benefit from a three-week schedule, which would allow flexibility and sufficient time for expansion of these topics as the interests of the class might warrant.

Evaluation of Progress

Since the main objective of this chapter is for the child to develop the ability to locate and graph points on the co-ordinate axes, observe the ability of each as you proceed through the lessons. The review at the end of the chapter is provided as an evaluation, but you may choose to use it as a review lesson and replace it with your own evaluative material.

Resources for Active Learning

General Activities

Freedom to Learn, "Length," pp. 121–140, Addison-Wesley. Some ideas for graphing.

Math Activity Cards, "Tic-Tac-Toe," A4, 8 and B4; "Go-bang," B5 (a popular game in Japan), Macmillan

Mathex: Matching and Graphing No. 1, "Co-ordinates," pp. 26-29 (pupil pages 43–50), Encyclopaedia Britannica Publications Ltd.

Nuffield Project: *Pictorial Representation* 1, Wiley. This entire book describes and illustrates ideas for graphing which lead to computational practice (review) and interesting topics related to the entire curriculum.

Manipulative Devices

Mirror Cards (Selective Educational Equipment; Webster, McGraw-Hill)

Sigma Chips (Sigma, Scott Scientific)

Commercial Games

Battleship (local retail outlet)

3-D Tic-Tac-Toe (Creative Publications; Math Media; Selective Educational Equipment)

PAGES 256–257

Objective

Given a picture of an object on a simple co-ordinate grid, the child will be able to give a number pair for the location of the object.

Preparation

To prepare for this lesson, briefly discuss the meaning of the word "location." For example, ask what is the location of the school, of your home, of a point on the chalkboard, etc. Help the children to realize that asking "What is the location of" is the same as asking "Where," "In what place," or "What is its position." They might also realize that there are different ways of describing a location. In this lesson, they will see whether number pairs can show locations.

Investigation

You might have the children work in groups of two or three for this investigation. Before they begin, teach them how to read the word "co-ordinates," but do not explain its meaning. Part of the challenge of the investigation is to discover what co-ordinates are.

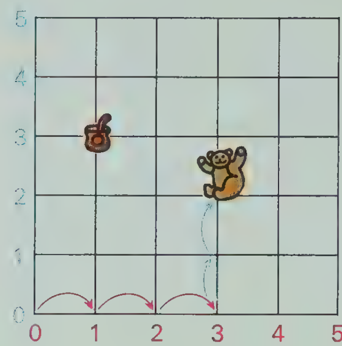
As the children work, move around the room and ask leading questions of any children who seem confused. For example, if a child does not describe the location of the honey according to the given pattern, ask him how far over is the honey, or how far up. Children should be able to describe the location of the objects in this manner even if they cannot readily identify the numbers as co-ordinates. Encourage them to discuss with each other what they think the co-ordinates are. You will be able to clarify their ideas of co-ordinates during the ensuing discussion.

11 Geometry and Graphing

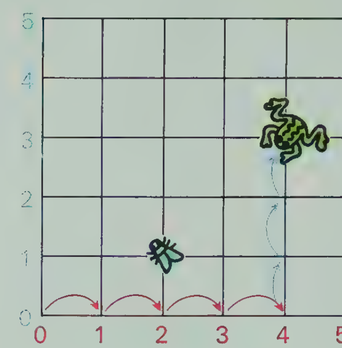
Can number pairs show location?

Investigating the Ideas

Study the graphs.



The bear is
"3 over and 2 up."
Its co-ordinates are (3,2).



The frog is
"4 over and 3 up."
Its co-ordinates are (4,3).

? Can you answer these questions about the graphs?

1. A Where is the honey?
   B What are its co-ordinates?
   A 1 over and 3 up B (1,3)

2. A Where is the fly?
   B What are its co-ordinates?
   A 2 over and 1 up B (2,1)

Discussing the Ideas

1. Does "2 over and 3 up" give the same location as (3,2)?
   No, (3,2) means 3 over and 2 up.
2. Explain how you would find the location for each of these co-ordinates.
   A (2,5) 2 over and 5 up B (5,4) 5 over and 4 up C (3,1) 3 over and 1 up

256

Discussion

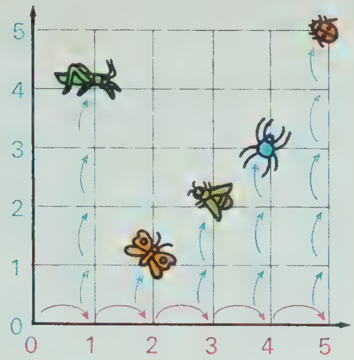
Overhead projection of a 10-by-10-unit grid on a transparency would facilitate discussion of this lesson. If no overhead projector is available, however, you can draw a grid system on the chalkboard or use a demonstration grid about 1 m by 1 m. Have several volunteers show where the honey, bear, frog, and fly would be located on the grid so that their co-ordinates are the same as those shown in the graphs on page 256. Relate the "over and up" phrase to the co-ordinates written in parentheses.

Be sure to discuss with the children the agreement to write the "over" number first. Develop this idea in discussion exercise 1, and ask one of the children to use the demonstration grid to show the difference between (2, 3) and (3, 2). Continue to use the demonstration grid to have children show the answers for exercise 2.

You might also find this an opportune time to show how a grid is formed, demonstrating the horizontal and vertical number lines (in this lesson, number "rays") and labelling points 0 through 9 for each. Complete the grid by drawing the interior horizontal and vertical lines, so that the children will see where the small squares come from.

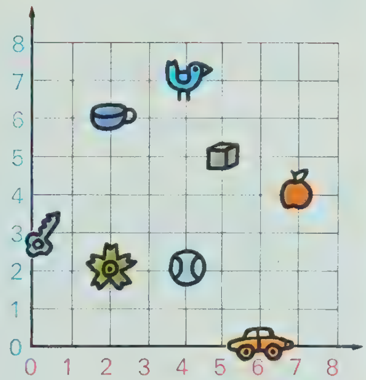
Using the Ideas

1. Give the missing numbers.
   - A The butterfly is 2 over and ? up. 1
   - B The grasshopper is ? over and 4 up. 1
   - C The bee is ? over and ? up. 3,2
   - D The spider is ? over and ? up. 4,3
   - E The ladybug is ? over and ? up. 5,5



2. Give the co-ordinates for each insect above.
   Grasshopper (1,4); butterfly (2,1); bee (3,2); spider (4,3); ladybug (5,5)

3. Give the missing numbers. Then give the co-ordinates.
   - A The ball is 4 over and ▒ up. 2
     The co-ordinates for the ball are ___ ? (4,2)
   - B The bird is ▒ over and 7 up. 4
     The co-ordinates for the bird are ___ ? (4,7)
   - C The apple is 7 over and ▒ up. 4
     The co-ordinates for the apple are ___ ? (7,4)
   - D The car is 6 over and ▒ up. 0
     The co-ordinates for the car are ___ ? (6,0)
   - E The block is ▒ over and 5 up. 5
     The co-ordinates for the block are ___ ? (5,5)



   - F The key is 0 over and ▒ up. 3
     The co-ordinates for the key are ___ ? (0,3)
   - G What are the co-ordinates for the cup ? (2,6)
   - H Give the co-ordinates for the flower. (2,2)

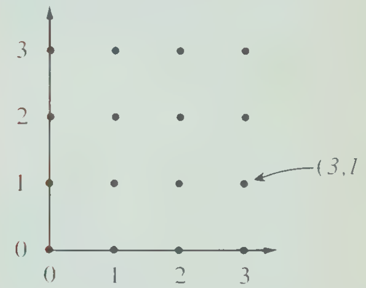
Possibly, though, you may prefer to use other approaches to the grid before presenting such an explanation. For example, you might use the suggestions in the follow-up before your discussion. Or you might present simplified copies of city maps, or talk about locations of desks in the classroom. Whatever approach you use, emphasize both the relation to the number line (but with the jumps in two directions, over and up) and the conventional agreement for the order of writing the co-ordinates.

Using the Exercises
Let the children try the exercises on page 257 on their own. When they are finished, check and discuss the answers together. Utilize the difference in color of the over and up numbers to stress the importance of order when writing the co-ordinates.

Assignments (page 257)*
Minimum: 1–3. Average: 1–3. Maximum: 1–3.

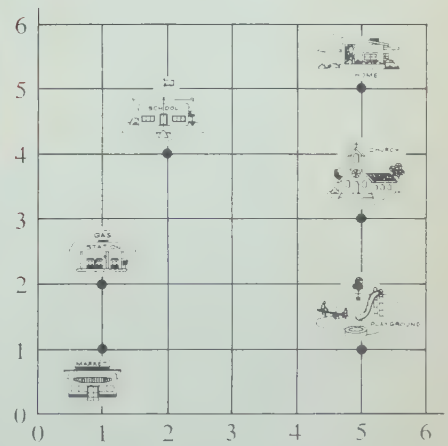
Mathematics
In this first treatment of graphing, only positive number rays are used, with whole-number points labelled.

Having chosen two rays, we can locate a point for any pair of whole numbers, providing we agree which number in the pair refers to which ray. Conventionally, the first number refers to the horizontal line or ray, and the second number refers to the vertical line or ray.



The conventional form for two number rays is shown. The arrow indicates the point for the number pair. The set of points in the co-ordinate plane which have whole-number pairs as co-ordinates are sometimes called the set of *lattice points.*

Follow-up
You might duplicate a "map" similar to the following and ask the children to give a number pair for each pictured location.



Resources for Active Learning
Math Activity Cards, "Addresses," A1, Macmillan.
Nuffield Project: *Graphs Leading to Algebra* 2, "Introduction to Coordinates," pp. 2–11, Wiley. [For this and the next two lessons, these ideas will be very useful.]

Objective

Given a pair of co-ordinates, the child will be able to find the corresponding point on the grid, and conversely.

Preparation

To prepare for this lesson, have children graph a few points on a demonstration grid. For example, say "The co-ordinates are (2,4)" and have a child graph the point. Use just three or four such examples as a brief review and continue into the investigation.

Investigation

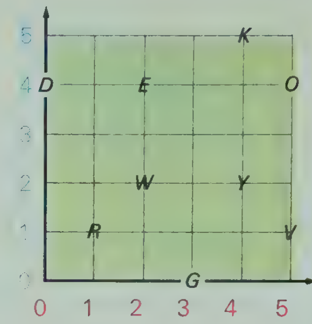
Encourage children to work individually on this investigation. Give guidance to those who seem to be having difficulty. For example, you may need to help them relate a co-ordinate pair to the phraseology used in the previous lesson, such as relating (5, 1) to over 5, up 1. Remind the children that, since the message is "secret," they should all figure it out for themselves without helping each other.

Can you find co-ordinates of points?

Investigating the Ideas

The letters on the graph form a three-word secret message.

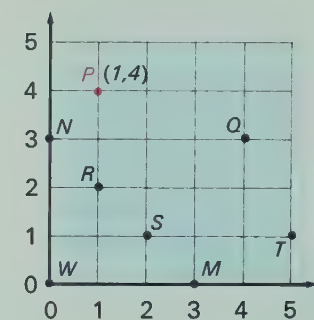
First Word:	(5,1), (2,4), (1,1), (4,2)
Second Word:	(3,0), (5,4), (5,4), (0,4)
Third Word:	(2,2), (5,4), (1,1), (4,5)



? Can you find the secret message? VERY GOOD WORK

Discussing the Ideas

1. The co-ordinates of the point *P* are (1,4).
   - A What are the co-ordinates of point *Q*? (4,3)
   - B Which point has co-ordinates (2,1)? S
   - C Which point has co-ordinates (1,2)? R
   - D What are the co-ordinates of point *W*? (0,0)
   - E Which point has co-ordinates (0,3), *M* or *N*? N



2. A What can be said about the locations of points *N, R, S,* and *M*? They all lie on the same line.
   - B What can be said about the co-ordinates of these points? See Discussion.

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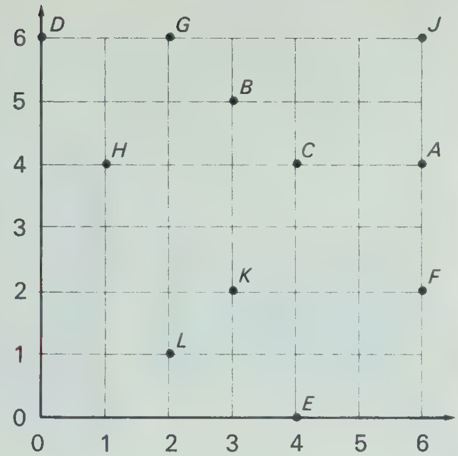
Discussion

In the last lesson, children matched a grid point with a number pair; in this lesson, they are given co-ordinates and must find the matching point. After everyone has agreed on the secret message, work through the discussion exercises. Stress the conventional agreement whereby the "over" number is given first in a pair. Also have the children note the (0, 0) pair for *W*; explain that often a graph is labelled with just one zero. Points *M* and *N* should be used to stress both the difference in the over and up co-ordinates and the fact that it is necessary to use the zero symbol in the writing of co-ordinates to identify points that lie on either axis.

For exercise 2B, you may need to list the co-ordinates (0,3), (1,2), (2, 1), (3, 0) on the chalkboard in order to help the children observe the pattern: as the first number increases by 1, the second number decreases by 1.

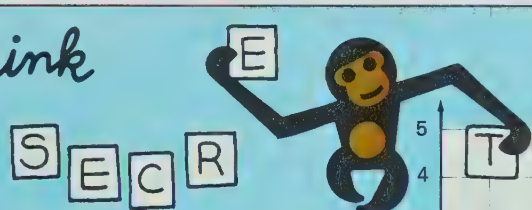
Using the Ideas

1. Answer the questions.
   - A What letter is 6 over and 4 up? A
   - B What letter is 3 over and 5 up? B
   - C What letter has co-ordinates (6,4)? A
   - D What letter has co-ordinates (3,5)? B
   - E What letter is at (4,4)? C
   - F What letter is at (1,4)? H
   - G What letter has co-ordinates (4,0)? E
   - H Co-ordinates (0,6) locate what letter? D
   - I What do you find at (2,6)? G
   - J Co-ordinates (6,2) locate what letter? F

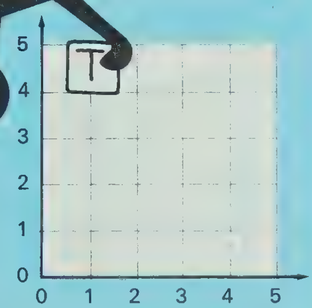


2. What are the co-ordinates of J, K, and L in exercise 1?
   J(6,6) K(3,2) L(2,1)

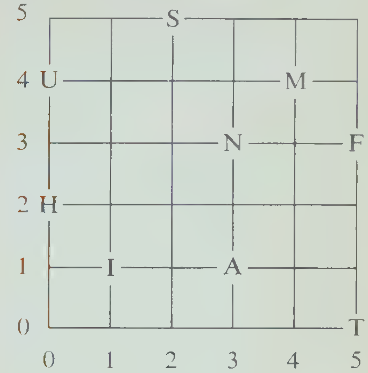
think



Can you write a secret message on a grid, as in the Investigation? Give the co-ordinates for your secret message to a classmate. See if he can find your message.
Answers will vary.



Follow-up
Here is a sample message to accompany page 259.



(4,4); (3,1); (5,0); (0,2); (1,1); (2,5); (5,3); (0,4); (3,3)

(MATH IS FUN)

Resources for Active Learning
Math Activity Cards, "Addresses," B1, Macmillan.
Mathex: Graphing and Probability No. 6 "Co-ordinates," pp. 3–7 (pupil pages 4–14), Encyclopaedia Britannica Publications Ltd. [This reference contains two good games.]

Duplicator Masters, page 53
Workbook, page 95

Using the Exercises
According to the ability of the children, assign these exercises on page 259 as independent work. When the children finish, allow sufficient time for discussion and checking papers.

Encourage all the children to try the *Think* problem. You might want to give the children a secret message you make up, such as the one suggested in the follow-up section.

Assignments (page 259)*
Minimum: 1–2. Average: 1–2.
Maximum: 1–2.

Objective

Given co-ordinates and a co-ordinate system on graph paper, the child will be able to graph the point corresponding to the co-ordinates.

Preparation

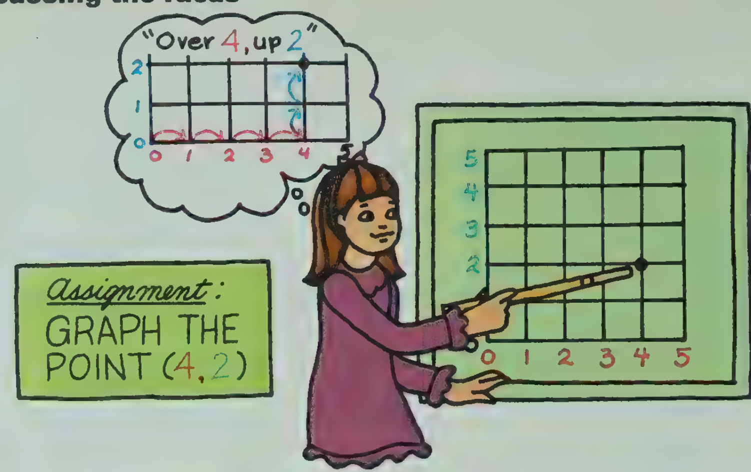
Materials

Graph paper, commercial 1-cm grid or grids prepared on duplicating masters

To prepare for this lesson, write the word *graph* on the chalkboard. Explain that this word can be used with a couple of meanings which they will learn in this lesson.

How do you graph a point?

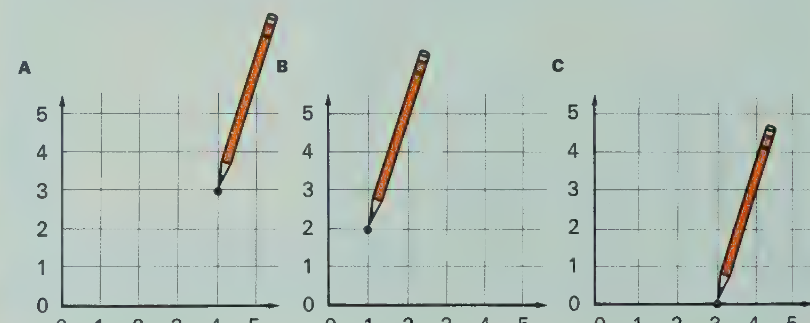
Discussing the Ideas



1. Jill started at 0, counted over 4 and then up 2 to find the location of the point. Could you show her an easier way?

Sample answer: She could look for the point where the "over 4" and "up 2" lines meet.

2. Here are some other points Jill graphed. Explain how she might have counted to decide where to mark each point. What are the co-ordinates of each point?



A: Over 4 and up 3 (4,3)
B: Over 1 and up 2 (1,2)
C: Over 3 and up 0 (3,0)

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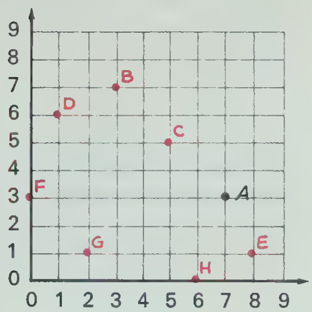
Discussion

Help the children interpret Jill's assignment and her way of thinking about it. They should see that Jill started at 0 and then counted over and up. If children have a hard time finding an easier way, suggest that they think of the grid lines. Help them note that the point where the "over 4" line and "up 2" line meet is the point that Jill graphed. Explain that the point is called the graph of (4, 2). Work through the graphs of exercise 2 similarly. Stress that the numbers next to the number lines make counting unnecessary.

Using the Ideas

1. Label your graph paper like this. Then graph each of these points and write the letter beside it. See graph.

$A(7,3)$	$E(8,1)$
$B(3,7)$	$F(0,3)$
$C(5,5)$	$G(2,1)$
$D(1,6)$	$H(6,0)$

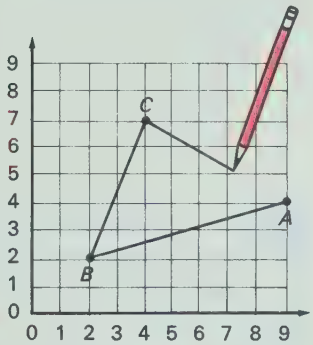


2. **A** Give the co-ordinates of points *A, B,* and *C.* A(9,4); B(2,2); C(4,7)

B What figure is formed? Triangle

C On your paper, graph three other points to form a triangle. Give the co-ordinates of your points. Answers will vary.

D Give the co-ordinates of three points that **cannot** be connected to form a triangle. Any three points that lie in the same line



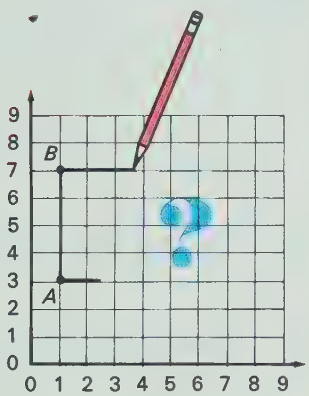
3. **A** Give the co-ordinates of points *A* and *B.* A(1,3) B(1,7)

B Give the co-ordinates of two more points needed to form a square. (5,7) (5,3)

C On your paper, graph four other points to form a square. Give the co-ordinates of your points.

★ **D** Give the co-ordinates of the points needed to form another figure. Graph and name the figure.

Answers for C and D will vary.



Using the Exercises

As you guide the children in labelling their graph paper for exercise 1, explain that each number line is called an *axis*; taken as a pair that intersect at right angles at (0,0), they are called *co-ordinate axes.* They are to *graph* the points whose co-ordinates are given. If possible, use an overhead projector and lead the children in drawing and labelling the axes. You might want also to guide the children in graphing several of the points. If you choose to have the children work independently, give individual help as needed. In exercises 2 and 3, parts C and D, the children will find it easier to draw the points in position on the grid and then find the corresponding co-ordinates. Starred exercise 3D is for the more capable children. They might extend this exercise and graph several figures giving the co-ordinates for each. You might then duplicate some of these for others in the class to use for finding co-ordinates.

Assignments (page 261)*
Minimum: 1–2. Average: 1–3C.
Maximum: 1–3.

Mathematics

The previous lesson emphasized the use of number pairs to locate points in the plane. This lesson asks the children to graph, or picture, points determined by certain number pairs. This background is vital for work with more advanced graphing concepts in which *sets* of number pairs are pictured in the plane.

Connecting pairs of points with segments can help the children see that patterns may exist in a given set of number pairs. In the future, observing and drawing these patterns from special sets of number pairs will constitute the most important mathematical use of graphing.

Each number line is an *axis*. We locate points on the plane with a pair of axes that intersect in a right angle at (0, 0). Such pairs of axes are called *co-ordinate axes.* On a grid composed of lines parallel to each axis, we *graph* points, or locations, in space. Note that the word *graph* can function as either a noun or a verb. As a noun, it indicates (1) the picture of a point or of a set of points, or (2) the entire co-ordinate axes showing the number lines and the points involved. As a verb, *graph* means to mark a point, or to picture, each number or pair of numbers in a given set.

Follow-up

You might suggest that the children graph the following sets of points on different grids.

A (7, 0), (7, 1), (7, 2), (7, 3), (7, 4)
B (0, 3), (1, 3), (2, 3), (3, 3), (4, 3)
C (0, 0), (1, 1), (2, 2), (3, 3), (4, 4)

Graph these sets on one grid.

D (8, 2), (7, 3), (6, 4), (5, 5)
E (5, 2), (6, 3), (7, 4), (8, 5)

Resources for Active Learning

Developmental Math Cards, E¹12, E²6, Addison-Wesley.

Mathex: Matching and Graphing No. 1, "Dotto Game (co-ordinates)," pupil pages 51–52, Encyclopaedia Britannica Publications Ltd.

PAGES 262–263

Objective

Given co-ordinates and a co-ordinate system, the child will be able to graph the points and connect them with segments to make a picture.

Preparation

Materials

graph paper, 1 cm grid

It would be appropriate in this lesson to begin immediately with the investigation. However, if you prefer, review the vocabulary which has been introduced in the preceding lessons: graph, co-ordinates, co-ordinate axes, axis.

Investigation

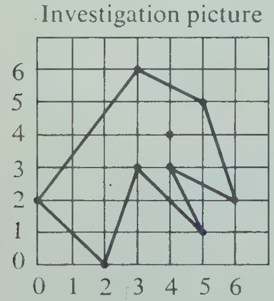
Read the directions with the children. Tell them to study how the teepee was made and then to graph and connect the points given in the text. If necessary, help the children in making the co-ordinate axes. On the chalkboard you might list other sets of points for the children to graph and connect, such as the following:

(2, 6), (3, 5), (6, 4), (4, 2), (3, 0), (0, 0), (2, 5), (2, 6) (Scottish terrier)

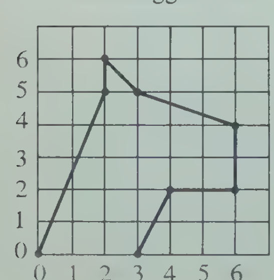
(1, 2), (3, 2), (3, 3), (4, 3), (4, 4), (5, 4), (5, 3), (6, 3), (6, 2), (9, 2), (8, 0), (2, 0), (1, 2) (Ship)

Encourage the children to take care in graphing the points; if they graph the point incorrectly, the picture will be distorted.

Investigation picture



Other Suggestions

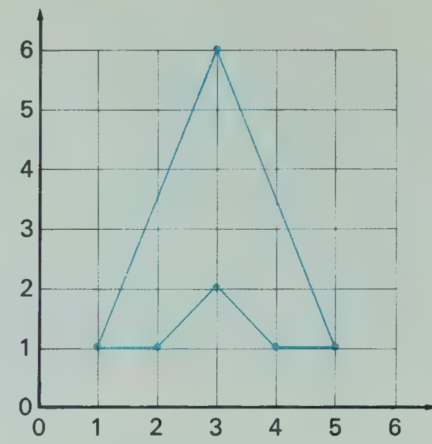


Scottish terrier

How can you make point pictures?

Investigating the Ideas

This teepee was made by graphing and connecting points with these co-ordinates.
(1,1) → (3,6) → (5,1) → (4,1) → (3,2) → (2,1) → (1,1)
The arrows show the order of the points.



? Can you make a picture by graphing and connecting these points in order? Try it.
(0,2) → (3,6) → (5,5) → (6,2) → (4,3) → (5,1) → (3,3) → (2,0) → (0,2)

See Investigation.

Discussing the Ideas

1. Could the points for the teepee be graphed and connected in a different order? Explain. See Discussion.

2. What point do you need to complete the picture you made in the Investigation? What are its co-ordinates? See Discussion.

3. Can you figure out the co-ordinates of points that will give you one of these familiar geometric figures?
   A square B parallelogram C rhombus

   Answers will vary.

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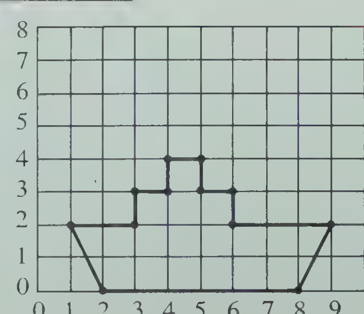


Ship

Discussion

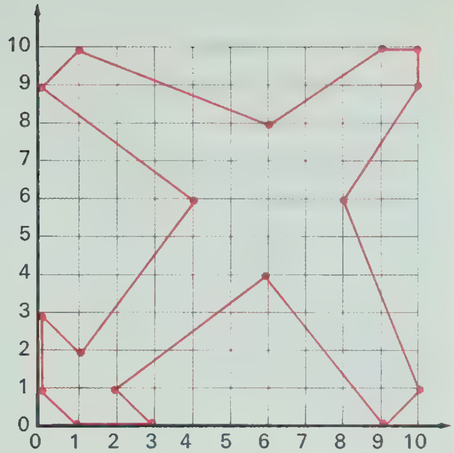
Besides giving children practice in graphing, this lesson should help them realize the importance of order in graphing. The children should include in their explanation of question 1 the idea that, if a different starting point is chosen but the same order is kept, the picture will not change. But if the order in which one point is connected to another is changed, the picture is changed. To be sure they see this, you might have them connect the points in this order:

(3, 2), (5, 1), (4, 1), (2, 1), (1, 1), (3, 2), (3, 6)

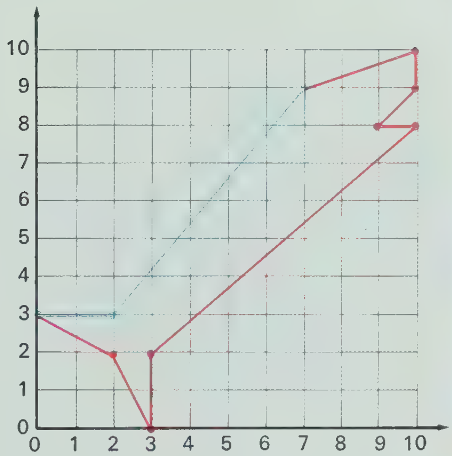
In exercise 2, children may differ

Using the Ideas

1. If these remaining 6 points are graphed and connected, the airplane will be finished.
(9,0) → (6,4) → (2,1) → (3,0) → (1,0) → (0,1)
Copy and complete the picture on your graph paper.



2. The first three points have been graphed and connected. Copy and complete the picture on your graph paper.
(0,3) → (2,3) → (7,9) → (10,10) → (10,9) → (9,8) → (10,8) → (3,2) → (3,0) → (2,2) → (0,3)



★ **3** Draw a picture by connecting points on your graph paper. Give the co-ordinates, in order, to a classmate and see if he can draw the picture. Answers will vary.

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in their opinions as to which point should be the "eye," but most will probably agree that either (5, 4) or (4, 4) would be suitable.

Using the Exercises
Have available an adequate supply of graph paper and encourage the children to copy and complete the pictures. Remind them to make and carefully label a pair of co-ordinate axes for each picture. Also some may enjoy coloring the pictures or designing them further. Starred exercise 3 challenges the children's creativity.

Assignments (page 263)*
Minimum: 1. Average: 1–2.
Maximum: 1–3.

Follow-up
The children might enjoy making figures on the grid and exchanging with classmates their lists of co-ordinates to connect.

If the children show a great interest, encourage them to make grid pictures on their own, or to make a list of points according to some pattern of numbers and graph them to see if any figure emerges. For example, suggest one of the following patterns.

A (1, 2), (2, 4), (3, 6), (4, 8), (5, 10), (10, 5), (8, 4), (6, 3), (4, 2), (2, 1), (1, 2)

B (1, 2), (3, 4), (5, 6), (7, 8), (9, 10), (8, 7), (6, 5), (4, 3), (2, 1), (1, 2)

C (1, 10), (2, 9), (3, 8), (4, 7), (5, 6), (6, 5) (7, 4), (8, 3), (9, 2), (10, 1), (1, 1), (1, 10)

Resources for Active Learning
Math Activity Cards, "Plotting Points," B12, Macmillan.

Duplicator Masters, page 54

PAGES 264–265

Objective

Given the graph of one half of a symmetric figure, the child will be able to find the points and give the co-ordinates of the other half of the figure.

Preparation

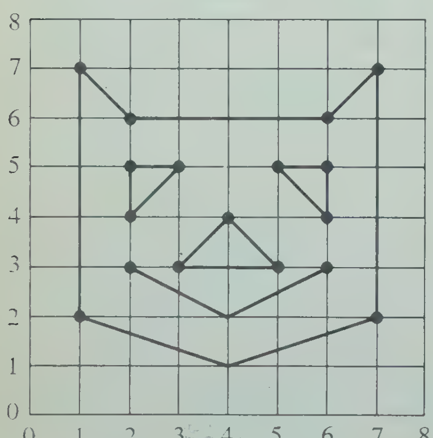
Materials

graph paper, 1-cm grid

If some children are still having difficulty finding points and giving co-ordinates, spend a few minutes reviewing how to use the co-ordinate axes, which number to write first, and the role of the grid lines. If you feel that the majority of children do not need such a review, write the word *symmetry* on the chalkboard and briefly recall its meaning and remind the children that they have previously studied symmetric folds of a square, circle, etc., in Chapter 8.

Investigation

Allow the children freedom to approach this investigation in whatever manner they choose. Some may copy the half that is shown and then figure out what points to use to make the other half. Some may try to figure out the hidden half before copying, using the centre points (4, 4), (4, 3), (4, 2), and (4, 1) as guides.



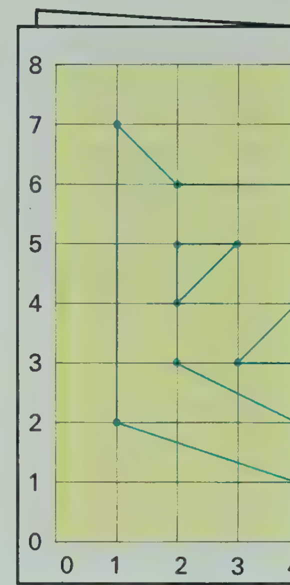
If some children finish very quickly challenge them to fold their graph paper and draw a half of a figure which they think would be symmetric and then find the points for the hidden half.

Let's explore symmetry on graph paper.

Investigating the Ideas

The paper has been folded so that only half of a symmetric figure can be seen.

Copy this part of the figure on your graph paper.



? Can you show the other half of the picture on your graph?

See Investigation.

Discussing the Ideas

1. A What are the co-ordinates of the point at the tip of the cat's ear shown above? (1,7)
   B What are the co-ordinates of the point at the tip of the cat's other ear? (7,7)

2. Pick other points and give their co-ordinates. Then give the co-ordinates of the matching points in the other half of the picture. (1,2) & (7,2); (2,3) & (6,3); (2,4) & (6,4); (2,5) & (6,5); (2,6) & (6,6); (3,3) & (5,3); (3,5) & (5,5)

3. Where is the line of symmetry of the figure? Along the "over 4" line

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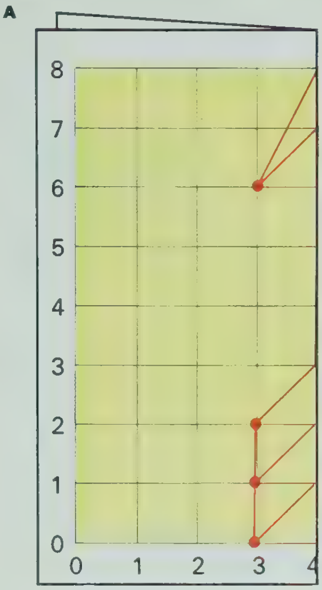
Discussion

As you discuss these exercises, help the children notice what numbers in the co-ordinate pairs are the same. For example, the left ear is point (1, 7), the right ear is (7, 7). Similarly, the left tip of the mouth is (2, 3) and the right tip is (6, 3). In both cases the second number, the "up" number, is the same in both pairs of the matching points. Ask the children whether they think this is always the case for the matching points of symmetric figures. While discussing exercise 3, have them note the centre points of the figure: (4, 4), (4, 2), and (4, 1). Help them see that an imaginary line may be thought to be drawn on the grid line where the "over" number is 4. Since this line is the centre line (that is, the matching points of both halves are equally distant from it), it is called the *line of symmetry*.

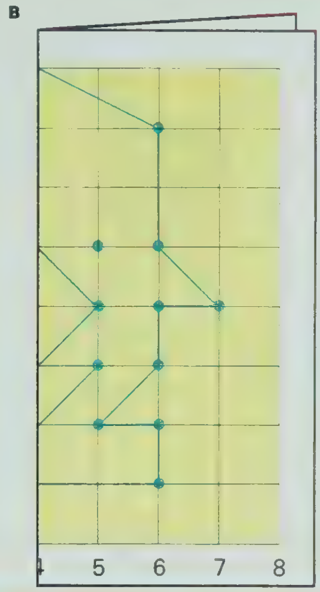
Using the Ideas

1. Use your graph paper to show what each symmetric picture will look like when the paper is unfolded. See Using the Exercises.

A



B



2. A Give the co-ordinates of each of the four points marked in 1 A.
(3,0) (3,1) (3,2) (3,6)

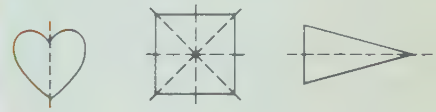
B Give the co-ordinates of four matching points in the other half of the picture for 1 A.
(5,0) (5,1) (5,2) (5,6)

★ 3. Use graph paper to make a symmetric picture of your own. Then fold it in half and see if a classmate can draw the complete picture.
Answers will vary.

You think about 10 hundreds.
100 tens will do.
That's enough to find my name.
You need no other clue.
WHO AM I? 1000

Mathematics

As was noted in the introduction to Chapter 8, the idea of line symmetry, taken intuitively, is a rather simple concept. A figure has a line of symmetry if it can be "folded" along that line in such a way that one half of the figure falls exactly on the other half. When a figure has one or more lines of symmetry it is said to be symmetrical.

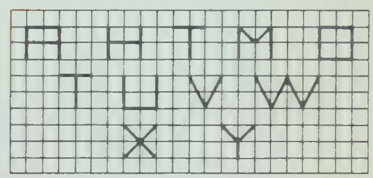


A more sophisticated mathematical definition of symmetry is not needed to explore many interesting concepts associated with this idea.

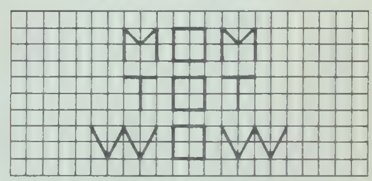
Follow-up

Starred exercise 3 may be extended as a follow-up activity. Or you might have children print capital letters on graph paper to see how many symmetric letters they can find. Some may even find some symmetric words.

Symmetric letters



Symmetric words

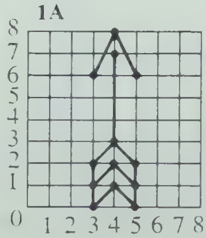


Using the Exercises

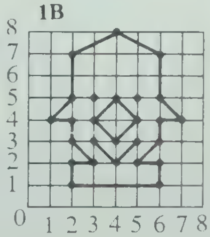
Let the children work independently on the exercises on page 265. If they draw the matching points attentively, they will enjoy watching the figures emerge. Check exercise 2 carefully; one of the main goals of this chapter is for the children to develop the ability to give the co-ordinates of points on a graph.

Answers, exercise 1, page 265

1A



1B



Assignments (page 265)*

Minimum: 1–2. Average: 1–2.
Maximum: 1–3.

Resources for Active Learning

Developmental Math Cards, D²3 Addison-Wesley.

Franklin Series: *Mirror Magic*, "Symmetry," pp. 73–83. Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Mirror Cards, Teacher's Guide and Materials, Webster, McGraw-Hill. [If you have not already used these, now would be a good time to begin.]

[See also those resources for symmetry listed in Chapter 8.]

Workbook, page 96

PAGES 266–267

Objective

Given a figure on a graph and directions for a translation, the child will be able to "move" the figure from one position on the graph to another.

Preparation

Materials

graph paper, 1 cm grid

Unless the children are having difficulty with some aspect of the topics presented previously in this chapter, proceed without a specific preparation.

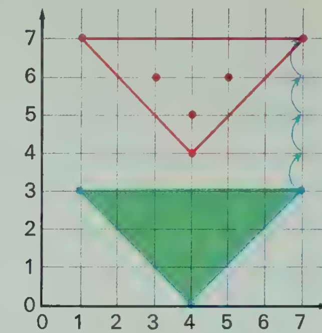
Investigation

Encourage the children to follow the directions in the text without your guidance. Remind them to label the co-ordinate axes carefully. Observe the children's work and when you see errors, encourage children to study the directions again to correct the errors. Let those who finish quickly continue on to the exercises on page 267. However, when all children have finished the investigation translation, direct them to return their attention to it and spend a few minutes in discussion.

How can figures be moved on a graph?

Investigating the Ideas

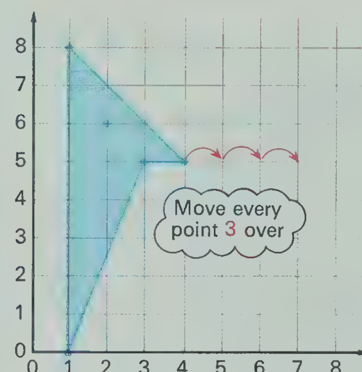
Jon

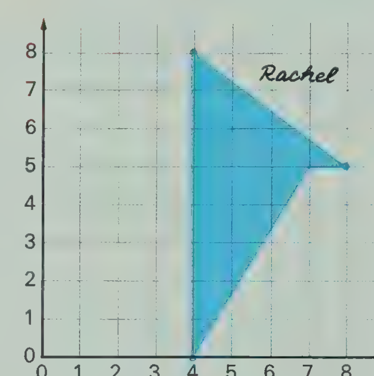


? Can you show on your graph paper where the figure will be after Jon moves all the points and connects them?

Discussing the Ideas

1. Look at the two figures. Did Rachel follow the directions?
   NO, she moved points (3,5) and (4,5) 4 over.





2. Can you show on your graph paper how Rachel should have drawn the figure? See above.

Discussion

This lesson develops at a simple level the idea of geometric translation. As the children discuss what they have done in the investigation, help them realize that each point is moved the same distance along the parallel grid lines. Consequently every new line of the figure is parallel to its corresponding line in the original figure. Depending upon the maturity of your class, you may or may not use the term translation with the children; it is not necessary to do so. After discussing and completing the translation for exercises 1 and 2, it would be helpful for the capable children to make a chart showing the related points for the translations in both the investigation and the discussion sections. These charts would be as follows:

Jon's figure

(1, 3)	→	(1, 7)
(4, 0)	→	(4, 4)
(7, 3)	→	(7, 7)
(3, 2)	→	(3, 6)
(4, 1)	→	(4, 5)
(5, 2)	→	(5, 6)

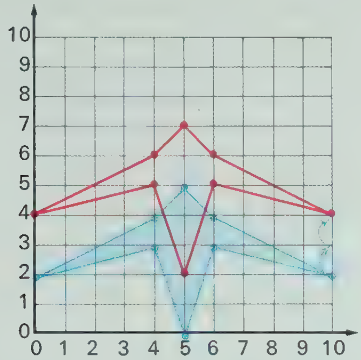
Rachel's figure

(1, 0)	→	(4, 0)
(1, 8)	→	(4, 8)
(3, 5)	→	(6, 5)
(4, 5)	→	(7, 5)
(2, 6)	→	(5, 6)

Using the Ideas

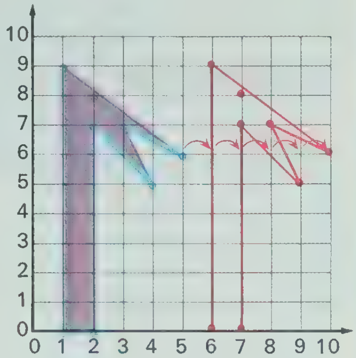
Use the moves given and show the final position of each picture on your graph paper.

1.



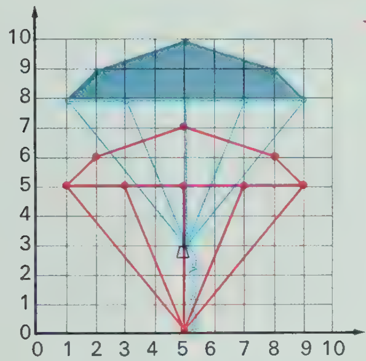
Move every point 2 up.

2.



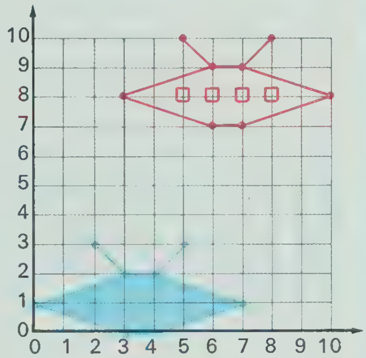
Move every point 5 over.

3.



Move every point 3 down.

★ **4.**



Move every point 3 over and 7 up.

★ **5.** Make a figure of your own on graph paper and then move it 3 over **and** 2 up. Answers will vary.

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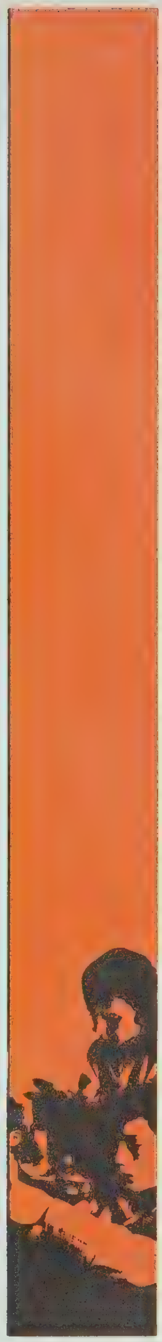


Mathematics
Technically, a translation is a kind of transformation of the points in the plane that preserves distances. However, as mentioned in the introduction to this chapter, it may be thought of simply as a distance-preserving movement in which all points of a given figure are moved the same distance and in the same direction.

Follow-up
The children would enjoy seeing figures that they have graphed displayed around the room. Have them show not only a figure they have made but also the corresponding co-ordinates. Some may show a figure in an original position and the same figure after it has been "moved" to a different position on the grid.

Using the Exercises
You may choose to treat this section as an extension of the investigation. Encourage all to do the work carefully or the figure will be distorted. Suggest that those who do this work quickly and easily either make other figures and give a rule for moving them or show a chart of related points as suggested in the discussion.

Assignments (page 267)*
Minimum: 1–2. Average: 1–3.
Maximum: 1–5.

PAGES 268–269

Objective

Given a function rule, the child will be able to write a number pair for the input and output numbers and graph the point corresponding to these co-ordinates.

Preparation

Materials

graph paper, 1-cm grid

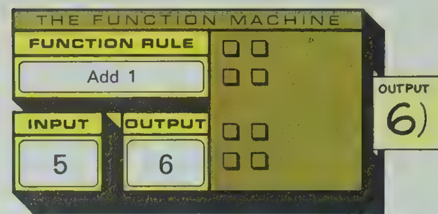
If you think it necessary, review the function rule orally by giving a rule and several input numbers for which the children are to respond with the appropriate output number. However, you might choose to begin immediately with the investigation.

Investigation

It would be appropriate to have the children work in groups for this investigation. But first direct them to label a pair of axes on graph paper, using the text illustration as their model. Suggest that they record their input and output numbers in a function table, as on page 269. Some groups might need the guidance of a model exhibited on the chalkboard to show how they might set up both their graph and the table. For those who finish quickly, write one or two other function rules on the chalkboard, such as "Add 4" or "Add 2" and ask them to prepare function tables for those rules.

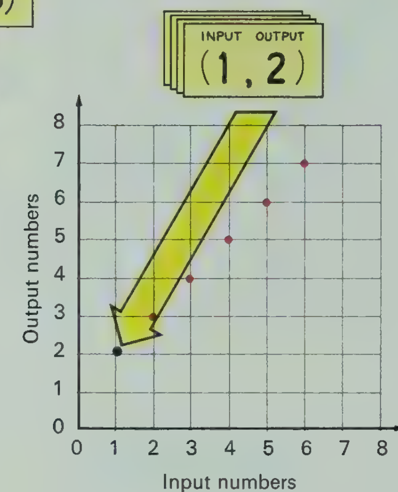
Can input-output pairs be shown on a graph?

Investigating the Ideas



This machine puts out an **input-output** card each time it operates.

The point for the first card is shown on the graph.



Can you write the number pairs for 5 more input-output cards and show the point for each pair on a graph?

Sample pairs: (2,3), (3,4), (4,5), (5,6), (6,7); see graph above.

Discussing the Ideas

1. If the input number were 0, where would you mark the point on your graph? (0,1)
2. Did all the points you graphed lie in a straight line? Yes
3. Do you think the points would lie in a straight line no matter what rule was used? See Discussion.

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Discussion

Although children were directed in the investigation to choose their own input numbers, many will have the same pairs. Therefore it would be helpful to expand the pattern of exercise 1 by asking what the output number is if the input number is 1, 2, 3, 4, and so on. The results of exercise 2 might surprise some children when they discover that their numbers do indeed form a straight line.

Discussion of the third question may yield a variety of opinions. The rules children use in this lesson do result in straight-line graphs, but other rules (such as "multiply the input number by itself") may not. It would probably be best simply to point out that all the rules used in this lesson *do* yield a straight line, even though the direction of lines may vary. (Note in this regard the direction of the graph for exercise 2C on page 269.)

You might want to cover a demonstration grid with acetate and select a different colored felt pen for each set of co-ordinates, to show that you can graph several functions on one pair of axes.

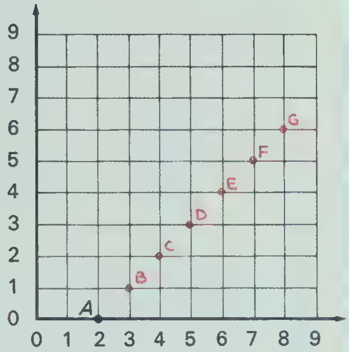
Using the Ideas

1. Use the function table to write the co-ordinates for points B through G. Then graph the point for each of the co-ordinates on your graph paper. Point A is graphed correctly.

Function Rule

Subtract 2

Input	Output		Coordinates	Point
2	0	→	(2, 0)	A
3	1	→	(3, ___) 1	B
4	2	→	(4, ___) 2	C
5	3	→	(___, ___) 5, 3	D
6	4	→	(___, ___) 6, 4	E
7	5	→	(___, ___) 7, 5	F
8	6	→	(___, ___) 8, 6	G



2. Think about input-output cards and graph the points for each table. See Answers.

A Function Rule

Subtract 1

Input	Output	
1	___	0
2	___	1
3	___	2
4	___	3
5	___	4
6	___	5

B Function Rule

Multiply by 1

Input	Output	
0	___	0
1	___	1
2	___	2
3	___	3
4	___	4
5	___	5

C Function Rule

Subtract from 7

Input	Output	
0	7	
1	6	
2	5	
3	4	
4	___	3
5	___	2

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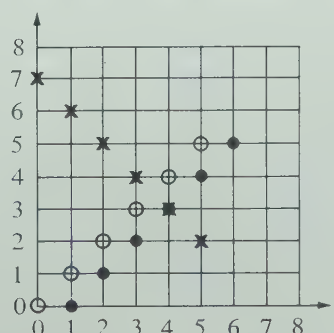
Mathematics
Each function rule produces a set of number pairs, and the graph of these pairs reveals a unique pattern. This idea of picturing or graphing a special set of number pairs associated with a function is one of the most important in mathematics. Notice that the pattern displayed in the graph of each function in this lesson is a straight line. Such functions are called *linear* functions, and they receive much attention in elementary algebra. More complicated functions and relations may give such patterns as parabolas, hyperbolas, circles, and ellipses.

Follow-up
Tic-tac-toe, adapted to a co-ordinate system, can be great fun as a class activity. Using only 0 and the positive integers 1 through 5, prepare a co-ordinate system on a large chart that can be covered with acetate or on a separate transparency that can be slipped under the roll of an overhead projector or on the chalkboard.

Divide the class into two evenly matched teams. Choose a number between 1 and 20 and ask the leaders of each team to try to guess the number. Let the one who comes the closest take the first turn, and designate his group as the "×" team. Call the other team the "○" team. The object of the game is for one team to get five ×'s or ○'s in a row, column, or diagonal by naming the co-ordinates of each × or ○. One at a time, in turn, the members of each team should have a chance to name any number pair they wish. It is usually wise to insist that each child make his own choice, whether it is the best choice or not, without the help of anyone else on his team. Strategies to block the other team, while continuing to build a string of ×'s or ○'s, will probably arise as the children play the game. Eventually, they should become skillful enough to make every contest a draw.

Using the Exercises
Use exercise 1 as a model for setting up parts of exercise 2. Distribute enough graph paper so that the children can make a separate graph for each part of exercise 2. When the children have finished, discuss again the question on page 268. After graphing all parts of exercise 2, they should see that the pattern is a straight line for all of their function rules but that the direction of each line is unique.

Answers, exercise 2
2. **A** Points are graphed with ●.
B Points are graphed with ○.
C Points are graphed with ×.



Assignments (page 269)*
Minimum: 1. Average: 1–2.
Maximum: 1–2.

Workbook, page 97

PAGES 270–271

Objective

Given an incomplete vertical bar graph and relevant information, the child will be able to complete the graph.

Preparation

Materials

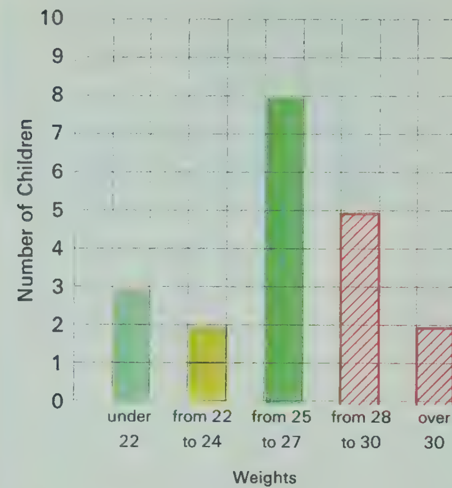
graph paper, 1-cm grid

Because of the nature of the investigation, no formal preparation is necessary.

Investigation

Guide the children in reading the text and studying the graph and chart. Minimize your introductory discussion so that children have the opportunity to make full use of their own potential for observation and discovery. Since the weights in the chart are listed in order, most of the children will be able to see the relationship between the weights listed and the lengths of the bars on the graph. If any children do have difficulty with this, give them individual help rather than discussing it with the entire class.

Investigating the Ideas



Information			
Amy	20	Kent	27
Beth	21	Lisa	27
Carl	21	Mike	27
Don	23	Nan	28
Eric	24	Orin	29
Fran	25	Paul	29
Gail	26	Ray	29
Hal	26	Sara	30
Ivan	27	Ted	31
Jill	27	Val	32

Lisa colored each bar on her graph paper to show how many children had the weight given below the bar.

? Can you tell how to make the last two bars to finish Lisa's graph? See graph.

Discussing the Ideas

1. Do you think the bar graph is a good way to show the information about weights? Why? See Discussion.

2. How can you tell from the bar graph how many students weigh from 25 to 27 kilograms? See Discussion.

3. Collect information about the weights of children in your class. How would the bar graph for your class look? See Discussion.

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Discussion

When the children have completed the investigation, spend a short time discussing their answers. Relate the graph to the co-ordinate axes, with which the children are by now familiar. Point out that the "horizontal axis" has been labelled in such a way that all the weights in the chart can be included.

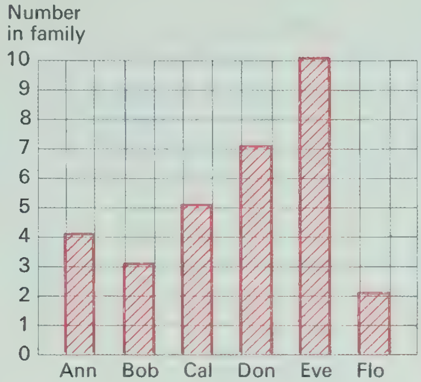
In discussing exercises 1 and 2, be sure the children observe that, since the numbers in the chart are in order, one may easily count on the chart how many children are within a certain range of weight. Also explain that a graph such as this one is called a bar graph.

Exercise 3 may be handled in a variety of ways. You might have the children gather the information regarding their weights in advance, or you might obtain a scale and have them weigh themselves and chart their weights during the investigation. This exercise might also be treated as a group investigation project. No matter how the data is collected, however, the important point is for the children to have experience in building the graph.

Using the Ideas

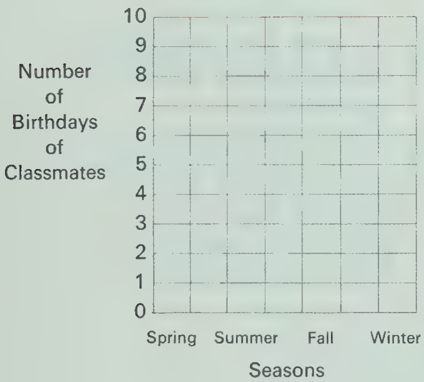
1. Draw and color bars on graph paper to show the number of people in the family of each student.

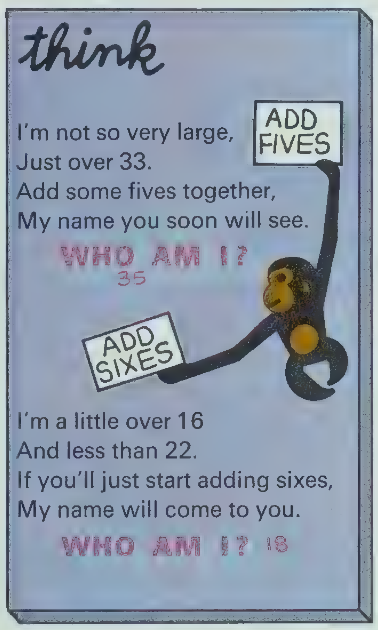
Name	Number of people in family
Ann	4
Bob	3
Cal	5
Don	7
Eve	10
Flo	2



2. Make a bar graph like the one in exercise 1 for eight students in your class. Answers will vary.

3. Make a bar graph using the idea suggested by the picture. Answers will vary.





Using the Exercises

Allow the children sufficient time to make at least one of the graphs suggested. You might choose to have them work in groups. For exercise 3 you will have to have the children write their birthdays on a chart so that all can use the information. In this case it will be necessary to tell the children what dates are in which season, unless you tally the totals yourself.

Follow-up

Depending on your time schedule and the interest of the children, you might have the children do more work with the bar graph. You might suggest that they use their geography studies to find material for a bar graph. For example:

Alternatively, they might use information pertaining to school, such as the number of children in each class or in each grade.

Resources for Active Learning

Developmental Math Cards, G²15, Addison-Wesley.

Math Activity Cards, "Shadows" and "Bar Graphs," A6,7, Macmillan.

Mathex: Graphing and Probability No. 6, "Bar and Line Graphs," pp. 1–3 (pupil pages 1–3), Encyclopaedia Britannica Publications Ltd.

Mathex: Matching and Graphing No. 1., "Further Stages in Graphing," pp. 20–25 (pupil pages 26–40), Encyclopaedia Britannica Publications Ltd.

Assignments (page 271)*
Minimum: 1. Average: 1–2.
Maximum: 1–3.

Objective

Given the four quadrants of the co-ordinate plane, the child will be able to give the co-ordinates of points using both positive and negative numbers.

Preparation

To prepare for this lesson, draw the quadrant and co-ordinate axes that the children have been using. Point out the rays extending to the right and up from the point (0, 0). Also, review the use of the grid lines for locating a point. Then tell the children that in the investigation for this lesson they will find out how we graph points that cannot be located on the grids that they have used up to now.

Investigation

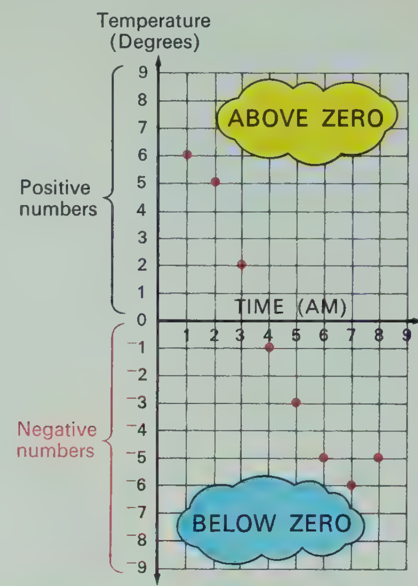
Guide the children in reading the text and studying the illustration for the investigation. Then ask them to observe what the numbers on the horizontal and vertical axes represent (hours of the day and temperatures, respectively). Call particular attention to the fact that this graph differs from those the children have studied previously in that the vertical axis is extended below the horizontal axis to show below-zero temperatures. Have the children attempt on their own to make a record of the information shown on the graph, but if any have difficulty getting started, you may wish to suggest that they use a chart with two column headings: *Hour of day* and *Temperature*. Others may use the same form that they used earlier for giving co-ordinates: (1, 6), (2, 5), (3, 2), etc. Later, they can be given the opportunity to explore methods for recording the information more clearly.

How can we use negative numbers in graphing?

Investigating the Ideas

Alice made this graph to show the temperature from 1:00 A.M. to 8:00 A.M. on a very cold day. She used **negative** numbers to show temperatures **below** zero and positive numbers to show temperatures **above** zero.

For negative numbers, we read:
Negative one, negative two, negative three, . . .



? Can you record the temperatures shown for each of the hours on the graph? See Investigation and Discussion.

Discussing the Ideas

1. A What was the temperature at 1:00 A.M.? 6°
   B What was the temperature at 2:00 A.M.? 5°
   C When was the temperature 2 degrees? 3:00 A.M.
   D What do the co-ordinates (4, −1) tell you? At 4:00 A.M. it was 1° below zero.
   E At what time was the temperature 3 degrees below zero? 5:00 A.M.
   F What is the coldest temperature shown on the graph? $^-6$°
   G Look at the graph and **guess** what the temperature would be at 9:00 A.M. See Discussion.
2. Can you think of some other ways that negative numbers might be used? See Discussion.

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Discussion

Allow time for a discussion of the various ways children may have chosen for recording the information given on the graph presented in the investigation. Let a few volunteers whose methods differed show them on the chalkboard. Some children may simply have presented the data as co-ordinates, some may have used a vertical, tabular presentation, and a few might even have used a horizontal table something like this:

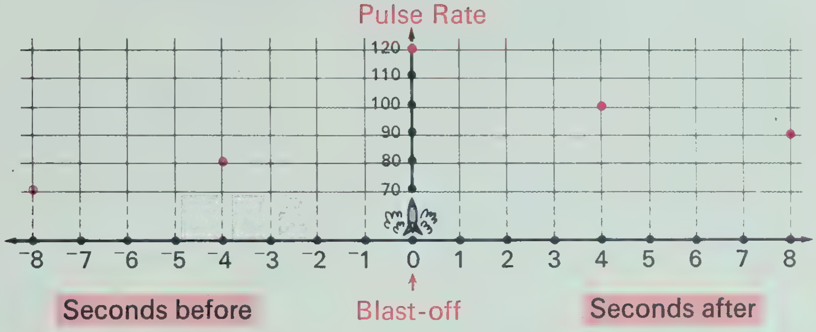
Time (A.M.)	1	2	3	4	5	6	7	8
Temperature	6	5	2	$^-1$	$^-3$	$^-5$	$^-6$	$^-5$

Encourage children to use their own record to answer the questions in exercise 1. As you discuss the questions, stress the proper reading of the negative numbers ("negative one," "negative two," etc.). For part G, it may be argued that the temperature would probably rise at the same rate at which it fell in the preceding two hours, and thus, at 9 A.M. would be −3. Note, however, that this is just a "best guess" based on the information available.

For exercise 2, children may suggest using negative numbers to show how much money someone owes, to indicate floors below

Using the Ideas

1. Ted made this graph to show the changes in pulse rate of an astronaut around blast-off time. He used **negative** numbers for the seconds **before** blast-off and positive numbers for the seconds **after** blast-off.



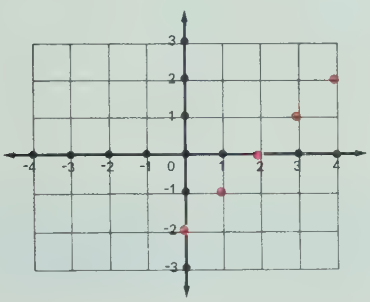
A What was the pulse rate 8 seconds before blast-off ? 70 What are the co-ordinates of that point ? (-8,70)

B Give the co-ordinates for the pulse rate 4 seconds before blast-off. (-4, 80)

C What was the pulse rate at blast-off ? Give the co-ordinates of that point. 120; (0,120)

D Give the co-ordinates for each of the other points on Ted's graph. (4,100), (8,90)

2. Give the missing numbers in the table. The graph may help you.



Function Rule: **Subtract 2**

Input	Output
4	2
3	1
2	0
1	[blank] -1
0	[blank] -2

ground level in some buildings, to show distance below sea level on maps, to show scores below zero in certain games, etc.

Using the Exercises

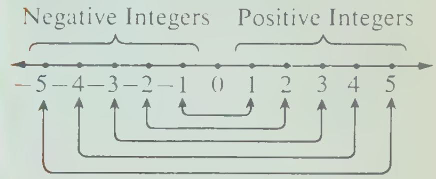
You will probably want to use these exercises as a basis for further demonstration and discussion. However, let the children do as much of their own thinking as they can. Emphasize the extension of the horizontal axis to the left, and relate this idea to the downward extension of the vertical axis which was presented in the investigation.

Assignments (page 273)* —
Minimum: 1, oral. Average: 1–2.
Maximum: 1–2.

Mathematics

There are two important mathematical concepts presented in this lesson. The first is the introduction of negative integers; the second is the use of these integers to extend graphing to the entire plane, instead of confining it to just one fourth of the plane.

One convenient way to think of the negative integers is to consider them "reflections" of the whole numbers on the number line.



The preceding diagram shows how to pair each negative integer with a corresponding whole number (positive integer). As indicated on the number line, we consider the negative integers to be those less than zero. One of the most elementary mathematical uses of this set of numbers is in solving the following type of equation.

$$5 + n = 0$$
$$n = {}^{-}5$$

Integers are used in this lesson as co-ordinates to indicate various locations on the plane.

Follow-up

If the children enjoyed the tic-tac-toe game suggested in the follow-up section on page 269 and have become skillful at it, consider extending the co-ordinate axes to include the negative integers. Start with a grid like the one on the bottom of page 273. Remind the children that they must distinguish between positive and negative integers by giving only the number when the integer is positive and by using the term "negative" when the integer is negative.

Resources for Active Learning

Mathex: Matching and graphing No. 1, "Graphs Leading to Integers," pupil page 41, Encyclopaedia Britannica Publications Ltd.

Nuffield Project: *Graphs Leading to Algebra* 2, "Coordinates . . . Integers," pp. 37–43, Wiley.

PAGES 274–275

Objectives

The child will demonstarate his ability to work with the concepts presented in this chapter.

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

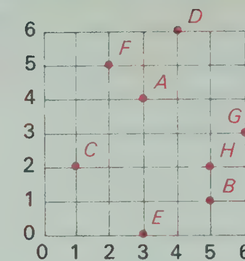
Materials

graph paper (2 sheets per child)

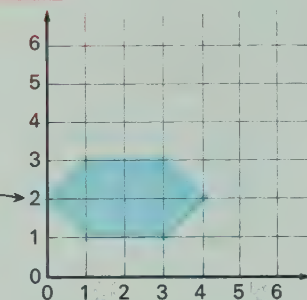
Review any topic which seemed to cause particular difficulty during the study of this chapter. For example, review the relation of function tables and graphing input and output number pairs. You might also use a short, brisk drill to review basic multiplication facts.

Reviewing the Ideas

1. What letter is 1 over and 2 up ? C
   The point lettered *G* has what co-ordinates ? (6,3)
   What are the co-ordinates for point *A* ? (3,4)
   What is the letter for the point with co-ordinates (2,5) ? F
   Are the co-ordinates of point *E* (3,0) or (0,3) ? (3,0)



2. Graph the points in the order listed and connect them to form a picture. See Discussion.
   A (3,0) → (3,1) → (2,1) → (1,3) → (2,2) → (4,2) → (6,4) → (7,3) → (6,3) → (5,1) → (4,1) → (4,0)
   B (2,0) → (2,1) → (1,3) → (2,5) → (2,6) → (3,5) → (4,5) → (5,6) → (5,5) → (6,3) → (5,1) → (5,0)
3. On graph paper, show how the figure would look for each move. See Discussion.
   A Move every point 2 over.
   B Move every point 3 up.



4. Complete the function tables and graph the input-output pairs. See Discussion.

A Function Rule: **Subtract 2**

Input	Output	
8	6	
7	5	
6	▮	4
5	▮	3
4	▮	2

B Function Rule: **Multiply by 2**

Input	Output	
0	0	
1	2	
2	▮	4
3	▮	6
4	▮	8

C Function Rule: **Add 0**

Input	Output	
1	▮	1
2	▮	2
3	▮	3
4	▮	4
5	▮	5

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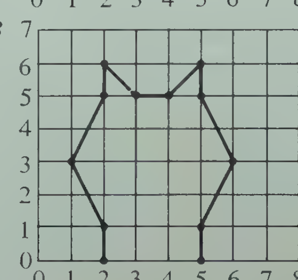
Discussion

The exercises on page 274 review the basic concepts treated in this chapter. You might choose to use them as an evaluation of the children's understanding or as a basis for a teacher-directed review. Provide the children with sufficient graph paper for these exercises. Children may need guidance in drawing their co-ordinates axes for exercises 2, 3, and 4; a pair of axes which extends 7 units is necessary for exercise 2A, but 6 units is sufficient for exercises 2B, 3A, and 3B. Exercise 4A requires a graph which includes 8 units.

Answers, exercises 2–4, page 274

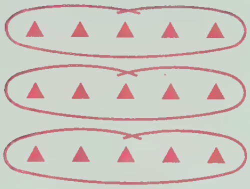
2A

2B

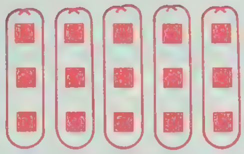


Keeping in Touch with Addition, Subtraction, Multiplication, Division

1. Write a multiplication equation for each exercise.

A 

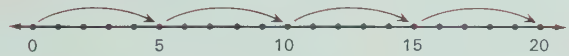
$3 \times 5 = 15$

B 

$5 \times 3 = 15$

C $6 + 6 + 6 + 6 + 6$

$5 \times 6 = 30$

D 

$4 \times 5 = 20$

2. Write 2 division equations for each exercise.

A $4 \times 5 = 20$ — $20 \div 4 = 5$, $20 \div 5 = 4$

B $7 \times 9 = 63$ — $63 \div 7 = 9$, $63 \div 9 = 7$

C $17 \times 28 = 476$ — $476 \div 17 = 28$, $476 \div 28 = 17$

3. Find the products.

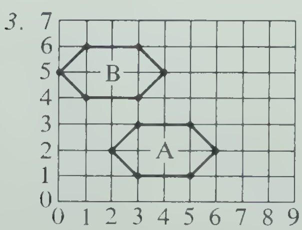
A $7 \times 6 = 42$	B $8 \times 5 = 40$	C $3 \times 8 = 24$	D $9 \times 7 = 63$	E $5 \times 7 = 35$	F $4 \times 6 = 24$	G $6 \times 8 = 48$	H $7 \times 7 = 49$
I $9 \times 6 = 54$	J $7 \times 4 = 28$	K $4 \times 9 = 36$	L $9 \times 8 = 72$	M $7 \times 8 = 56$	N $8 \times 4 = 32$	O $8 \times 8 = 64$	P $5 \times 6 = 30$

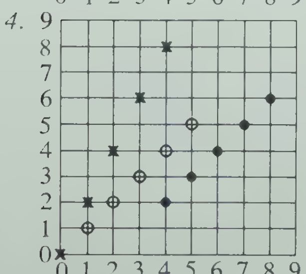
4. Find the sums and differences.

A $27 + 44 = 71$	B $64 - 31 = 33$	C $24 + 65 = 89$	D $64 - 34 = 30$	E $64 - 38 = 26$	F $59 + 63 = 122$
G $47 + 56 = 103$	H $93 - 26 = 67$	I $127 - 54 = 73$	J $141 - 95 = 46$	K $65 - 19 = 46$	L $48 + 16 = 64$

You are invited to explore

ACTIVITY CARD 13 Page 315





Part A graph denoted by •.
Part B graph denoted by ×.
Part C graph denoted by ○.

Using the Exercises

Assign the exercises on page 275 as independent work. Allow time to discuss the solutions while checking papers. Remind children to record any of the basic facts they missed in exercise 3 so that they can study them for quick recall.

Follow-up

Children may enjoy graphing the following sequences of co-ordinates. Direct them to make co-ordinate axes for the positive integers and number them vertically from 0 to 36 and horizontally from 0 to 26. Graph paper with 1-cm grid would be most suitable. Do not reveal the name of the object which will emerge; let children discover it on their own.

1. Gingerbread man:
(0, 8); (1, 6); (4, 7); (7, 4) (12, 8); (16, 6); (15, 2); (18, 0); (21, 1); (22, 3); (20, 4); (21, 8); (17, 12); (18, 15); (21, 15); (24, 17); (25, 19); (24, 21); (22, 21); (19, 19); (16, 20); (19, 22); (20, 25); (19, 30); (16, 32); (12, 32); (9, 29); (9, 24); (11, 21); (8, 21); (5, 23); (3, 23); (2, 21); (3, 19); (6, 17); (9, 17); (7, 14); (6, 10); (3, 12); (0, 8)
2. Ice cream cone:
(0, 0); (7, 4); (14, 8); (17, 7); (22, 9); (24, 14); (23, 17); (24, 20); (23, 22); (21, 24); (17, 25); (14, 24); (14, 21); (11, 22); (9, 21); (7, 18); (8, 16) (8, 14); (10, 10); (14, 8); (10, 10); (8, 14); (5, 9); (0, 0)
3. Umbrella:
(0, 0); (23,15); (16, 12); (20, 17); (15, 13); (8, 18) (13, 14); (23, 25); (20, 28); (18, 26); (19, 25); (20, 26); (21, 25); (12, 15); (15, 20); (11, 16); (13, 22); (9, 17); (10, 23); (0, 0)

Resources for Active Learning

Mathex: Geometry No. 4, "Using a Geoboard – Activity 1," p. 23, Encyclopaedia Britannica Publications Ltd.

Workbook, page 98

CHAPTER 12 | Dividing

Pages 276–308

General Objectives

To maintain understanding of division concepts

To develop the division algorithm for 1-digit divisors

To provide a variety of word-problem situations

To provide experiences in estimation

The main objective of this chapter is to lead the child through a conceptual development of the long-division algorithm. A carefully planned, step-by-step procedure is designed to lead children to the final algorithm. At each stage of development, the meaning of division is maintained through sets of word problems which focus attention on the current phase of the development and help clarify the ideas of division. The objective is not that children master the division algorithm; the goal remains that of fostering an understanding of basic concepts.

First, division concepts and the multiples of 10 and 100 are briefly reviewed. Following this, the children are asked to work with various types of repeated subtraction that lead to the idea of subtraction in the division algorithm. Quotients associated with multiples of 10 and 100 are reemphasized. Practice with estimation is suggested to prepare children for making accurate estimates in the division algorithm.

This background leads the children step-by-step to the final algorithm for 1-digit divisors. Then the idea of remainders is introduced.

Mathematics

Although Chapter 12 is long, the only new mathematical concept introduced is division with remainders. The background developed in previous chapters makes it possible to approach the division algorithm using familiar mathematical concepts.

Recall the basic definition for division of whole numbers.

If a, b, and c are whole numbers, and if $a \times b = c$, and $b \neq 0$, then $a = c \div b$.

We need to recognize that the numbers a, b, and c may not be whole numbers, and that the definition should also hold for other numbers, for example, rational numbers. However, at this stage of development we deal only with whole numbers for a, b, and c; so we must extend our definition of division to include remainders:

If a, b, c, and d are whole numbers such that

$(a \times b) + d = c$, $b \neq 0$, and $d < b$,

we say that for $c \div b$, a is the quotient and d is the remainder.

Notice that, in this definition if $d = 0$, we have the original definition. That is, $a \times b = c$ or $c \div b = a$.

Making a statement of equality with regard to $c \div b$ requires rational numbers; that is,

$$c \div b = a + \frac{d}{b}.$$

An example of this is $14 \div 3$.

$$\begin{array}{r} 4 \\ 3\overline{)14} \\ \underline{12} \\ 2 \end{array} \quad \rightarrow \quad 14 \div 3 = 4 + \frac{2}{3} \quad \text{or} \quad 4\frac{2}{3}$$

When we encounter rational numbers in future work, we will return to the idea of remainders in division and show exactly how nonzero remainders give a quotient which is not a whole number.

Teaching the Chapter

Materials

Counters (50 per child)
Felt shapes or objects
Flannelboard
Graph paper
Index cards or paper, approximately 10 by 15 centimetres
Large multiplication table
Number line
Objects for set demonstrations (pencils, crayons, checkers)
Overhead projector
Rulers
Scissors
For Follow-up Activities:
Chart paper (approximately 75 by 100 centimetres)
Felt-tipped pen
Menus from local restaurants, cafeterias, or coffee shops
Dienes Blocks
Numeral cube or large die
Toy catalogues, sale circulars, Sunday supplements
Toy cars

Vocabulary

dividend	map scale
divisor	quotient
estimate	remainder

You will notice that most of the lessons of this chapter treat the concepts without using many manipulative materials. The children are thereby encouraged to work in a more abstract environment. Of course some set demonstrations may enhance presentation of the main topics of the chapter, and children may advantageously use certain materials such as counters, strips, or the number line in connection with a few topics. At other times, however, encourage most children to work without materials.

Feel free to introduce the vocabulary gradually, using the terms in discussions prior to the lessons in which the children study them in the text. However, always make sure the children know what you are talking about and are not hindered by the use of the new words.

The word *dividend* is not introduced on the children's pages, but you may wish to use it in your discussions.

Lesson Schedule

Plan to cover this material in about four weeks. Because the chapter is long, you should maintain a fairly

rapid pace; the light touch recommended for this chapter should enable you to do this. From time to time, you will notice that activities have been incorporated to keep the children alert and thinking.

Since the gradual development and introduction of the division algorithm allows for rapid coverage of this material, we suggest that you attempt to get through each two-page lesson in about one day. You may need to spend an additional day on certain lessons such as those containing word-problem exercises.

Evaluation of Progress

Do not expect all children to master the division algorithm in this chapter. Most should gain some understanding of the methods used and also remember some of the ideas involved. These processes will be treated again in Book 4; however, the exposure is important to the children's later experiences with the algorithm. Keep in mind that it is more important to develop a good understanding of the estimation and subtraction ideas associated with division than to insist on mastery of the algorithm.

Keeping this objective in mind, you should include two phases in your evaluation: determining whether the children can work with the division algorithm, and assessing their understanding of the concepts involved. Your daily observation of the children's participation in and contribution to discussions is important in determining comprehension. Individual interviews are an excellent aid to understanding a child's particular learning difficulty. For example, have a child work a division problem orally for you, "thinking out loud" how he would solve it by himself.

Give the children many opportunities to create and analyze story problems using the division operation. Pages 306–308 review the concepts and skills presented so far. The review on pages 304 and 305 should help you test the children on this chapter.

Resources for Active Learning

General Activities

[Use the games and activities from earlier chapters which the students enjoyed. Use some of them for evaluation now if time is available.]

Nuffield Project: *Computation and Structure* 2, "Simple Sharing," pp. 61–64, Wiley

Toward Improving Computation, "Sequences . . . ," pp. 42–48; "Reinforcement," pp. 10–14, Curriculum Development Associates. This reference contains ideas for self-correcting material and assignments.

Manipulative Devices

Dienes Multibase Arithmetic Blocks (Herder and Herder)

SEE Calculator (Selective Educational Equipment)

Commercial Games

TUF (Creative Publications; Cuisenaire Co.; TUF). A game useful for evaluating number facts.

PAGES 276–277

Objective

Given one of several ways of thinking about division, the child will be able to write and solve a simple division equation.

Preparation

Materials

counters (38 per child)

It would be appropriate to prepare for this lesson with a short, brisk oral drill to review basic multiplication facts by finding missing factors. For example, say: "I'm thinking of the number which when multiplied by 7 gives 35. What's my number?" Keep drills such as this short and fast-moving to help children reach the level of quick recall.

Investigation

Guide the children in reading the text and studying the illustrations for the investigation. Then ask them to try to answer the question on their own. Allow them freedom to use any one of the three suggested methods for finding the solution, but encourage the faster children to try all three approaches. Provide counters for those children who wish to work the problem using method A.

12 Dividing

What are some ways to think about division?

Investigating the Ideas

You can use sets, the number line, or subtraction to help you find quotients.

A Sets — 38 counters

2, 4, 6, . . .

B Number line

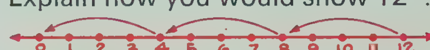
C Subtraction

$$\begin{array}{r} 38 \\ -\ 2 \\ \hline 36 \end{array} \quad \begin{array}{r} 36 \\ -\ 2 \\ \hline 34 \end{array} \quad \begin{array}{r} 34 \\ -\ 2 \\ \hline \end{array} \quad \ldots$$

? Can you use one of these methods to help you find how many twos are in 38? 19; see Investigation.

Discussing the Ideas

1. How would you show 24 ÷ 6 using method A?
   Count out sets of 6 counters until none are left.
2. Explain how you would show 12 ÷ 4 on a number line.



3. Give each difference for finding 36 ÷ 9 by subtraction.

$$\begin{array}{r} 36 \\ -9 \\ \hline 27 \end{array} \quad \begin{array}{r} 27 \\ -9 \\ \hline 18 \end{array} \quad \begin{array}{r} 18 \\ -9 \\ \hline 9 \end{array} \quad \begin{array}{r} 9 \\ -9 \\ \hline 0 \end{array}$$

4. What division problem can you solve if you know the missing factor in the equation?
   30÷5=6

$$? \times 5 = 30$$

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Discussion

After all of the children have had time to find the answer for the investigation question by at least one of the methods, ask volunteers to demonstrate the three interpretations before the class.

Work through the discussion questions with the children, using counters, a demonstration number line, and the overhead projector or chalkboard as appropriate. If time permits, extend exercises 1 through 3 to include demonstrations of each of the three interpretations suggested by the investigation. In connection with exercise 4, the children might enjoy, and benefit from, showing how the division problems in the first three exercises could be presented as missing factor problems:

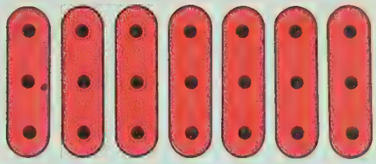
24 ÷ 6	? × 6 = 24
12 ÷ 4	? × 4 = 12
36 ÷ 9	? × 9 = 36

Using the Ideas

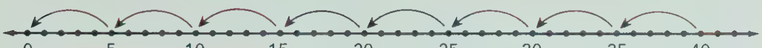
1. Write a division equation for each exercise.

A

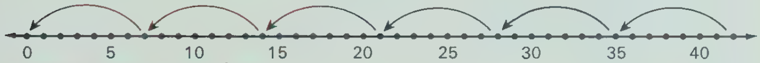

18 stars
18÷3=6 or 18÷6=3

B


21 dots
21÷7=3 or 21÷3=7

C


40÷5=8 or 40÷8=5

D


42÷7=6 or 42÷6=7

E	F	G
30 − 6 = 24	**24** − 8 = 16	**20** − 5 = 15
24 − 6 = 18	16 − 8 = 8	15 − 5 = 10
18 − 6 = 12	8 − 8 = **0**	10 − 5 = 5
12 − 6 = 6	24÷8=3 or 24÷3=8	5 − 5 = **0**
6 − 6 = **0**		20÷5=4 or 20÷4=5
30÷6=5 or 30÷5=6		

H $3 \times 4 = 12$ 12÷3=4 or 12÷4=3

I $9 \times 8 = 72$ 72÷9=8 or 72÷8=9

J $5 \times 7 = 35$ 35÷5=7 or 35÷7=5

2. Find the products.

A 3 × 8 24	E 4 × 8 32	I 2 × 9 18	M 1 × 7 7	Q 1 × 9 9
B 6 × 7 42	F 0 × 8 0	J 3 × 7 21	N 6 × 4 24	R 8 × 3 24
C 5 × 5 25	G 5 × 4 20	K 5 × 6 30	O 5 × 9 45	S 9 × 4 36
D 3 × 9 27	H 6 × 6 36	L 7 × 3 21	P 4 × 7 28	T 6 × 8 48

3. Find the quotients.

A 18 ÷ 9 2	E 42 ÷ 7 6	I 24 ÷ 8 3	M 24 ÷ 6 4	Q 36 ÷ 4 9
B 7 ÷ 7 1	F 0 ÷ 8 0	J 24 ÷ 4 6	N 21 ÷ 7 3	R 45 ÷ 9 5
C 20 ÷ 4 5	G 21 ÷ 3 7	K 36 ÷ 6 6	O 30 ÷ 6 5	S 18 ÷ 2 9
D 25 ÷ 5 5	H 48 ÷ 6 8	L 27 ÷ 9 3	P 28 ÷ 7 4	T 32 ÷ 8 4

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Using the Exercises
Assign the exercises as independent work. Then allow time for checking papers and discussing the ideas. Again, stress finding quotients by thinking of missing factors. This is extremely important at this stage, to prepare for division involving larger numbers. As indicated by the answers, either division fact is acceptable for each part of exercise 1, since the child may think about finding the size of the unit *or* about the number of units, depending upon which way he looks at the figures.

Assignments (page 277)
Minimum: 1–3. Average: 1–3.
Maximum: 1–3.

Follow-up/Blue and Green Relay
Make a large chart (approximately 75 by 100 centimetres) similar to that illustrated below. At the right side of the 8-by-25-cm sections, write the numbers 0 through 10 at random, concentrating on fives, sixes, and sevens. Divide the class into two teams and line them up so that all children can see the chart. Give a green crayon or felt-tip pen to the leader of one team and a blue pen or crayon to the leader of the other. At your signal, the leaders should go to the chart and quickly write a division equation to match any quotient they choose on the chart. Then each must hand the chalk to the next person on his team and retire to the end of the line. Play should continue until all the spaces on the chart have a completed division equation. Direct the children in checking and counting the correct responses in each color, and then compare the scores to determine the winning team.

← 25 cm →
8 cm

7	9	18 ÷ 3 = 6
20 ÷ 4 = 5	2	4
3	7	5
6	8	10

Blue and Green Relay

Resources for Active Learning
Developmental Math Cards, H-1, G-16, G-13, Addison-Wesley.

Workbook, page 99

PAGES 278–279

Objective

Given a division equation which is related to a multiplication equation in which one factor is a multiple of ten, the child will be able to find the quotient by thinking of missing factors.

Preparation

Materials

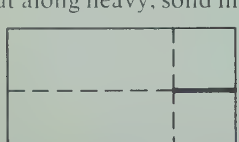
scissors; cards 10 × 15 (8 per child)

In order to benefit from the investigation in this lesson, children need to be reminded of how the basic facts are used when multiplying multiples of ten. For example, in order to multiply 5 × 90, one must think, "What is 5 × 9?" To review this process, conduct a short oral drill on multiplying with multiples of 10. Say for example, "Since 4 × 8 is ____ (child's response), we know that 4 × 80 is ____ (child's response)."

Investigation

Some children may need considerable guidance in making the cards. Read the directions with them and work through making a model card together. Then encourage them to make the 7 other cards on their own. As children finish, let them choose a partner who is also finished and begin to question one another. If you choose, remind them to keep a record of their trials and their correct answers. However, stress that the principal goal is not simply to have a high score but to give correct answers based on understanding the concepts involved. Circulate throughout the room to see that children have written a multiple of ten as one factor on each card. Encourage any children having difficulty to use the multiplication table. For this investigation, you might find it easier to duplicate forms than to have each child make his own cards.

Cut along heavy, solid line.

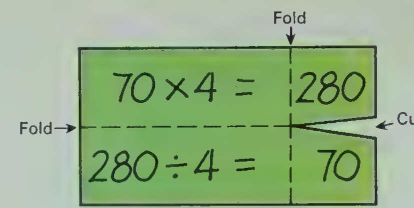


Fold along dotted lines.

● *Can products be used to find quotients?*

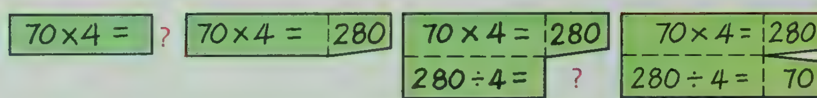
Investigating the Ideas

Use 3-by-5 cards or pieces of heavy paper to make cards for a Product-Quotient Quiz. Make up 8 equations like the ones shown, but with different numbers.



As one factor, use a number ending in 0, such as 70, 80, 500, or 700. For the other factor, choose from the digits 2, 3, 4, 5, 6, 7, 8, and 9. Cut and fold your 8 cards as shown.

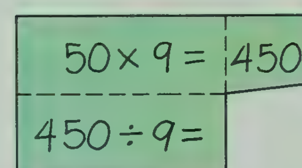
? Can you follow the directions below to quiz one of your classmates? See Investigation.



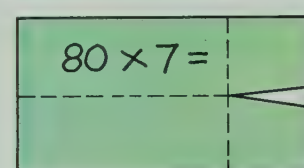
Question 1 Check. Question 2 Check.

Discussing the Ideas

1. How can you use the multiplication equation to help you solve the division equation? Think of the missing factor, 50.



2. How would you complete this card? 80 × 7 = 560 560 ÷ 7 = 80



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Discussion

Work through the discussion exercises with the children. Then lead them in a general discussion of how helpful the multiplication equation is in finding quotients. Stress this relation and the relation of the basic facts to multiplying when one factor is a multiple of 10. Illustrate the relationships by writing examples on the chalkboard.

7 × 3 = 21	21 ÷ 3 = 7
70 × 3 = 210	210 ÷ 3 = 70
700 × 3 = 2100	2100 ÷ 3 = 700
6 × 9 = 54	54 ÷ 6 = 9
6 × 90 = 540	540 ÷ 6 = 90
6 × 900 = 5400	5400 ÷ 6 = 900

It would be helpful at this time to have the children identify which multiplication facts still cause difficulty. These facts should be displayed on a bulletin board so that not knowing them will not hinder the child's learning of the division algorithm. Encourage the children to keep working on these harder combinations. They might complete a personal multiplication table to use only when necessary, or they might practice with a friend by taking turns holding flash cards and checking products for each other. You might also provide brief oral practice daily for several weeks until they know the combinations.

Using the Ideas

1. Find the products.

A 5 × 90 450	E 70 × 6 420	I 9 × 70 630	M 50 × 7 350	Q 8 × 80 640
B 80 × 8 640	F 6 × 50 300	J 60 × 6 360	N 6 × 80 480	R 4 × 80 320
C 7 × 80 560	G 5 × 50 250	K 8 × 50 400	O 30 × 2 60	S 6 × 40 240
D 70 × 7 490	H 3 × 80 240	L 30 × 9 270	P 60 × 3 180	T 5 × 60 300

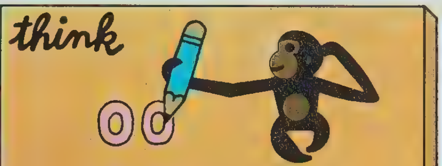
2. Find the quotients.

A 450 ÷ 5 90	D 300 ÷ 6 50	G 270 ÷ 9 30	J 630 ÷ 9 70
B 640 ÷ 8 80	E 250 ÷ 5 50	H 180 ÷ 3 60	K 480 ÷ 6 80
C 560 ÷ 7 80	F 360 ÷ 6 60	I 490 ÷ 7 70	L 320 ÷ 4 80

3. Find the products.

3600 A 9 × 400	I 7 × 700 4900
2400 B 300 × 8	J 300 × 3 900
3500 C 5 × 700	K 3 × 500 1500
2700 D 3 × 900	L 600 × 8 4800
4000 E 800 × 5	M 400 × 8 3200
2500 F 500 × 5	N 4 × 500 2000
6400 G 8 × 800	O 6 × 300 1800
5600 H 700 × 8	P 6 × 600 3600

think



Now here's a simple rule
That should prove quite
a friend.
When you multiply by me,
Put two zeros on the end.

WHO AM I? 100

4. Find the missing factors.

A $5 \times n = 350$ 70	F $n \times 8 = 2400$ 300	K $9 \times n = 450$ 50
B $n \times 7 = 490$ 70	G $7 \times n = 5600$ 800	L $n \times 8 = 6400$ 800
C $4 \times n = 200$ 50	H $n \times 8 = 3200$ 400	M $3 \times n = 2100$ 700
D $n \times 6 = 420$ 70	I $5 \times n = 4500$ 900	N $n \times 5 = 300$ 60
E $9 \times n = 630$ 70	J $n \times 2 = 600$ 300	O $6 \times n = 2400$ 400

5. Find the quotients.

A $400 \div 8 = n$ 50	F $600 \div 3 = n$ 200	K $900 \div 3 = n$ 300
B $180 \div 6 = n$ 30	G $2400 \div 4 = n$ 600	L $2500 \div 5 = n$ 500
C $450 \div 5 = n$ 90	H $3500 \div 5 = n$ 700	M $3600 \div 6 = n$ 600
D $560 \div 7 = n$ 80	I $2000 \div 4 = n$ 500	N $3000 \div 5 = n$ 600
E $630 \div 7 = n$ 90	J $6300 \div 9 = n$ 700	O $4000 \div 5 = n$ 800

More practice, page A-34, Set 45

Using the Exercises

Instruct the children to do the exercises independently. Since the *Think* problem is intended as enrichment, some may have difficulty arriving at the correct answer; however, most children will understand the reasoning when the correct answer is discussed. Be sure that everyone who wants to try the *Think* problem has time to work on it before you present the answer.

Follow-up/Mental Chain Games

Prepare several chain reaction games which require the children to compute mentally. For example:

"Start with 6 . . . Multiply by 2 . . . Subtract 4 . . . Multiply by 6 . . . Subtract 3 . . . Divide by 9 . . . Write your answer." (5)

Choose several children to give their answers and then ask if anyone disagrees. If so, show the progressive work on the chalkboard, leaving blanks for the answers and asking the class to do the operations. Write out these chains beforehand so that, if children lose track of the sequences, any resulting differences of opinion can easily be resolved. If you write chain games on the board, use the form suggested below, to make sure the children understand that the operations are between pairs of numbers.

Start

4	×70		÷40		×30		→

End

÷7		×6		÷2	90

Start

10	×70		÷7		−90		→

End

×50		−300		÷20	10

Start

8	×30		÷6		−4		→

End

÷6		×2		÷4	3

Resources for Active Learning

Mathematics in Modules, WN5, Addison-Wesley.

Mathex: Numeration No. 7, "Activity 1," p. 20, Encyclopaedia Britannica [Fibonacci sequences]

Duplicator Masters, page 55
Workbook, page 100
***Skill Masters*, page 55**

Assignments (page 279)
Minimum: 1–2. Average: 1–5.
Maximum: 1–5.

PAGES 280–281

Objective

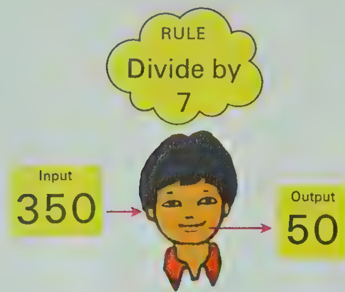
Given appropriate input or output numbers, the child will be able to perform multiplication and division operations with multiples of 10 according to a given function rule.

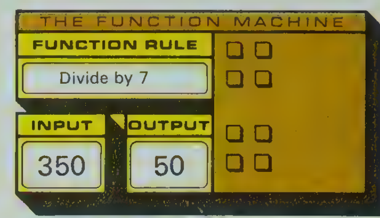
Preparation

Review the idea of the function machine by playing a function game with the children. Think of a rule and ask the children for a number, and then give them the corresponding number according to your rule. For example, you might use the rule "Multiply by 3." If a child gives you the number 6, respond with the number 18. As some children discover the rule, call on them to give the answer in response to the number suggested by one of the other children.

What's the input, output, or rule?

Study the picture. Then give the number or function rule for each gray space in exercises 1 through 6.





1. Function Rule: Multiply by 10

	Input	Output	
	15	150	
A	24		240
B	37		370
C	48		480
D		720	72

2. Function Rule: Multiply by 100

	Input	Output	
	8	800	
	23	2300	
A	65		6500
B	83		8300
C		2800	28

3. Function Rule
A: Multiply by 40

	Input	Output	
	9	360	
	4	160	
	6	240	
B	5		200
C		280	7

4. Function Rule: Divide by 4

	Input	Output	
	80	20	
A	280		70
B	240		60
C	120		30
D	200		50

5. Function Rule: Divide by 3

	Input	Output	
	60	20	
	180	60	
A	240		80
B	120		40
C		30	90

★ **6.** Function Rule
A: Multiply the number by itself

	Input	Output	
	4	16	
	5	25	
	10	100	
B	8		64
C		49	7

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Discussion

One of the main purposes of pages 280 and 281 is to provide the children with meaningful practice of multiplication and division with multiples of 10.

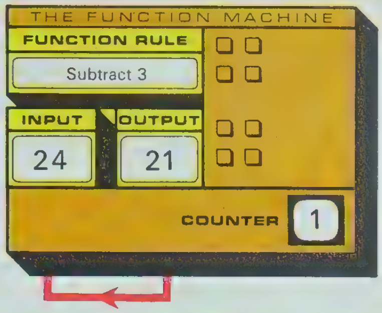
Discuss the illustration at the top of page 280, mentioning that the child in the picture hears 350, uses the rule "Divide by 7," and says "50." Relate this to the illustration of the function machine, which has an input of 350, the rule "Divide by 7," and an output of 50.

Page 281 presents optional material. Depending on the ability of your class, however, you might want to conduct a discussion of these ideas with some or all of the children.

As you discuss the function machine at the top of page 281, point out that the counter is a new feature on the machine. Ask the children to think about putting the number 24 into the machine, applying the rule "Subtract 3," and getting an output of 21. The number 21 should then be put into the machine on the *input* side, as the arrow at the bottom of the machine indicates. Point out that the counter tells how many times the function rule is used. This is a way of leading the children to think about the number of times 3 must be sub-

Let's use a special function machine.

This special function machine uses the output number as a new input number and keeps operating. A counter tells how many times the rule is used.



Here is a record of the machine's operations.

Function Rule	Input	Output Numbers							Counter
Subtract 3	24	21	18	15	12	9	6	3	0	8

Here are more records. Give what you think should go in each gray space, then write a division equation for the exercise.

1. Function Rule: Subtract 4 — Input: 24, 20, A, 12, B, 4, C — Counter: 6
   24÷4=6   A: 16   B: 8   C: 0

2. Function Rule: Subtract 6 — Input: 42, 36, A, B, 18, 12, C, 0 — Counter: D
   42÷6=7   A: 30   B: 24   C: 6   D: 7

3. Function Rule: Subtract A (5) — Input: 40, B, 30, C, 20, 15, 10, D, 0 — Counter: E
   40÷5=8   B: 35   C: 25   D: 5   E: 8

4. Function Rule: Subtract 3 — Input: A, 30, 27, B, C, D, 15, 12, 9, 6, 3, 0 — Counter: E
   33÷3=11   A: 33   B: 24   C: 21   D: 18   E: 11

5. Function Rule: Subtract A (9) — Input: 45, B, C, D, E, 0 — Counter: F
   45÷9=5   B: 36   C: 27   D: 18   E: 9   F: 5

281

tracted from 24 before we arrive at 0.

Draw attention to the example under the machine with the rule "Subtract 3." Point out that the counter registers the number of subtractions. In other words, to get from 24 to 0, we must subtract three 8 times. This tells us that there are 8 threes in 24, or that $24 \div 3 = 8$.

Ask the children to complete the first exercise by giving the correct numbers for boxes A, B, and C. Emphasize that the number 4 was subtracted 6 times to get from 24 to 0; therefore, there are 6 fours in 24, or $24 \div 4 = 6$.

Discuss the difficult exercises when the children complete them. Give the children opportunities to talk about the reasoning they used to arrive at correct answers. Give particular attention to exercises 3A and 5A on page 281. The child must examine the pattern that appears in the output numbers before he can determine what number is being subtracted.

Assignments (page 281)*

Minimum: 1–5. Average: 1–5. Maximum: 1–5.

Follow-up

More capable children might enjoy making up exercises which show more records of the operations of the special function machine on page 281.

Others would benefit from a worksheet with exercises similar to those on page 280.

Function Rule: Divide by 8

Input	Output
320	
64	
	60
400	
	3

Function Rule: Multiply by 7

Input	Output
30	
500	
	5600
	490
20	

Resources for Active Learning

Developmental Math Cards, G[1] 13, Addison-Wesley. [Puzzles]

Toward Improving Computation, "Protect . . . ," pp. 15–22, Curriculum Development Associates. [Ideas for "chain reaction"]

Workbook, page 101

PAGES 282–283

Objectives

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Given a map drawn to scale, the child will be able to solve word problems about it by referring to the scale of the map.

Preparation

Materials

ruler (1 per child)

Since there are a few exercises which contain 2-digit multipliers, you might very briefly review the multiplication algorithm—both the longer method and the shortcut. Write on the chalkboard an example such as the following:

```
  53      53
×  7    ×  7
  21     371
 350
 371
```

Then give the class two or three similar exercises to work through orally.

Keeping in Touch with

Subtraction
Multiplication
Measurement
Story problems

1. Give the mark (>, <, =) that should go in each ◍.

A 70×3 ◍ 3×70 =	E 5×50 ◍ 6×40 >	I 30×7 ◍ 70×3 =
B 3×40 ◍ 6×20 =	F 98×7 ◍ 700 <	J 800×9 ◍ 900×8 =
C 70×5 ◍ 9×40 <	G 8×60 ◍ 70×7 <	K 48×8 ◍ 8×53 <
D 99×4 ◍ 4×100 <	H 700×8 ◍ 600×9 >	L 34×5 ◍ 35×4 >

2. Find the missing factors.

A $n \times 6 = 240$ 40	G $n \times 7 = 350$ 50	M $3 \times n = 270$ 90
B $n \times 4 = 240$ 60	H $7 \times n = 420$ 60	N $3 \times n = 120$ 40
C $4 \times n = 320$ 80	I $5 \times n = 400$ 80	O $n \times 8 = 560$ 70
D $8 \times n = 480$ 60	J $5 \times n = 450$ 90	P $n \times 6 = 480$ 80
E $n \times 9 = 360$ 40	K $n \times 7 = 210$ 30	Q $n \times 6 = 180$ 30
F $n \times 9 = 270$ 30	L $n \times 6 = 300$ 50	R $3 \times n = 150$ 50

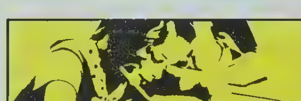
3. There are no whole number answers for 3 of these exercises. List these. Then find the differences in the other exercises. C, F, I

A $65 - 23$ 42	B $49 - 26$ 23	C $31 - 42$	D $548 - 234$ 314	E $156 - 133$ 23
F $657 - 756$	G $329 - 216$ 113	H $254 - 237$ 17	I $472 - 481$	J $355 - 167$ 188

4. Find the area of each region. The unit is ▢.

A
28


B
22

You are invited to explore

ACTIVITY CARDS 14, 15
Page 316

282

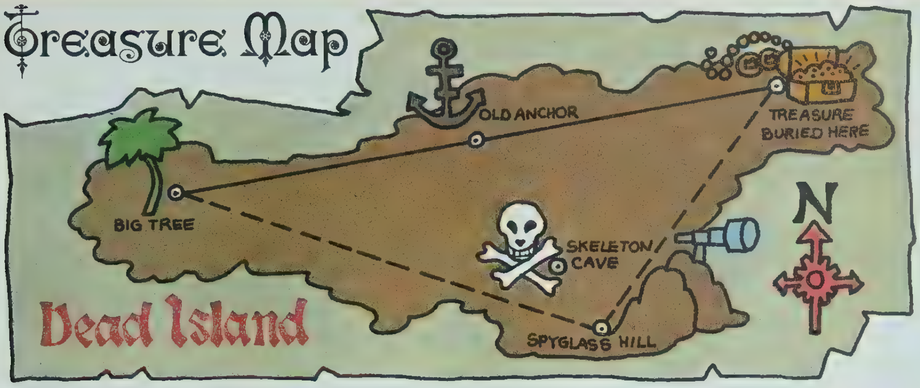
Discussion

You may wish to discuss the Treasure Map, exercise 5, on page 283 before having the children begin these pages.

For page 283, everyone will need a ruler to make the appropriate measurements on his map. Ask the children to study the map and to read the material that precedes the exercise set. Then discuss with them the meaning of Scale A with respect to the map. Help them see how to find certain distances on the island in terms of kilometres by measuring to determine the number of centimetres.

You may wish to work through exercises 5A and 5B with the class, discussing each one and answering questions before proceeding to read and discuss the material concerning Scale B. Point out that when Scale B is used, each centimetre on the map means two kilometres on the island.

Assign the remaining exercises on 283 and those on page 282 as independent work. When the class finishes, allow time to check papers and discuss any exercises which may have caused difficulty. As you discuss page 282, pay particular attention to exercise 2. A thorough review of this type of exercise will enhance the children's chances for



5. John found this map in an old trunk in his attic. One piece of paper in the trunk where the map was found looked like this.

Scale A: 1 centimetre
Each centimetre on the map means 1 kilometre on the island.

Use scale A to answer these questions.

A How many kilometres is it from Big Tree to Spyglass Hill ? 7½ km

B How many kilometres is it from Spyglass Hill to the treasure ? 5 km

There was another scrap of paper in the trunk. Use scale B to answer the rest of the questions.

Scale B: 1 centimetre
Each centimetre on the map means 2 kilometres on the island.

C How many kilometres is it from Big Tree to Spyglass Hill ? 15 km

D How many kilometres is it from Spyglass Hill to the treasure ? 10 km

E How many kilometres would you walk if you went from Big Tree to Spyglass Hill and then to the treasure ? 25 km

F How many kilometres is it from Big Tree straight to the treasure ? 20 km

G How many kilometres is it from Old Anchor straight to Skeleton Cave ? 5 km

H How far is it from Skeleton Cave to Spyglass Hill ? 2½ km

I Which is farther from Big Tree, Spyglass Hill or the treasure ? Treasure
How much farther ? 7 km

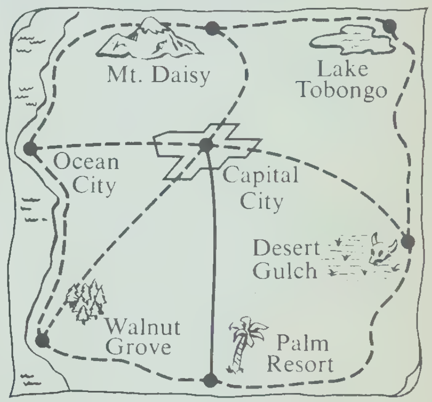
283

success with the division algorithm.

Follow-up

An activity that combines practice in estimation, map work, and measurement makes a good review device. On a master sheet, create your own, larger version of the map sample below. Names that parody those in your area should provide some humor for the children. Use a scale such as "1 centimetre equals 20 kilometres" to make it easier for them to estimate distances. Ask them to measure the distances between cities, to estimate the distances in kilometres, and to write their estimates on their maps. You may have them work out a table of equivalents like the following to help find the approximate distances.

Measure	1 cm	2 cm	3 cm	4 cm	5 cm
Kilometres	20	40	60	80	100



Prepare questions like the following to be answered by the children after they write in the estimated distances on their maps.

1. Mike's family lives in Walnut Grove. They want to visit Mt. Daisy on their vacation. How far will they travel to Mt. Daisy if they go through Capital City?
2. How far will Mike's family travel from Walnut Grove to Mt. Daisy if they go through Ocean City?
3. Which way is farther? How much farther?

Resources for Active Learning

Developmental Math Cards, H-13, Addison-Wesley.

Nuffield Project: *Computation and Structure* 2, "Length," pp. 6–23; "Multiplication with Measures," pp. 56–60, Wiley.

PAGES 284–285

Objective

Given exercises involving repeated subtraction, the child will be able to solve a related division equation.

Preparation

Plan a short oral warm-up for this lesson. You might wish to give the children some mental chain-games such as these:

"Start with 7 . . . Multiply by 6 . . . Subtract 2 . . . Divide by 4 . . . Your answer is . . ." (10)

"Start with 6 . . . Multiply by 8 . . . Add 2 . . . Divide by 10 . . . Your answer is . . ." (5)

"Start with 56 . . . Divide by 7 . . . Multiply by 4 . . . Subtract 2 . . . Divide by 3 . . . Your answer is . . ." (10)

You might pattern the last few to lead into the discussion in the text. For example:

"Start with 21 . . . Subtract 3 . . . Subtract 3 . . . Subtract 3 . . . Subtract 3 . . . Subtract 3 . . . Your answer is . . ." (6)

"Start with 15 . . . Subtract 5 . . . Subtract 5 . . . Subtract 5 . . . Your answer is . . ." (0)

Can subtracting help you find the quotients?

Discussing the Ideas

24 ÷ 4?

How many fours in 24?

1. I'll subtract 1 four at a time to find out.

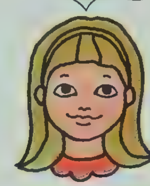
Fred

$$\begin{array}{r}24\\-4\\\hline 20\end{array}\quad\begin{array}{r}20\\-4\\\hline 16\end{array}\quad\begin{array}{r}16\\-4\\\hline 12\end{array}\quad\begin{array}{r}12\\-4\\\hline 8\end{array}\quad\begin{array}{r}8\\-4\\\hline 4\end{array}\quad\begin{array}{r}4\\-4\\\hline 0\end{array}$$

A How many fours did Fred find in 24? 6

B Solve: 24 ÷ 4 = *n* 6

2. I'll subtract 2 fours at a time to find how many fours are in 24.



Sandy

$$\begin{array}{r}24\\-8\\\hline 16\end{array}\leftarrow 2\text{ fours}\quad\begin{array}{r}16\\-8\\\hline 8\end{array}\leftarrow 2\text{ fours}\quad\begin{array}{r}8\\-8\\\hline 0\end{array}\leftarrow 2\text{ fours}$$

A How many fours did Sandy find in 24? 6

B Whose method is shorter? Sandy's

3. I can do the problem by subtracting 3 fours at a time.

Jerry

$$\begin{array}{r}24\\-12\\\hline 12\\-12\\\hline 0\end{array}\begin{array}{l}\\\leftarrow 3\text{ fours}\\\\\leftarrow 3\text{ fours}\\\\\end{array}$$

A How many fours did Jerry find in 24? 6

B Explain two ways Jerry's method is shorter than the others. See Discussion.

284

Discussion

If you use the suggestions in the preparation, relate your chain games to Fred's method of finding how many fours are in 24. If necessary, work through the method used in each exercise on the overhead projector or chalkboard. Explain that for the second exercise Sandy is thinking about subtracting fours, not one at a time, but two at a time. Help the class understand that all three exercises show the same problem but that Jerry's way of subtracting is shorter.

In answer to exercise 3B, the children should realize that his technique is shorter because (1) he subtracts a greater number of fours each time, and (2) by writing his work in vertical form, he does not recopy the problem each time. You may need to work through another sample problem such as 24 ÷ 3, first subtracting 1 three at a time, then 2 threes, and finally 4 threes.

Using the Ideas

1. Find the differences. Then solve the division equation.

A	25 − 5 = 20	20 − 5 = 15	15 − 5 = 10	10 − 5 = 5	5 − 5 = 0		$25 \div 5 = n$ 5
B	18 − 3 = 15	15 − 3 = 12	12 − 3 = 9	9 − 3 = 6	6 − 3 = 3	3 − 3 = 0	$18 \div 3 = n$ 6

2. Find the differences. Then solve the division equation.

A 21 − 7 = 14, 14 − 7 = 7, 7 − 7 = 0

$21 \div 7 = n$ 3

B 16 − 8 = 8, 8 − 8 = 0

$16 \div 8 = n$ 2

C 27 − 9 = 18, 18 − 9 = 9, 9 − 9 = 0

$27 \div 9 = n$ 3

3. Find the differences. Then solve the division equation.

A 48
−16 ← 2 eights = 32
−16 ← 2 eights = 16
−16 ← 2 eights = 0

$48 \div 8 = n$ 6

B 35
−14 ← 2 sevens = 21
−14 ← 2 sevens = 7
−7 ← 1 seven = 0

$35 \div 7 = n$ 5

C 30
−18 ← 3 sixes = 12
−12 ← 2 sixes = 0

$30 \div 6 = n$ 5

285

Using the Exercises

Direct the children to do the exercises on page 285 independently. Remind them that exercise 3 illustrates a shorter method than exercise 1 or exercise 2.

When the children complete their work, allow time for questions, discussion, and checking papers.

Assignments (page 285)

Minimum: 1–3. Average: 1–3. Maximum: 1–3.

Follow-up

This would be a good time to encourage any children who are having difficulty with the concept of division as repeated subtraction to work with counters or with the Dienes blocks, if available. Give them division problems similar to those on page 285 or pose the problems as written questions like these: How many fours can you subtract from 32? How many fives can you subtract from 75? How many threes can you subtract from 21?

Resources for Active Learning

Developmental Math Cards, 1¹1, Addison-Wesley.

Workbook, pages 102, 103

Objective

Given a division equation with a 2-digit dividend and a single-digit divisor, the child will be able to find the quotient by using repeated subtraction.

Preparation

Materials

counters (to be used as needed)

The investigation in this lesson may be undertaken without any formal preparation. You might explain that counters are available for those children who want to use them, but many will prefer to do their subtracting without this aid.

Investigation

Children who wish to use the counters might work as partners or individually and separate 48 into sets of 3 as they count. Whether the children are using counters or not, emphasize that they may subtract more than 1 three at a time. Then direct their attention to the problem of finding how many twos are in 136. For the children who finish quickly, write on the chalkboard other subtractions and related division equations:

Can you find how many fours there are in 96? $96 \div 4 = ?$

Can you find how many threes there are in 45? $45 \div 3 = ?$

Can you find how many fives there are in 100? $100 \div 5 = ?$

As the children work on the investigation questions, move around the room asking questions like these: "Is it quicker to subtract threes one at a time, or four at a time?" "Is it faster to subtract twos five at a time; or ten at a time? Encourage the children to do their subtracting carefully to avoid mistakes.

Can larger quotients be found by subtraction?

Investigating the Ideas

Find how many threes there are in 48 by starting with 48 and subtracting as many threes as you like each time.

HOW MANY THREES IN 48 ?

$$\begin{array}{r} 48 \\ -\ ? \\ \hline \\ -\ ? \end{array}$$

? Can you find how many twos there are in 136 ?
68 ; see Investigation.

Discussing the Ideas

A How many twos in 34 ? $34 \div 2$	B How many threes in 42 ? $42 \div 3$	C How many fours in 60 ? $60 \div 4$
34 −20 ← 10 twos 14 −14 ← 7 twos 0	42 −30 ← 10 threes 12 −12 ← 4 threes 0	60 −40 ← 10 fours 20 −20 ← 5 fours 0
$34 \div 2 = n$ 17	$42 \div 3 = n$ 14	$60 \div 4 = n$ 15

1. What is the first step in each of the examples above ? Subtracting 10 times the divisor
What is the second step ?
Subtracting the divisor as many times as possible

2. How can you use these two steps to find each quotient ?
Add the numbers that show how many times the divisor was subtracted.

286

Discussion

Although the concept of subtracting the divisor in groups of ten is introduced here, it is not essential that the children grasp it thoroughly at first encounter. It is more important to stress the basic concept of interpreting division as repeated subtraction. For this purpose, use the chalkboard or the overhead projector and work through one of the discussion exercises by subtracting the divisor in groups of less than 10 at a time. Then discuss the method shown by the examples in the text, in which multiples of 10 times the divisor are subtracted. Work through several examples until the children are able to do problems like those in exercise 1A on page 287.

Using the Ideas

1. Use the subtractions to help you find each quotient.

A $36 \div 3 = n$ 12

$$\begin{array}{rl} 36 & \\ -30 & \leftarrow 10 \text{ threes} \\ \hline 6 & \\ -6 & \leftarrow 2 \text{ threes} \\ \hline 0 & \end{array}$$

B $52 \div 4 = n$ 13

$$\begin{array}{rl} 52 & \\ -40 & \leftarrow 10 \text{ fours} \\ \hline 12 & \\ -12 & \leftarrow 3 \text{ fours} \\ \hline 0 & \end{array}$$

C $84 \div 6 = n$ 14

$$\begin{array}{rl} 84 & \\ -60 & \leftarrow 10 \text{ sixes} \\ \hline 24 & \\ -24 & \leftarrow 4 \text{ sixes} \\ \hline 0 & \end{array}$$

2. Find the differences. Then find each quotient.

A $46 \div 2 = n$ 23

$$\begin{array}{rl} 46 & \\ -20 & \leftarrow 10 \text{ twos} \\ \hline 26 & \\ -20 & \leftarrow 10 \text{ twos} \\ \hline 6 & \\ -6 & \leftarrow 3 \text{ twos} \\ \hline 0 & \end{array}$$

B $75 \div 3 = n$ 25

$$\begin{array}{rl} 75 & \\ -30 & \leftarrow 10 \text{ threes} \\ \hline 45 & \\ -30 & \leftarrow 10 \text{ threes} \\ \hline 15 & \\ -15 & \leftarrow 5 \text{ threes} \\ \hline 0 & \end{array}$$

C $126 \div 6 = n$ 21

$$\begin{array}{rl} 126 & \\ -60 & \leftarrow 10 \text{ sixes} \\ \hline 66 & \\ -60 & \leftarrow 10 \text{ sixes} \\ \hline 6 & \\ -6 & \leftarrow 1 \text{ six} \\ \hline 0 & \end{array}$$

3. Find the quotients.

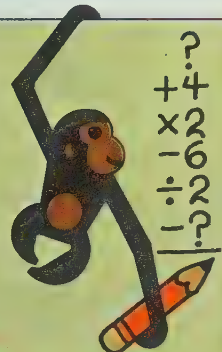
A $92 \div 4 = n$ 23
B $93 \div 3 = n$ 31
C $148 \div 4 = n$ 37
D $78 \div 2 = n$ 39
E $115 \div 5 = n$ 23
F $177 \div 3 = n$ 59
G $96 \div 8 = n$ 12
H $185 \div 5 = n$ 37
I $162 \div 6 = n$ 27
J $224 \div 7 = n$ 32

think

1. Pick a number.
2. Add 4.
3. Multiply by 2.
4. Subtract 6.
5. Divide by 2.
6. Subtract the number you started with.

Do you think you will always end with 1 ? Yes

Try this several times. See Using the Exercises.



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Using the Exercises

As you assign the exercises on page 287, remind the children to find the quotient by determining how many of a given divisor have been subtracted. Note that in exercise 3 they must convert the horizontal equation to the vertical subtraction notation. You may choose to work through parts of exercise 3 with the class.

To show the children why the answer to the *Think* problem will always be 1, ask them to work through the steps several times, starting with a different number each time. Then have them work through the steps again, letting a □ stand for any starting number, as suggested in the chart.

Directions	□ (any number)
Add 4.	□ + 4
Multiply by 2.	2 × (□ + 4) = 2□ + 8
Subtract 6.	2□ + (8 − 6) = 2□ + 2
Divide by 2.	(2□ + 2) ÷ 2 = □ + 1
Subtract the number.	(□ + 1) − □ = 1

Assignments (page 287)
Minimum: 1–2. Average: 1–3.
Maximum: 1–3.

Follow-up

To continue improving the children's computation skills with multiples of 10, give them a worksheet like this one.

Study each set of numbers.
Fill in each blank.

___, ___, 30, 40, ___, ___, 70, ___, 90.
40, 80, ___, ___, 200, 240, ___, 320.
50, 100, ___, 200, 250, ___, ___, 400.
70, ___, 210, 280, ___, 420, ___, 560.

Multiplication tables like those below might provide an adequate review.

Write the products in the tables.

×	3	5	7	9
20				
40				
60				
80				
100				

×	2	4	6	8
10				
30				
50				
70				
90				

Workbook, pages 104, 105

PAGES 288–289

Objective

Given an appropriate division equation, the child will be able to find the quotient by using repeated subtraction and estimating products.

Preparation

Plan and conduct an oral review of finding products with factors of ten and multiples of ten. Then switch to asking how many sixes in 12 and in 120, or how many fours in 32 and in 320.

Let's explore shorter ways for finding quotients.

Discussing the Ideas

1. Copy each problem on paper or on the chalkboard. Subtract as many threes each time as shown by the number in the ring.

A	B	C
$96 \div 3 = n$ 32	$96 \div 3 = n$ 32	$96 \div 3 = n$ 32
96	96	96
30 − (10) threes	60 − (20) threes	90 − (30) threes
66	36	6
30 − (10) threes	30 − (10) threes	6 − (2) threes
36	6	0
30 − (10) threes	6 − (2) threes	
6	0	
6 − (2) threes		
0		

Did you get 0 in the red screen each time? Yes

2. A What is the quotient for each part above? 32
   B Did you get the same quotient for each part? Yes
   C Which way was easiest? See Discussion.
   D Which way was shortest? Method C

3. If you start with 48, can you subtract as many as
   A 10 twos? Yes B 20 twos? Yes C 30 twos? No

4. A How many twos in all can you subtract from 48? 24
   B What is the quotient $48 \div 2$? 24

288

Discussion

Display each of the three sample problems on the chalkboard or overhead projector as children write them as directed in the text. As you work through the three different examples together, give the children sufficient time to do the subtracting on their own. During the discussion of questions 2C and 2D, do not expect all children to give the same response; some may see that method C is shorter but may not agree that it is easier.

You may want to provide other examples for discussion. If you do so, be sure that the examples have a zero remainder, since we are not ready to introduce division with a remainder other than zero. Because the object of this lesson is to develop understanding rather than skill, continue to ask the children for answers but help them when necessary.

Using the Ideas

1. Follow the directions. Give any missing numbers. Then find the quotients.

A

Start with 48.

↓

Subtract (10) threes.

↓

Subtract (6) threes to end with 0.

$48 \div 3 = ?$ 16

B

Start with 96.

↓

Subtract (20) fours.

↓

Subtract (?) fours to end with 0. 4

$96 \div 4 = ?$ 24

C

Start with 138.

↓

Subtract (?) sixes. 20

↓

Subtract (3) sixes to end with 0.

$138 \div 6 = ?$ 23

2. Find the product. This will help you decide how many sevens you can subtract at first when finding the quotient. Write each quotient.

A $20 \times 7 = n$ 140 $154 \div 7 = n$ 22
B $50 \times 7 = n$ 350 $371 \div 7 = n$ 53
C $80 \times 7 = n$ 560 $567 \div 7 = n$ 81

3. Find these quotients.

A $46 \div 2$ 23
B $115 \div 5$ 23
C $155 \div 5$ 31
D $132 \div 4$ 33
E $138 \div 3$ 46
F $126 \div 6$ 21
G $224 \div 7$ 32
H $336 \div 8$ 42

think

Study the first four equations. Then solve the last one.

$(1 \times 9) + 2 = 11$
$(12 \times 9) + 3 = 111$
$(123 \times 9) + 4 = 1111$
$(1234 \times 9) + 5 = 11\,111$
$(12\,345 \times 9) + 6 = n$ 111 111

More practice, page A-35, Set 46

289

Follow-up

The whole class can practice finding products when the factors are multiples of ten if you adapt a numeral cube from a Roll-a-Problem game (suggested in the follow-up section for page 241) or if you use a large die. Announce a rule such as "Multiply by 20" (or another multiple of ten), and then roll the cube and call out the numeral that appears. The children should try to do the multiplication in their heads, but allow them to write the product on scrap paper if necessary. Give them a little time to think about the product and record it; then direct them to divide that product by some number less than 10. (Be sure to choose a number that is a factor of the product the children find.) Again, allow the children a little time to think, but keep the pace lively. Call on a child to give the answer, and before you agree or disagree, ask several others whether they think that his answer is correct. If any children have difficulty understanding the correct answer, work through the problem on the board.

Resources for Active Learning

Developmental Math Cards, H14, Addison-Wesley.

Workbook, page 106

Using the Exercises

You may choose to treat several of these exercises as a basis for discussion, particularly exercises 1 and 2. Some children will solve exercise 1C only after much trial and error. In exercise 2, help them see how estimations can help them in division.

Most of the children should be able to study the pattern of the *Think* problem and predict a 6-digit answer in which all of the digits are ones, in other words, $n = 111\,111$. Ask the faster children to try to extend the pattern by predicting the next several lines.

Assignments (page 289)
Minimum: 3. Average: 1–3.
Maximum: 1–3.

PAGES 290–291

Objective

Given word problems which involve division, the child will be able to solve the problems by finding the quotients.

Preparation

A short review of estimating and multiplying by multiples of ten would be a suitable preparation for this lesson. You might begin by asking the children if they know their 29's or their 58's. Remind them that they can make a rough guess by first thinking of 29 as 30, or of 58 as 60, and then multiplying. Mention again that this "good guessing" is called *estimation*. Ask them to estimate 29×3. They should quickly get 90 by mentally computing 30×3. Follow through with a brief, well-paced practice session on mental estimation.

Solving Story Problems

1 300 baseball cards.
5 on each page.
How many pages? 60

2 320 airplane cards.
4 cards on each page.
How many pages? 80

3 96 marching-band players.
8 rows. Same number in each row. How many in each row? 12

4 To the zoo in buses.
210 children. 7 buses.
Same number in each bus.
How many children in each bus? 30

5 108 boys.
9 boys on each team.
How many teams? 12

6 Square dancing. 112 girls.
8 girls make a "square."
How many squares? 14

7 192 trading stamps. 8 pages.
Same number on each page. How many stamps on each page? 24

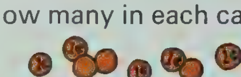
8 240 doll-picture cards.
6 on each page.
How many pages? 40

9 450 kilometres in 5 hours.
Same number of kilometres each hour. How many kilometres each hour? 90

10 216 pieces of candy.
6 pieces in each bag.
How many bags? 36

11 168 pennies.

Same number in each of 8 cans.
How many in each can? 21



12 Light bulb.
Blinks 9 times each minute.
378 blinks. How many minutes have passed? 42

290

Discussion

It would be helpful to work through several of these problems using the problem-solving guidelines. For example, problem 2 might be developed as follows:

"What do I know?" We know that 320 cards are separated into groups of 4 on a page.

"What must I find?" We must find how many groups or pages of 4 there are.

"What do I do?" This is a division situation, and our equation is $320 \div 4 = n$. To solve this equation, subtract fours from 320; or recall that since $4 \times 8 = 32$, then $4 \times 80 = 320$; so $320 \div 4 = 80$.

"Does my answer make sense?" Upon rereading the problem, we agree that 80 is a reasonable answer.

Guide the children through other problems similarly. Exercise 12 might be difficult for some, so begin by having everyone estimate the number of minutes that pass while the light blinks 378 times. Then, some of the children can subtract multiples of ten times nine to find the exact number of minutes.

You may choose to assign the problems on page 291 as individual work or read them with the class. If time permits, let the children make up other word problems

At the Scout Camp

1. 8 Girl Guides slept in each cabin. How many cabins were used by the 56 girls from Pine City? 7
2. 54 Boy Scouts went to Camp Eagle in 6 station wagons. There were the same number of scouts in each station wagon. How many went in each station wagon? 9
3. There were 270 Girl Guides at Camp Sunrise. There were 9 troops of the same size. How many were in each troop? 30
4. One week 208 scouts came to Camp Eagle. They lived in tents. 4 scouts slept in each tent. How many tents were used? 52
5. 200 Girl Guides ate meals in a large cabin. 8 girls sat at each table. How many tables were there? 25
6. 72 scouts planned to take a boating trip from Camp Eagle to Camp Sunrise. If each boat could hold 6 scouts, how many boats were needed? 12

7. On July 1, a scoutmaster at Camp Eagle bought a bottle of pop for each camper. There were 540 Boy Scouts and Girl Guides at the campfire party. How many cartons of 6 bottles did he buy? 90
8. During the summer 288 campers visited an Indian museum. The guide took the campers through the museum in groups of 9. How many trips did he make in all? 32

More practice, page A-35, Set 47

Follow-up

To give children practice in estimating with multiples of ten, prepare a worksheet similar to this:

Estimate the answer to each equation by thinking with multiples of 10.	
Sample $4 \times 59 = ?$ Think: $4 \times 60 = 240$ so the estimate is 240.	
Equation	Estimate
$5 \times 38 = ?$	
$3 \times 29 = ?$	
$8 \times 31 = ?$	
$7 \times 52 = ?$	
$9 \times 27 = ?$	
$6 \times 41 = ?$	

which require division. Some children would benefit from trying to rewrite the stories on page 291 in short-story form. Such practice should help children identify the essential information in a story problem, and it should make problem analysis easier. You may do this as a group activity with less capable children and have them set up the equations to solve the problems but omit the computation.

Assignments (page 291)
Minimum: 1–8, oral. Average: 1–8.
Maximum: 1–8.

PAGES 292–293

Objectives

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Given problems involving money, the child will solve them and demonstrate his ability to use decimal notation for dollars and cents.

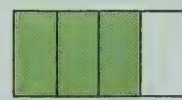
Preparation

Since these pages are intended primarily for review, you might begin immediately with their discussion. However, if you prefer, use an oral warm-up to review regrouping or basic multiplication facts.

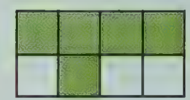
Keeping in Touch with Addition, Subtraction, Multiplication, Fractions, Story problems

1. Write a fraction that shows the part of each region that is shaded.

A 3/4



B 5/8



C 1/5

2. Find the products.

A 34 × 2 = 68	B 26 × 3 = 78	C 42 × 4 = 168	D 37 × 5 = 185	E 73 × 6 = 438
F 68 × 5 = 340	G 59 × 6 = 354	H 74 × 7 = 518	I 83 × 8 = 664	J 92 × 9 = 828

3. Answer "more than 100" or "less than 100" for each product.

A 3 × 33 Less than	D 4 × 33 More than	G 6 × 20 More than	J 11 × 11 More than
B 3 × 34 More than	E 5 × 21 More than	H 10 × 9 Less than	K 9 × 11 Less than
C 4 × 22 Less than	F 5 × 19 Less than	I 10 × 11 More than	L 9 × 12 More than

4. Find the sums.

A 15 + 32 + 40 = 87	B 62 + 24 + 46 = 132	C 74 + 37 + 53 = 164

★ **5.** Find the missing digits.

A 3 ▥5 + 4▥ 6 = 81

B 7 ▥8 + 3▥ 4 = 112

C 6▥ 7 − 2 ▥8 = 39

D 5 ▥4 − 1▥ 6 = 38

E 4 ▥6 × 2 = 92

F 3▥ 7 × 4 = 148

think

There were two stacks of checkers on the table. 6 checkers were removed from one stack and placed on the other. Then the stacks had the same number of checkers. Before moving the 6 checkers, how many more did the taller stack have than the shorter one? 12

292

Discussion

You may choose to work through the exercises on page 292 with the children as review, or you may have them work the exercises first on their own and then discuss them as you check the work. Stress any concept or skill with which the class has had difficulty. The children may enjoy discussing exercise 5, in which they are to find the missing digits. This type of exercise is particularly stimulating if the children approach it as they would a challenging game.

It would be helpful to discuss parts of page 293. For instance, in exercise 6, read through the examples with the children and then ask them to complete the exercises. Point out that converting both amounts to cents will give two addends. After finding the sum of the addends, they can convert back to dollars-and-cents notation. This procedure forestalls preoccupation with regrouping over the decimal point. Allow time for checking papers; and if time permits, suggest that the children make up word problems involving money. For this lesson, it might be helpful to have a few children practice writing dollar notation on the chalkboard as others name amounts.

Give a stack of checkers to each

6. Give the missing numbers.
Examples:
$3.47 means 3 dollars and 47 cents.
$3.47 means 347 cents.

- **A** $5.39 means ▯ dollars and ▯ cents. 5, 39
- **B** $3.86 means ▯ dollars and ▯ cents. 3, 86
- **C** $4.23 means 3 dollars and ▯ cents. 123
- **D** $7.33 means ▯ cents. 733

1 dollar
is worth
100 pennies

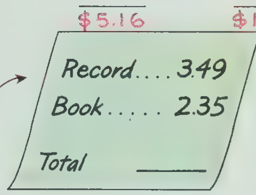
7. Find the total amounts.
Example: $1.34 and $3.23 is $4.57.

- **A** $3.32 and $2.45 $5.77
- **B** $12.57 and $8.32 $20.89
- **C** $3.64 and $4.58 $8.22
- **D** $8.64 and $1.36 $10.00
- **E** $ 7.32 + 12.68 = $20.00

8. Find the difference in the amounts.

	Example	A	B	C	D
	$4.65	$5.86	$9.47	$8.52	$4.38
	3.13	2.42	2.13	3.36	2.53
	$1.52	$3.44	$7.34	$5.16	$1.85

9. **A** Susan bought a record and a book. The clerk wrote the costs down like this. Find the total amount. $5.84



B Susan gave the clerk 6 dollars. How much change did she get? $.16

C Susan had $8.67 when she left home. She spent $6.34. How much did she have left? $2.33

10. Ken bought a baseball and a bat. Find the total cost. $5.25

You are invited to explore

ACTIVITY CARD 16
Page 317

293

Follow-up

Toy catalogues, sale circulars, and Sunday supplements offer material for the class to use in devising their own money problems. Ask the children to cut out two items priced under $10 that they might like to receive for Christmas or a birthday, and direct them to paste these items on separate sheets of tagboard with price tags. After many of the pupils are finished, prop their pictures on the chalk tray, and ask the children to pretend that they have saved $10 with which to shop for a special gift for their brother's birthday. Then instruct the children to select four possible gifts and to tell how much money would remain from the ten-dollar bill after the purchase of each item.

Resources for Active Learning

Developmental Math Cards, G⁵9, F⁵21, Addison-Wesley. [Arithmetic evaluations]

Mathex: Measurement and Estimation No. 5, "Penny Bank Game," pp. 43–44, Encyclopaedia Britannica Publications Ltd.

Nuffield Project: *Computation and Structure* 2, "Money," pp. 91–102, Wiley.

child or to a small group of children who try to solve the *Think* problem on page 292. After they have had time to experiment with these, ask someone who has solved the puzzle to tell the class how he arrived at his solution and to defend it, if he needs to. If no one solves the problem, pick out two stacks of checkers, such as stacks of 3 and 9, and demonstrate removing 6 of the 9 checkers to see whether the two stacks will then match. Solicit the children's ideas on the size of other stacks to try, until you arrive at the correct number in each stack by trial and error.

PAGES 294–295

Objective

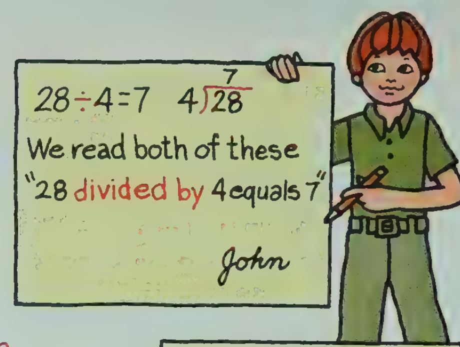
Given a division equation, the child will be able to write the problem using the traditional division notation ($\overline{)\quad}$).

Preparation

Use a game like "What's My Rule" (see page 69) to provide the children with practice in finding quotients of the basic facts. Remind them that every quotient may be thought of as a missing factor.

Let's explore ways to write division exercises.

Discussing the Ideas



John made a poster to show a new way to write division exercises.

1. Show how to write the following division equation using the new way.

   $45 \div 5 = 9$ $\quad 5\overline{)45}$ with 9 above

For $24 \div 3 = 8$
we write
$8\overline{)24}$ with 3 above
Charles

2. Explain what Charles did wrong on his paper.
   He wrote the problem as if it were $24 \div 8 = 3$ instead of $24 \div 3 = 8$.

3. Which letter—A, B, or C—represents the quotient? B

   $\boxed{A}\overline{)\boxed{C}}$ with $\boxed{B}$ above

4. Fran wanted to find how many pages she would need for 78 doll pictures, if she put 3 pictures on each page. She wrote her work like this.
   - A How many threes did Fran subtract the first time? 10
     the second time? 10
     the third time? 6
   - B How many threes did she subtract altogether? 26
     What is the quotient? 26
     How many pages does she need for the doll pictures? 26

$$
\begin{array}{r l}
 & 26 \\
3\overline{)78} & \\
30 & (10) \\
\hline
48 & \\
30 & (10) \\
\hline
18 & \\
18 & (6) \\
\hline
0 &
\end{array}
$$

FRAN

294

Discussion

As you discuss the illustration in the text, write a division equation such as $24 \div 6 = 4$ on the chalkboard and show the corresponding traditional notation as indicated:

$$24 \div 6 = 4 \qquad 6\overline{)24} \text{ with 4 above}$$

When you discuss exercise 2, help the children see that Charles wrote $24 \div 8 = 3$ instead of $24 \div 3 = 8$. It is not necessary to teach the term *divisor* here; it will be treated in a subsequent lesson.

Guide the children through exercise 4. Present other examples of this form of the algorithm, remembering to use only those which have no remainder. For instance, write an example like $3\overline{)96}$ on the chalkboard and ask the class how many threes (using a multiple of ten) to subtract. Then ask how many more threes to subtract. Complete the subtractions, add the number of threes subtracted, and show the children where to write the quotient. In your demonstrations, use arrows as shown in the illustration for exercise 4.

Using the Ideas

1. Write each of these exercises using the new method.

A $12 \div 4 = 3$ (3, $4\overline{)12}$) B $14 \div 2 = 7$ (7, $2\overline{)14}$) C $48 \div 6 = 8$ (8, $6\overline{)48}$) D $63 \div 9 = 7$ (7, $9\overline{)63}$)

2. Find the quotients.

A $2\overline{)6}$ 3 B $2\overline{)14}$ 7 C $8\overline{)24}$ 3 D $8\overline{)40}$ 5 E $8\overline{)56}$ 7

F $5\overline{)25}$ 5 G $4\overline{)16}$ 4 H $4\overline{)32}$ 8 I $6\overline{)36}$ 6 J $7\overline{)56}$ 8

3. Find the quotients.

A $2\overline{)46}$ 23
20 (10)
26
20 (10)
6
6 (3)
0

B $5\overline{)85}$ 17
50 (10)
35
35 (7)
0

C $4\overline{)144}$ 36
80 (20)
64
40 (10)
24
24 (6)
0

D $3\overline{)114}$ 38
90 (30)
24
24 (8)
0

4. Copy each exercise and give the missing numbers.

A $3\overline{)42}$ 14
30 (?) 10
12
12 (?) 4
0

B $6\overline{)78}$ 13
60 (?) 10
18
18 (?) 3
0

C $4\overline{)92}$ 23
80 (?) 20
12
12 (?) 3
0

D $7\overline{)105}$ 15
70 (?) 10
35
35 (?) 5
0

5. Find the quotients.

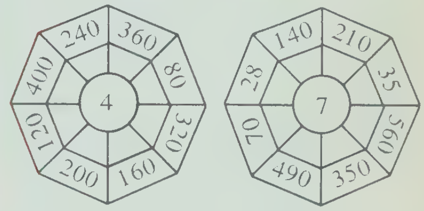
A $2\overline{)34}$ 17 B $5\overline{)65}$ 13 C $3\overline{)48}$ 16 D $6\overline{)72}$ 12 E $4\overline{)96}$ 24 F $7\overline{)84}$ 12

G $8\overline{)96}$ 12 H $9\overline{)108}$ 12 I $4\overline{)68}$ 17 J $7\overline{)98}$ 14 K $6\overline{)90}$ 15 L $3\overline{)51}$ 17

More practice, page A-36, Set 48

Follow-up/Practagons

For those who need to improve their skill in finding missing factors, fill in some of the blank practagons on the master worksheet as shown below and instruct the children to find the missing factors. Leave at least two of the practagons blank so that the children can create and solve problems of their own.



Resources for Active Learning

Toward Improving Computation, "Multiplication and Division," pp. 73–75; "Division," pp. 66–70; "Division with Bean Sticks," pp. 56–58, Curriculum Development Associates.

Duplicator Masters, pages 56, 57
Workbook, pages 107, 108
Skill Masters, pages 56, 57

Using the Exercises

You might choose to guide the children through several of these exercises. However, some children should be encouraged to try to do them independently. For exercises 3 to 5, ask the children not to put the arrows in their work. If children understand the division process, the 3-digit numerals in exercises 3C and 3D should not present any special difficulty. You may choose to work through the first parts of exercise 5 with the children. Encourage them to think of subtracting multiples of 10 times the divisor, but of course it is acceptable to use any repeated subtraction which will yield the correct quotient.

After they have finished the exercises, have several children present their techniques for finding quotients in exercise 5, either to small groups or to the whole class.

Assignments (page 295)*
Minimum: 1–3. Average: 1–5.
Maximum: 1–5.

PAGES 296–297

Objective

Given a simple if-then division statement, the child will be able to write and complete a division problem that would solve it.

Preparation

Materials

counters (50 per child)

No specific preparation is needed before undertaking the investigation in this lesson.

Investigation

If the number of counters available is not sufficient to allow children to work individually, have them work in pairs, or if necessary, in groups of three. As you read the directions, point out that in the first problem it is necessary to agree that each box contains the same number of books. Caution the children to separate the counters carefully. Remind them that for each problem they must write a division problem using the new notation they learned in the last lesson. On the chalkboard you might write a few other examples like those below, for the children who finish quickly.

IF	THEN
85 beads fill 5 boxes	? beads fill 1 box.
64 apples fill 8 boxes	? apples fill 1 box.
6 bottles fill 1 case	42 bottles fill ? cases.

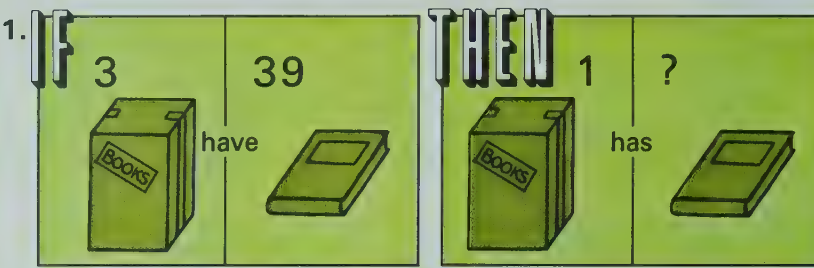
As you move around the room, try to lead the child's interest from the counters to the written division problems.

● *Let's use division to solve problems.*

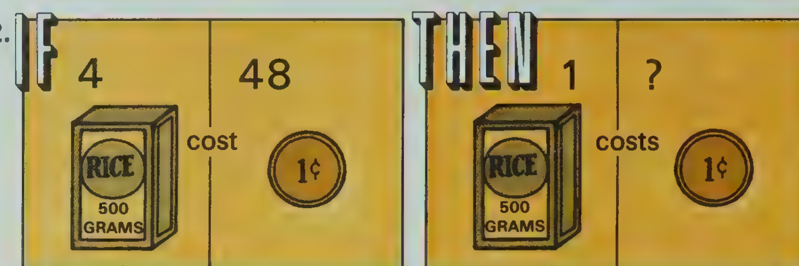
Investigating the Ideas

Divide sets of counters to find the answers to these problems.

1.



2.



1. $\overset{13}{3\overline{)39}}$
2. $\overset{12}{4\overline{)48}}$

? Can you write and complete a division problem that would help you answer the questions above? See Investigation.

Discussing the Ideas

1. There is a total of 54 washers in 3 boxes. Write and solve a division problem that will tell you how many washers are in each box. Why is division used to solve the problem? See Discussion. $\overset{18}{3\overline{)54}}$

2. Can you make up a problem like those above and explain how to solve it by using division? Answers will vary.

296

Discussion

Let volunteers discuss how they would solve exercise 1. Some may begin by writing $54 \div 3 = n$. Some may immediately use the new notation $3\overline{)54}$. Both are correct, although the equation should be followed by the other notations when the problem is worked out. All the children should realize why division is used. Again mention that it is necessary to agree that each box contains an equal number of washers. Relate this problem to the first problem in the investigation, and ask volunteers to write their completed division problems from the investigation on the chalkboard. Review the use of the division algorithm. As the children make up the problems for exercise 2, you might have volunteers work the problems on the chalkboard, explaining why division is used in each case. Be sure that the numbers you give the children to use will not involve a remainder other than zero.

Using the Ideas

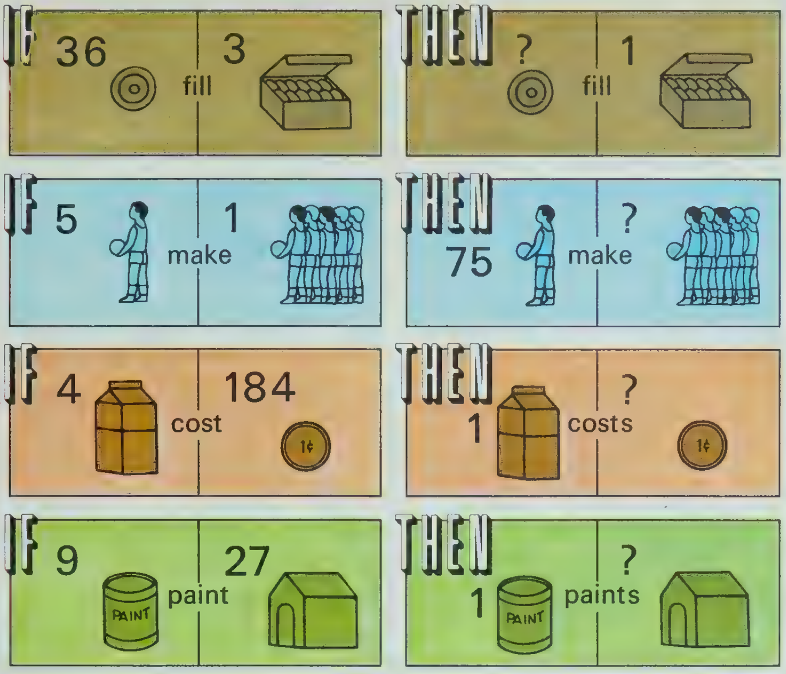
Solve the problems.

1. $3\overline{)36}$ 12

2. $5\overline{)75}$ 15

3. $4\overline{)184}$ 46

4. $9\overline{)27}$ 3



think

There are 3 stacks of checkers on a table.
6 checkers were removed from 1 stack and divided equally between the other 2 stacks.
Then all the stacks had the same number.
At the beginning, how many more checkers were in the tall stack than in the shorter ones? 9

297

Follow-up

To give children practice with the division algorithm, prepare a worksheet similar to the following:

Complete each example. Study the difference between the longer method and the shorter.

1. $4\overline{)96}$ 40 ○ 56 40 ○ 16 16 ○	$4\overline{)96}$ 80 ○ 16 16 ○
2. $6\overline{)78}$ 30 ⑤ 48 ____ ○	$6\overline{)78}$ 60 ○ 18 ____ ○
3. $4\overline{)84}$ 40 ○ 44 40 ○ 4 ____ ○	$4\overline{)84}$ 80 ○ 4 4 ○
4. $3\overline{)72}$ 30 ○ 42 ____ ○ 12 ____ ○	$3\overline{)72}$ ____ ○ 12 ____ ○

Resources for Active Learning

Math Workshop: Games and Enrichment Activities, "Three Boxes," pp. 10–11, Encyclopaedia Britannica Educational Corp. [An If-Then situation]

Using the Exercises

Assign the problems on page 297 as independent work. Remind the children to write out the division problem for each. Encourage those who finish quickly to make up problems of their own. To insure that the numbers they choose can be divided evenly, you might ask that they use only numbers from a set you have written on the chalkboard, such as:

81, 3; 74, 2; 38, 2; 64, 4;
88, 4; 76, 2

The *Think* problem is intended as enrichment. Give those who wish to try it some checkers to use for experimenting with different groupings. It would be helpful to pass out both red and black checkers. Suggest that they make one stack red and the other two black, or vice versa.

Assignments (page 297)

Minimum: 1–4, oral. Average: 1–4.
Maximum: 1–4.

PAGES 298–299

Objective

Given short story problems, the child will demonstrate his ability to use any of the four basic operations to solve them.

Preparation

If you wish to treat page 298 as independent work, it would be appropriate to use a short game to review all basic operations. Choose any review game that the children particularly enjoy. The "What's My Rule" game, when played with the grid on the chalkboard as suggested on page 69, may be used to show the results of different operations:

a	7	5	8	4	6	9
$a+9$	16	14	17	?	?	?
$a \times 4$	28	20	32	?	?	?
$(a \times 4) - 9$	19	11	23	?	?	?
$(a \times 4) \div 2$						

(The rules are given for your convenience; they need not be written for the children.)

Solving Short Stories

1 78 children.
9 more came later.
How many children in all? 87

2 162 children.
Same number of children in each of 6 groups.
How many in each group? 27

3 82 children. 37 boys. How many girls? 45

4 315 chairs.
7 rows.
Same number in each row.
How many in each row? 45

5 6 baskets.
48 kilograms per basket.
How many kilograms in all? 288

6 Satellite makes 558 orbits.
9 orbits each day.
How many days? 62

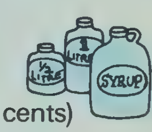
7 108 eggs.
9 cartons.
Same number in each carton.
How many in each carton? 12

8 7 days in a week. 364 days.
How many weeks? 52

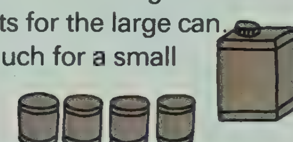
9 7 boxes weigh 112 grams.
How many grams does 1 box weigh? 16

10 Walk 5320 metres in one hour.
How far in 2 hours? 10640

11 87 cents for 3 litres of ice cream.
How much per litre? 29¢



12 4 cans of oil in a large can.
96 cents for the large can.
How much for a small can? 24¢



★ 13 $1.92 (192 cents) for a jug of syrup.
4 litres in a jug.
How much for a half litre? 24¢

298

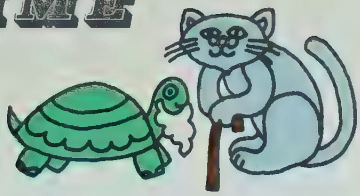
Discussion

Page 298 offers a good opportunity to let the children demonstrate their understanding of the four basic operations. However, many children will need guidance in determining the problem situation. If you think your children are capable of doing many of these problems independently, let them do so before discussing them. If you think that most will need guidance, you may use the problems as a basis for class discussion or have the children work in groups of two or three. Many would benefit from having the problem-solving guidelines written on the chalkboard as an outline to follow in attacking each problem:

What do I know?
What must I find?
What do I do?
Does my answer make sense?

The short stories on page 299 should be treated the same as those on the preceding page. If your class is capable, encourage them to work independently. Otherwise, work through several problems together. Keep in mind that the discussion which accompanies a page of problems like these is an important part of the lesson, since the problems are designed to stimulate children's interest in arithmetic and its applications.

TIME

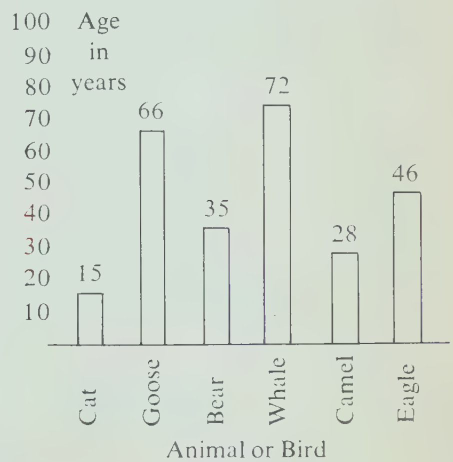


Animals, trees, birds, and insects grow old. Exercises 11 through 18 tell how old they sometimes grow.

1. 4 o'clock now. Sleep for 7 hours. What time will it be? 11 o'clock
2. Walk a kilometre in 15 minutes. Run a kilometre in 6 minutes. How much quicker to run? 9 min
3. 60 minutes in 1 hour. 8 hours. How many minutes? 480
4. Machine runs 424 minutes a day. Makes one thing each 8 minutes. How many things? 53
5. 1 day has 24 hours. 9 days. How many hours? 216
6. 126 days make 9 fortnights. 1 fortnight is how many days? 14
7. Total sleep, 736 hours. 8 hours each day. How many days? 92
8. 658 days. How many weeks? 94
9. Turbojet goes 928 km/h (kilometres per hour). Gas-engine plane goes 527 km/h. How much faster is the jet? 401 km/h
10. 420 seconds. Same as 7 minutes. How many seconds in a minute? 60
11. An old cat: 15 years old. An old turtle: 10 times as old. How old is an old turtle? 150 yr
12. An old rabbit: 6 years old. An old goose: 11 times as old. How old is an old goose? 66 yr
13. An old bear: 35 years old. An old camel: 28 years old. How much older is an old bear than an old camel? 7 yr
14. An old reindeer: 12 years old. An old whale: 6 times as old. How old is an old whale? 72 yr
15. Spruce tree: 243 years old. Lives 339 more years. How old? 582 yr
16. An old eagle: 46 years old. An old elm tree: 7 times as old. How old is an old elm tree? 322 yr
17. An old elephant: 61 years old. An old cow: 24 years old. How much older is an old elephant than an old cow? 37 yr
18. An old butterfly: 8 weeks old. An old housefly: 6 weeks old. How many **days** older is the old butterfly? 14

Follow-up

Exercises 11 through 18 on page 299 might stimulate children's interest in finding out the ages of a variety of things they have read about or are studying. You might suggest that children try to find out the ages of some of your local landmarks. With such information, or even with information from the exercises, the children might make illustrated charts to show ages of things in various categories. If you remind them of the earlier lesson on bar graphs (pages 270, 271), some more capable children may make graphs to show the ages of a number of items in a given category such as old animals or public buildings and monuments. The graph below represents one such possibility.



Resources for Active Learning

Developmental Math Cards, G²15, Addison-Wesley.

Nuffield Project: Computation and Structure 2, "Weight," pp. 23–33; "Capacity," pp. 33–41; "Time," pp. 82–90, Wiley.

Duplicator Masters, page 58
Workbook, page 109

Assignments (page 298) Minimum: Even-numbered problems. Average: 1–12. Maximum: 1–13.

Assignments (page 299) Minimum: Odd-numbered problems. Average: 1–18. Maximum: 1–18.

PAGES 300–301

Objective

Given a division problem with a remainder other than zero, the child will be able to find the quotient and the remainder.

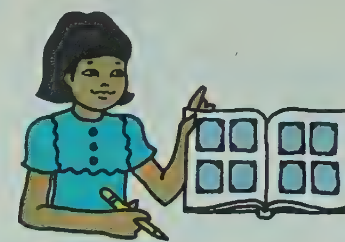
Preparation

Use a brief oral review of finding missing factors for the basic multiplication facts and for equations in which the product is a multiple of 10 or 100. For example, say, "What number times 50 equals 400?" or "What number times 7 equals 490?"

Is the remainder always 0?

Discussing the Ideas

Sara had 51 photographs to put into her new photo book. She could put 4 pictures on a page. She decided to use division to find how many pages she would need.



quotient → 12
4)51 ← divisor
40 (10)
11
8 (2)
3 ← remainder

1. When Sara divided 51 by 4, what did she get for the quotient? 12

2. After Sara used 12 pages of the book, how many photos would she have left? 3

3. Notice that 3 is called the **remainder** and 4 the **divisor**.
   A If 4 is the divisor, could you have a remainder of 4? Explain. No
   B Could you have a remainder greater than the divisor? No

4. Explain this rule. See Discussion.

   Always carry out the dividing until the remainder is less than the divisor.

5. Give the quotient, divisor, and remainder for each example. Is the remainder always less than the divisor? Yes

A
6 quotient
divisor 4)26
24
2 remainder

B
3 quotient
divisor 7)25
21
4 remainder

C
27 quotient
divisor 5)136
100
36
35
remainder 1

D
quotient 45
divisor 3)135
120
15
15
remainder 0

300

Discussion

Ask the children to study the example at the top of the page. It would be helpful to use felt objects on the flannelboard for a demonstration to illustrate the problem. Then discuss what the remainders are if you divide by 5, or by 3. Present other examples as needed. For instance, you might place a set of 19 felt objects on the flannelboard and ask a child to remove the objects 3 at a time to find out how many sets of 3 he can remove. When he has finished, observe with the class that he was able to remove 6 sets of 3 with 1 object left over. Now, show the corresponding division problem in vertical notation on the chalkboard or on the acetate of the overhead projector.

6 ← quotient
divisor → 3)19
18
1 ← remainder

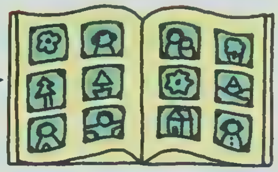
Explain that in this problem the 3 is called the divisor, 6 is called the quotient, and 1 is called the remainder. Label the corresponding numerals in your example as you talk, emphasizing the new words *divisor* and *remainder*. Point out that the remainder is less than the divisor; and for exercise 4, be sure children realize that if the re-

Using the Ideas

1. Find the quotients and remainders.

A $3\overline{)28}$ 9 R1	B $4\overline{)39}$ 9 R3	C $5\overline{)42}$ 8 R2	D $6\overline{)45}$ 7 R3	E $7\overline{)59}$ 8 R3
F $8\overline{)78}$ 9 R6	G $9\overline{)71}$ 7 R8	H $4\overline{)46}$ 11 R2	I $3\overline{)98}$ 32 R2	J $7\overline{)166}$ 23 R5
K $4\overline{)237}$ 59 R1	L $8\overline{)337}$ 42 R1	M $3\overline{)181}$ 60 R1	N $9\overline{)376}$ 41 R7	O $5\overline{)364}$ 72 R4

2. Susan had 57 pictures for her new album. She put 6 pictures on each page.
   A How many full pages could she get? 9
   B How many pictures would be left over for the last page? 3



3. Jim has 221 old pennies. His coin book has room for 9 pennies on each page.
   A How many full pages could he get? 24
   B How many pennies would be left over for the last page? 5

★4. Can you write and solve a division problem that will help you answer each question?

A	35 marbles	 8 8 8 8 ?	How many left over?	3
B	132 marbles	 6 6 6 ?	Less than 6 left over. How many bags? 22 How many left over?	0
C	127 marbles	 5 5 5 ?	Less than 5 left over. How many bags? 25 How many left over?	2

More practice, page A-36, Set 49

301

mainder in a problem were larger than the divisor, the quotient would be misleading, since it would not be the greatest quotient possible.

Work through other problems in which the remainder is not zero, in a similar manner, and then write the vertical notation on the chalkboard and call on children to point out the quotient, divisor, and remainder. You might also write some of these problems in equation notation and ask children to convert them into the traditional vertical notation before they point out the parts. Continue to emphasize that the remainder must be less than the divisor.

Using the Exercises

On page 301 you might find it worthwhile to work through a couple of parts of exercise 1 with the class before assigning the rest of the exercises as independent work. Starred exercise 4 is intended primarily for the faster children, but the entire class might benefit from a discussion of it later, after the written work is completed.

Assignments (page 301)*
Minimum: 1; 2–3, oral. Average: 1–3. Maximum: 1–4.

Follow-up

Now that the children have had experience in working division problems with a remainder, you might ask them to make up division problems and write them on cards; they can then exchange cards and try to solve the problems.

You might also duplicate on worksheets or write on the chalkboard a list of numbers to be divided by the same divisor. Such lists might be presented in the form suggested by the sample below.

Give the quotient and the remainder (if any).

÷	7	÷	4	÷	9
63	—	82	—	32	—
72	—	54	—	108	—
38	—	36	—	95	—
91	—	78	—	47	—
45	—	60	—	124	—
29	—	25	—	76	—

Resources for Active Learning

Mathex: Operations No. 3, pupil pages 46–47, Encyclopaedia Britannica Publications Ltd. [Using the "Hundred Square" to perform operations]

Workbook, page 110

PAGES 302–303

Objective

Given a division problem, the child will be able to find the quotient and remainder and check his answer by multiplying the quotient and divisor and adding the remainder.

Preparation

You may wish to begin immediately with the investigation in the text and save time to introduce a game as a follow-up activity. Or you might spend a few minutes in an oral review, such as a mental chain-game like this:

"Start with 54 . . . Divide by 6 . . . Multiply by 8 . . . Subtract 2 . . . Divide by 10 . . . Your answer is . . ." (7)

"Start with 42 . . . Divide by 7 . . . Multiply by 4 . . . Divide by 3 . . . Subtract 2 . . . Multiply by 7 . . . Your answer is . . ." (42)

Investigation

Before directing the children's attention to the investigation in their text, remind them of the relation between multiplication and division by displaying related equations on the chalkboard or overhead projector; for example:

$3 \times 8 = 24 \quad 24 \div 8 = 3 \quad 8\overline{)24}$ (quotient 3)

Next, help the children read through the directions for the investigation. When you feel that they understand what is expected of them, ask them to copy and solve the multiplication problems on their own and then "grade" Cynthia's paper as follows:

1. Copy the problems and the answers as Cynthia has written them.
2. Put a check mark beside each correct quotient.
3. Mark an *X* through each incorrect quotient and beside it write the correct quotient.

Move about the room observing the children as they work. If some children simply use the division algorithm to check the answers, remind them that they should try to find a related multiplication problem that will show them whether the quotient is correct.

How can answers in division be checked?

Investigating the Ideas

Find each product.

A 27×4 108 B 15×6 90 C 44×3 132

D 34×7 238 E 32×5 160 F 36×3 108

Cynthia

1. 238 ÷ 7 = 34 ✓
2. 160 ÷ 5 = 32 ✓
3. 108 ÷ 4 = ~~24~~ 27
4. 108 ÷ 3 = ~~34~~ 36
5. 90 ÷ 6 = 15 ✓
6. 132 ÷ 3 = ~~42~~ 44

? Can you use your answers to the exercises above to help you grade this paper? See Investigation.

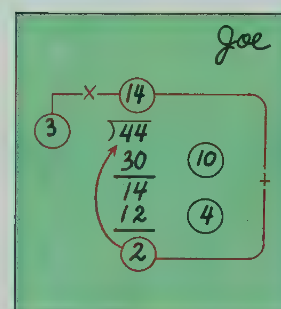
Discussing the Ideas

1. Explain how Patty can use multiplication to see if she has found the quotient for 48 ÷ 4. Multiply the quotient by the divisor.

Patty

$4\overline{)48}$ quotient 12; 40 (10); 8; 8 (2); 0

2. Joe made this poster to explain how to check division when the remainder is not zero.
   A What is the product of 3 × 14? 42
   B Explain why you must **add** the remainder to this product to get 44. See Discussion.
   C Explain how Joe's diagram shows a way you can check division. See Discussion.



302

Discussion

When the children have completed the investigation, let them discuss their work, and have volunteers identify the multiplication problem that is related to each division problem.

Work through the discussion exercises with the children. Emphasize the need for the addition step in checking division problems with a nonzero remainder, as illustrated in exercise 2. Present several more examples to help children understand how to check division. Stress problems in which the remainder is nonzero. Stress that the remainder must be added to the product of the divisor and quotient in order to get the dividend with which we started. Although the term *dividend* is not treated formally in the text, you may choose to introduce it as part of the oral vocabulary associated with division.

Using the Ideas

1. Find the quotients and remainders. Then check your answers.

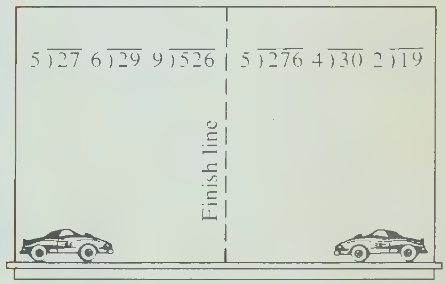
A $4\overline{)25}$ 6 R1	B $5\overline{)27}$ 5 R2	C $3\overline{)10}$ 3 R1	D $6\overline{)39}$ 6 R3	E $2\overline{)19}$ 9 R1
F $5\overline{)37}$ 7 R2	G $6\overline{)29}$ 4 R5	H $7\overline{)65}$ 9 R2	I $4\overline{)30}$ 7 R2	J $3\overline{)29}$ 9 R2
K $8\overline{)206}$ 25 R6	L $6\overline{)154}$ 25 R4	M $5\overline{)276}$ 55 R1	N $7\overline{)185}$ 26 R3	O $6\overline{)134}$ 22 R2

2. In a given part of this exercise, each bag contains the same number of marbles. Use division to help you find the answer. Check your work.

A	161 marbles	? ? ? ? ? ? ?	How many in each bag? 23
B	43 marbles	? ? ? ?	3 left over. How many in each bag? 10
C	39 marbles	7 7 ?	4 left over. How many bags? 5
D	27 marbles	? ? ? ?	3 left over. How many in each bag? 6
E	44 marbles	? 5	4 left over. How many bags? 8
F	60 marbles	9 ?	Less than 9 left over. How many bags? 6 How many left over? 6
G	184 marbles	8 ?	Less than 8 left over. How many bags? 23 How many left over? 0

More practice, page A-37, Set 50

Follow-up/"Chalkboard Auto Races"
The children might enjoy playing some "Chalkboard Auto Races." Before playing the game, you will need to have prepared two cardboard cars or two small toy cars of different colors. Place one of these at each end of the board, on the chalk tray. Divide the class into two teams. On each of the opposite ends of the chalkboard, write a row of division and/or multiplication problems. As play commences, a member of each team goes to the problems furthest from the centre "finish" line. For every problem correctly worked, the team's car moves forward to the next problem. If a child does not solve a problem correctly, another member of his team must try it. As "judge" of the race, you should observe the work and signal the appropriate car forward for every correctly worked problem. The team whose car arrives at the centre finish line first wins.



Duplicator Masters, pages 59, 60
Workbook, page 111
Skill Masters, pages 59, 60

Using the Exercises
When you assign exercise 1 on page 303, ask the children to copy each problem, find the quotient and remainder, and then check it. You may wish to give some children a prepared worksheet of problems that are complete or partially complete with quotients and remainders and ask them to complete and check the work.

Before assigning exercise 2, you might discuss the way the information is presented. For example, tell the class that the illustrations in parts F and G help them see that they must find the number of bags as well as the number left over. You may wish to guide the children in solving these examples as you discuss them together.

Assign the rest of the exercises, mentioning that in some exercises the left-over marbles are pictured as small dots. Later, help the children see that the left over marbles correspond to the remainder in the division problem.

Assignments (page 303)*
Minimum: 1. Average: 1–2.
Maximum: 1–2.

PAGES 304–305

Objective

The child will demonstrate his ability to work with the concepts presented in this chapter.

Preparation

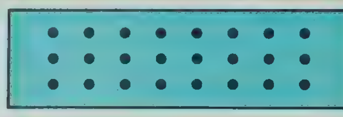
To prepare for these pages, you might use a mental-arithmetic game emphasizing estimation and multiplying with factors which are multiples of 10. For example, say: "I'm thinking of a multiple of 10 which is very near the product 3×49. What's my number?" (150) Or: "I'm thinking of a multiple of 10 which is very near the product 7×28. What's my number?" (210)

Reviewing the Ideas

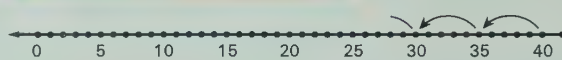
1. Write a division equation to answer each question.

A 

How many sets of 3 are in a set of 21? $21 \div 3 = 7$

B 

If we put 24 dots into 3 sets of the same number, how many dots are in each set? $24 \div 3 = 8$

C 

Starting at 40, how many jumps of 5 does it take to get to 0? $40 \div 5 = 8$

D $\begin{array}{r} 42 \\ -7 \\ \hline 35 \end{array} \quad \begin{array}{r} 35 \\ -7 \\ \hline 28 \end{array} \quad \begin{array}{r} 28 \\ -7 \\ \hline \end{array} \quad \ldots$

Starting with 42, how many times do we subtract 7 to get 0? $42 \div 7 = 6$

E $n \times 9 = 54$ What number times 9 gives 54? $54 \div 9 = 6$

2. Find the quotients.

Since $11 \times 23 = 253$, we know that $253 \div 23 = n$. 11
$253 \div 11 = n$. 23

3. Find the quotients.

A $64 \div 8$ 8	I $49 \div 7$ 7	Q $81 \div 9$ 9
B $28 \div 4$ 7	J $40 \div 8$ 5	R $18 \div 2$ 9
C $20 \div 5$ 4	K $48 \div 6$ 8	S $25 \div 5$ 5
D $18 \div 6$ 3	L $54 \div 6$ 9	T $45 \div 5$ 9
E $27 \div 9$ 3	M $24 \div 6$ 4	U $30 \div 5$ 6
F $45 \div 5$ 9	N $32 \div 8$ 4	V $36 \div 9$ 4
G $42 \div 7$ 6	O $56 \div 8$ 7	W $16 \div 4$ 4
H $36 \div 6$ 6	P $72 \div 9$ 8	X $35 \div 7$ 5

think

Brian's father weighs 50 kilograms more than Brian. Together they weigh 120 kilograms. How much does Brian weigh? 35 kg

304

Discussion

The material on page 304 deals with the concepts of division and division facts, rather than with the division algorithm. The page is fairly simple, and lengthy discussion should be unnecessary.

Be sure that you correct the papers yourself, so you can find any existing areas of weakness.

The *Think* problem on page 304 is intended for more capable children, but the entire class should benefit from a discussion of the trial-and-error approach to its solution. For example, you can illustrate building a table of guesses at the chalkboard or with an overhead projector. You might suggest starting with a guess that Brian weighs 25 kilograms. Then say, "If Brian's father weights 50 kilograms more, how much does his father weigh? How much do they weigh together? According to the problem, is this enough? Should your next guess for Brian's weight be larger or smaller?" Continue to build the table, using similar questions.

Brian's Weight	Father's Weight	Weight Together
25	50 + 25 = 75	25 + 75 = 100
30	50 + 30 = □	

4. Find the quotients.

A $5\overline{)315}$ 63	B $6\overline{)264}$ 44	C $3\overline{)228}$ 76	D $4\overline{)372}$ 93	E $2\overline{)168}$ 84
F $4\overline{)300}$ 75	G $7\overline{)700}$ 100	H $6\overline{)420}$ 70	I $8\overline{)272}$ 34	J $9\overline{)558}$ 62

5. Find the quotients and the remainders.

A $3\overline{)128}$ 42 R2	B $5\overline{)342}$ 68 R2	C $6\overline{)415}$ 69 R1	D $4\overline{)321}$ 80 R1	E $2\overline{)101}$ 50 R1
F $8\overline{)327}$ 40 R7	G $9\overline{)576}$ 64 R0	H $3\overline{)194}$ 64 R2	I $7\overline{)627}$ 89 R4	J $6\overline{)245}$ 40 R5

6.	258 marbles	? ? ? ? ? ?	How many in each bag? 43
7.	175 marbles	7 7 ?	How many bags? 25
8.	?	9 9 9 9 9 9 9	How many marbles? 63

9. Jan had 35 balloons for her party. There were 8 children at the party. Each child got the same number of balloons, and there were 3 left over. How many balloons did each child get? 4

10. Jim had 50 cents. Table tennis balls cost 9 cents each.
- A How many could he buy? How much money is left over? 5, 5¢
- B If he bought only 3 balls, how much money would he have left? 23¢

11. Sara had 75 cents when she went shopping.
- A How many pencils could she buy if they were 6 cents each? 12
  How much would she have left? 3¢
- B If pencils were 9 cents each, how many could she buy? 8
  How much money would she have left? 3¢

305

Follow-up

Create problems like the samples below and suggest that the children become "division detectives" and locate the missing digits. If you prepare a worksheet, order the problems according to difficulty and do not expect any but the more able children to work the harder problems.

Find the digit for each $\square$.

1. $\begin{array}{r} \square \\ 5\overline{)3\square} \\ 35 \\ \hline 2 \end{array}$ 2. $\begin{array}{r} 6 \\ 6\overline{)\square 9} \\ 3\square \\ \hline 3 \end{array}$

3. $\begin{array}{r} 9 \\ \square\overline{)1\square} \\ \square\square \\ \hline 1 \end{array}$ 4. $\begin{array}{r} \square \\ 7\overline{)6\square} \\ 63 \\ \hline 4 \end{array}$

5. $\begin{array}{r} 8\square \\ 5\overline{)\square\square\square} \\ \square\square\square \\ \hline 28 \\ 25 \\ \hline \square \end{array}$ 6. $\begin{array}{r} 4\square \\ \square\overline{)\square\square\square} \\ 24\square \\ \hline 3\square \\ \square 0 \\ \hline 4 \end{array}$

7. $\begin{array}{r} \square 0 \\ 7\overline{)\square 8\square} \\ \square\square\square \\ \hline 5 \end{array}$ 8. $\begin{array}{r} \square 6 \\ 7\overline{)\square 2\square} \\ \square\square\square \\ \hline \square\square \\ \square\square \\ \hline 4 \end{array}$

Resources for Active Learning

Discovery, Section II, Units 5/5; 6/4, 17/4, Encyclopaedia Britannica Educational Corp.

Workbook, page 112

PAGES 306–307

Objective

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

Choose an oral warm-up activity that reviews any of the text material which has been troublesome for the children. If you have detected no general weakness, use a game such as "What's My Rule" to review the four basic operations.

Keeping in Touch with Addition, Subtraction, Multiplication, Division, Story problems

1. Find the sums, products, quotients, and differences.

A $94 + 39$ = 133	B 68×3 = 204	C $78 - 52$ = 26	D $81 + 79$ = 160	E 63×5 = 315	F 79×6 = 474
G $27 + 88$ = 115	H 56×8 = 448	I $65 + 99$ = 164	J 93×7 = 651	K $142 - 80$ = 62	L $125 - 52$ = 73
M $65 - 23$ 42	N $350 \div 7$ 50	O $350 \div 5$ 70	P $540 \div 9$ 60	Q $120 \div 3$ 40	

2. Tell what operation ($+$, $-$, $\times$, $\div$) you think of for:

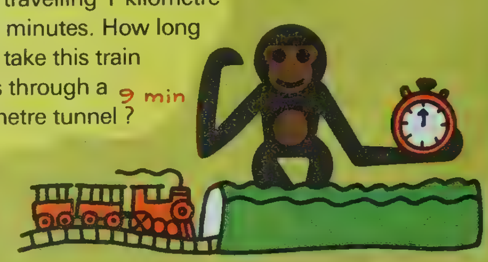
A putting 2 sets together and finding the total number. +
B finding how many are left after some have been taken away. −
C finding how many sets of a certain size we get from a set. ÷
D finding how many in a certain number of rows of the same number. ×
E finding how many more one set has than another. −
F finding how many ways we can pair objects in 2 sets. ×
G finding how many rows when we put a set into rows having the same number. ÷

3. Solve the equations.

A $n + 6 = 11$ 5
B $8 + n = 15$ 7
C $3 \times n = 18$ 6
D $n \times 8 = 24$ 3
E $50 \div 5 = n$ 10
F $8 - n = 6$ 2
G $10 \div n = 2$ 5
H $42 \div n = 6$ 7
I $n - 8 = 6$ 14
J $n \div 6 = 5$ 30
K $18 + n = 24$ 6

think

A train that is 1 kilometre long is travelling 1 kilometre each 3 minutes. How long does it take this train to pass through a 2-kilometre tunnel? 9 min



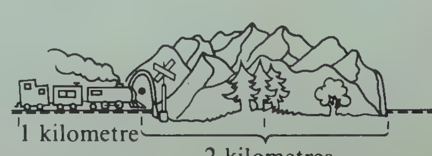
306

Discussion

As with all lessons of this type, you may choose to work through the exercises as a class activity, reviewing concepts and skills as you go along. Or you may assign them as independent work and discuss them when you have the children check their papers. Exercise 2 should serve as a basis for a discussion of the applications of the basic operations.

For page 307, read the problems with the children and ask them to write and solve equations for each exercise.

The *Think* problem on page 306 is worth discussing after the children have spent some time on it. If most of them give 6 as the answer, suggest that they draw a picture like the one below. Tell them to put a colored dot on the engine, or on the caboose, and to check the distance that the dot will have travelled when the train has gone completely through the tunnel. When the children see that this distance is 3 kilometres, then the correct answer, 9 minutes, will be a simple computation.

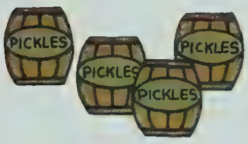


Solving Story Problems

1 Shirt costs $4.89.
Jacket costs $11.56.
How much less is the shirt? $6.67

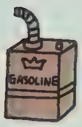
2 182 kilometres on Friday.
496 kilometres on Saturday.
527 kilometres on Sunday.
How far in the three days? 1205 km

3 128 pickles in a barrel.
4 barrels in storeroom.
How many pickles in all? 512



4 Peaches: 12 cents each.
Pears: 16 cents each.
Michael bought 7 peaches and 9 pears. How much did he spend? $2.28

5 138 hockey cards. 6 per package. How many packages? 23



6 513 pansies in cartons.
9 cartons.
How many pansies per carton? 57

7 Use 9 litres of gasoline to go 135 kilometres. How many kilometres travelled on each litre? 15

8 184 tickets. 8 bundles of the same size.
How many in each bundle? 23

9 Sixty-five 8-cent stamps.
Nine 11-cent stamps.
How much change from a 10-dollar bill? $3.81

10 Forty-eight 8-cent stamps.
Sixteen 10-cent stamps.
One hundred ninety-six 2-cent stamps.
How much change from a 10-dollar bill? $.64

Follow-up

If you wish to give the children more practice in problem solving, make up some word problems without numbers, and ask the children to tell you what must be known in order to find each solution. Create other story problems, this time using numbers, and ask what operation (or operations) can be used to solve the problems. Then make up more stories which give information, but ask no questions, and instruct the children to devise problems that could be completed from the given information.

If possible, secure menus and other information from restaurants or cafeterias in your area, especially those where the children are apt to go. Many phases of eating out, such as service, menus, and prices, adapt to story problems. Behind-the-scenes factors (e.g., cost of food supplies, preparation time, wages, and quantities of certain items like ice cream and hamburgers) also lend themselves to problem solving. In some cases, a reception for parents planned by the children or an outing, such as a field trip that includes lunch, can provide an excellent opportunity for problem analysis.

Resources for Active Learning

Developmental Math Cards, G5 19, Addison-Wesley.

Discovery, Section II, Units 1/3; 2/4, Encyclopaedia Britannica Educational Corp.

PAGE 308
The story problems on page 308 should be considered as an extension of the Keeping in Touch lesson on pages 306–307. You may prefer to treat this page as a discussion activity and point out the uses of different operations in the problems. Or you may prefer to assign the problems as independent work. Note that exercise 7B is starred and should be reserved primarily for faster pupils.

Notice also that children are referred to an Activity Card on page 317. You might give those who have not had a previous opportunity a chance to choose from among the activity cards any which they would like to try.

Eating at the Restaurant

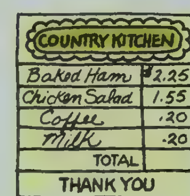
Kay and her mother went to a restaurant in the city.

1. There were 36 tables in the restaurant.
   There were 4 chairs at each table.
   How many chairs were there? 144

2. There were 9 waiters. How many tables might be assigned to each waiter? 4

3. The waiters used 1 pitcher of water for each 4 people.
   Kay counted 56 people. How many pitchers of water did they need? 14

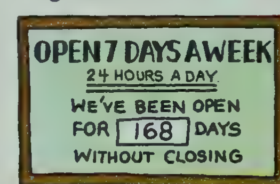
4. The waiter wrote this order for Kay and her mother. What was the total cost of their lunches? $4.20



5. Kay's mother gave the waiter $4.50.
   How much change did she get back? $.30

6. Their lunch cost about $4.00. Kay's mother decided to tip the waiter 15 cents for each dollar that the lunch cost.
   How much extra did she give the waiter for his good service? $.60

7. Kay saw this sign in the window.
   A How many weeks has the restaurant been open without closing? 24
   ★ B How many hours has it been open without closing? 4032



You are invited to explore

ACTIVITY CARD 17
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Mathematical Activities

How to Use the Activity Cards

Do you like to explore things for yourself? These Activity Cards will give you some exciting experiences with mathematics. Each card presents a different idea for you to explore. Often you will find that a card will give you ideas for additional activities on your own.

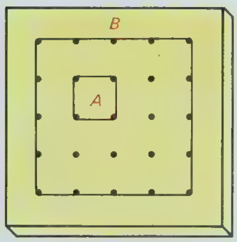
ACTIVITY CARD 1

A is the smallest **square** you can form on the geoboard with one rubber band.

B is the largest square you can form on the geoboard with one rubber band.

Six more squares of different sizes can be formed.

How many of them can you find and draw on dot paper?



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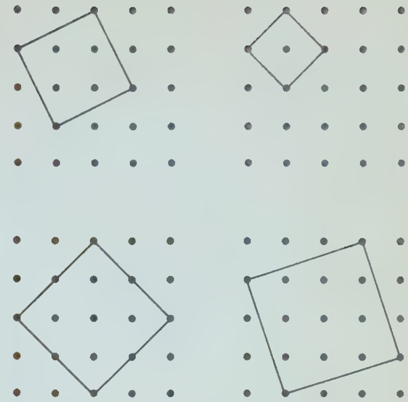
Activity Card 1

For the best results from cards 1 and 2, each child should have his own geoboard to work with. In addition, it would be helpful to make dittos of dot paper on which the children could record their results by drawing pictures of the figures they form. If geoboards are not available, it is possible to have the children do the work called for on these cards by using just the dot paper. In dittoing the dot paper, it is suggested that you fill the page with 5 x 5 arrays of dots. The dots can be spaced approximately 1 centimetre apart. By having the children record their various pictures on dot paper, you will be able to check their work more efficiently than by having to look at each geoboard each time a child forms a figure. The picture on card 1 shows the child two of the squares that can easily be formed on the geoboard. There are two more that are relatively simple—the 2×2 and the 3×3. This, now, gives a total of four of the squares that are relatively easy to find. The four pictures above, right, show the other four squares that can be formed.



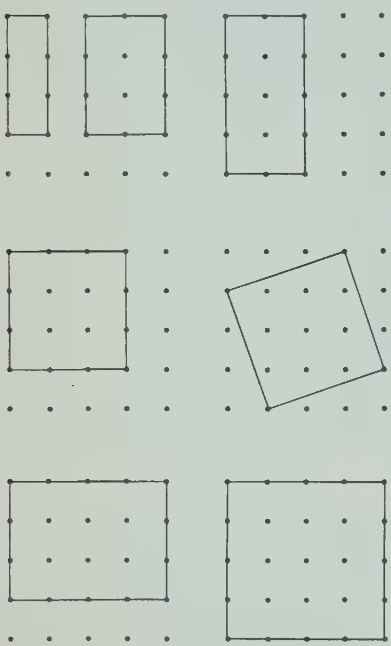
Certainly, it is not expected that all the children will be able to find all of these squares, but it will be worthwhile for them to attempt to find as many as they can. After adequate time has been given for them to explore and investigate these possibilities, you may want to show them the ones that they were not able to find.

Activity Card 2

As with card 1, it would be helpful to provide the children with dittoed dot paper containing 5 x 5 arrays of dots so that they can record the results of this activity. It may be helpful to review the concept of area and square units with the children prior to having them work on this card. They should understand what the unit is here, in order to determine the various areas. The answers to the questions on card 2 are as follows:

Can't: 5, 7, 11, 13, 14, 15

Can: 3, 6, 8, 9, 10, 12, 16 (See accompanying illustrations.)

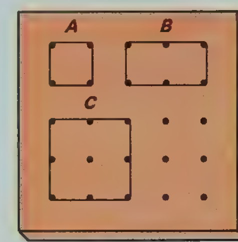


ACTIVITY CARD 2

Rubber band *A* encloses an **area** of 1 square.

Rubber band *B* encloses an area of 2 squares.

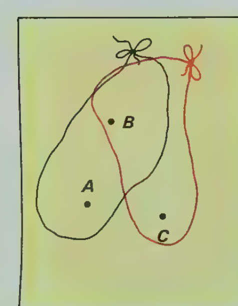
The area of *C* is 4 squares.



Can you show a **square** or a **rectangle** that has an area of 3? 5? 6? 7? 8? 9? 10? 11? 12? 13? 14? 15? 16?

ACTIVITY CARD 3

Mark 3 points, *A*, *B*, and *C*, on a sheet of paper. Use a black and a red loop of string. In the figure, *A* and *C* are inside only one loop. *B* is inside both loops.



How many ways can you place the string so that

▶ each dot is inside some loop
and
exactly one dot is inside both loops?

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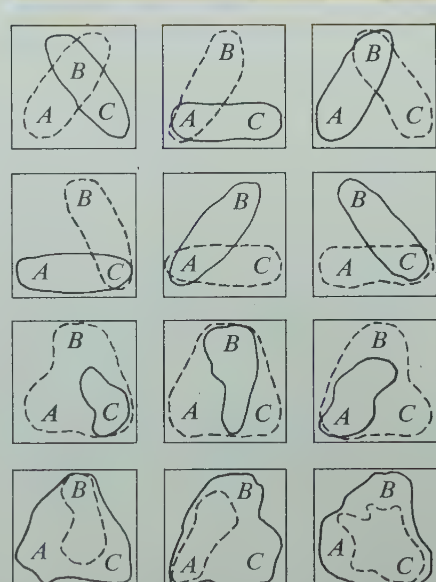
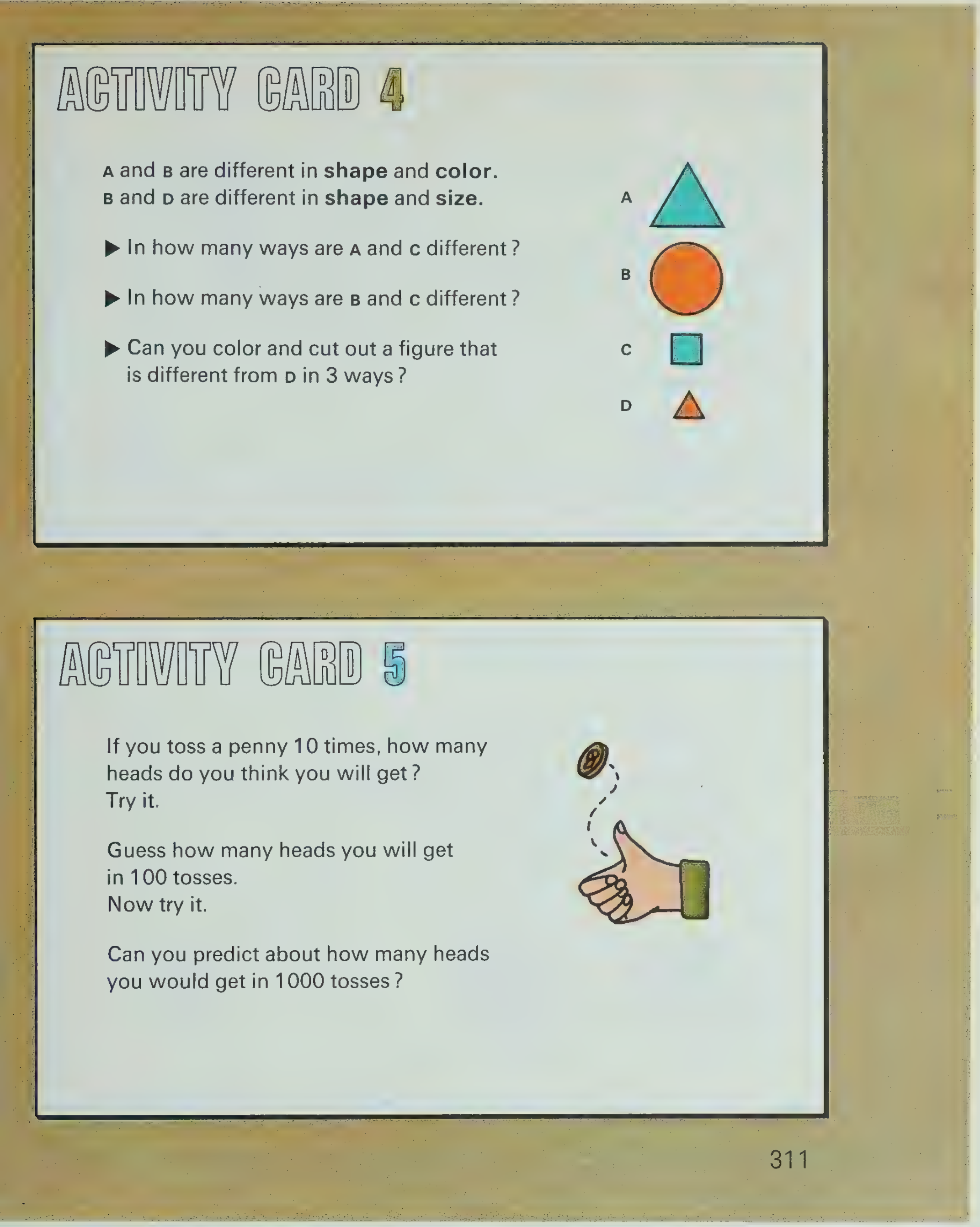
Activity Card 3

This card provides introductory work concerning ideas of Venn diagrams and the concept of closed curves, inside and outside. Also, the children are required to put the strings down on the paper in such a way that they must satisfy two different conditions at the same time. Notice in the figure on the card that both *A* and *B* are inside the black string while both *B* and *C* are inside the other string. This puts *B* inside both while *A* and *C* are inside just one of the loops. Dashed lines represent the red string in the solution illustrated at the right.



The children are not expected to be able to work out all 12 of these possibilities for satisfying the conditions. However, you should encourage them to experiment and find as many of them as they can. You might also find it worthwhile to have them draw pictures of the various ways they find to place the strings on the paper to satisfy the required conditions. Finally, you might wish to picture on the chalkboard any of the possible arrangements that have not been discovered by any of the children.



ACTIVITY CARD 4

A and B are different in **shape** and **color**.
B and D are different in **shape** and **size**.

▶ In how many ways are A and C different?

▶ In how many ways are B and C different?

▶ Can you color and cut out a figure that is different from D in 3 ways?

A

B

C

D

ACTIVITY CARD 5

If you toss a penny 10 times, how many heads do you think you will get?
Try it.

Guess how many heads you will get in 100 tosses.
Now try it.

Can you predict about how many heads you would get in 1000 tosses?

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Activity Card 4

This card may require some extra explanation from you, for the children must understand that the two large figures are considered to be the same size, and the two small figures are considered to be the same size. Thus, the children must cope with three conditions imposed on these figures—size, color, and shape. Notice that A and C are different in two ways (size and shape), while B and C are different in three ways (size, shape, and color). The final question on the card concerns constructing and cutting out a figure that is different from D in three ways. If a child constructs some figure that differs from the triangle in shape and is larger than the triangle and is a different color, it does not need to be one of the shapes shown here, or one of the sizes shown here, or one of the colors shown here, as long as it does differ from D in shape, color, and size. This is a lesson in helping the child sort figures according to their various properties. In this case, emphasis is placed upon size, shape, and color.

Activity Card 5

This card involves probability and the guessing should be relatively easy. For instance, the children will most likely see very quickly that about half the times they will get heads. However, it is an important experience for them to actually flip a coin 100 times and keep a record. It should be noted that it would be very unlikely for them to get 50 heads and 50 tails. Indeed, the results will vary around the room. However, if all of the results are put together, it is very likely that the average of the results will be somewhere near 50. Of course, the intended prediction for the second question on this card is that if the coin were tossed 1000 times the result would be about 500 heads. As a follow-up to this card you might have 10 of the children try tossing a penny 100 times each and then find the average number of heads reported by the ten children, since the total number of tosses would be 1000.

Activity Card 6

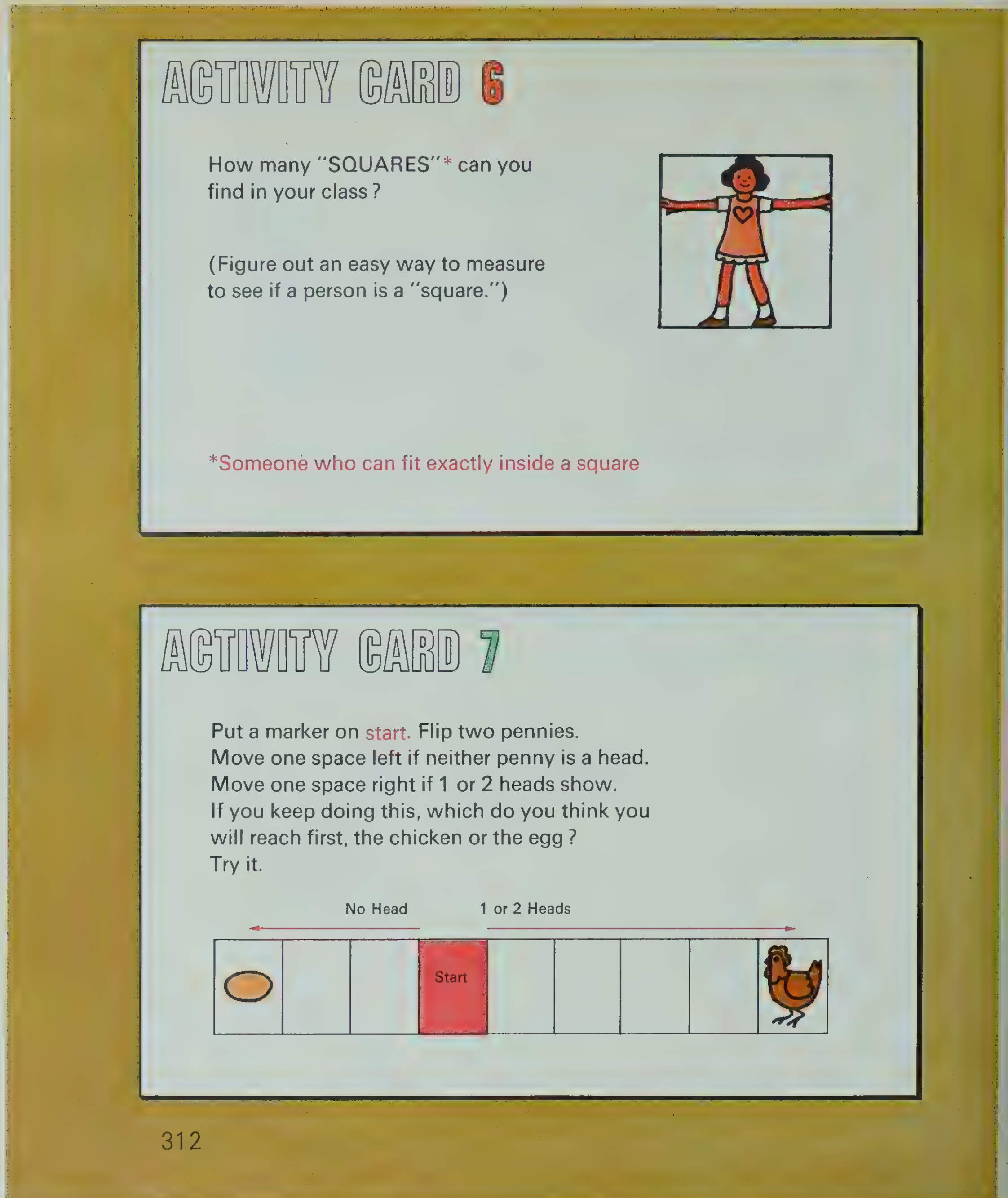
This card is one of the more open-ended cards that the child will be asked to work with. Here, the child is faced with a situation in which he is required to make a number of decisions on his own. First of all, he must decide from the picture what it means for a child to be a "square," and then he must find an efficient way to measure some of the children in the class to find out whether they are squares. This card is intended to be at least somewhat ambiguous so that the child is forced to make a number of decisions on his own as to how to proceed. You might suggest some things to the child, such as using string, a metre stick, the chalkboard, etc., in order to perform the various measurements. Note that many of the children will *not* be squares.

Activity Card 7

This card has a particularly interesting concept underlying it, in that, unless the child thinks carefully about flipping two different pennies, he might conclude that one third of the time he would have both tails, one third of the time he would have both heads, and one third of the time he would have one of each. Hence, since it is just three jumps to the egg and five to the chicken, he might expect to get to the egg first. However, the actual case is indicated in the accompanying table: you get one or two heads three fourths of the time; therefore, in most instances, you should arrive at the chicken before the egg.

Possible outcomes

First penny	H	H	T	T
Second penny	H	T	H	T



ACTIVITY CARD 6

How many "SQUARES"* can you find in your class?

(Figure out an easy way to measure to see if a person is a "square.")

*Someone who can fit exactly inside a square

ACTIVITY CARD 7

Put a marker on start. Flip two pennies.
Move one space left if neither penny is a head.
Move one space right if 1 or 2 heads show.
If you keep doing this, which do you think you will reach first, the chicken or the egg?
Try it.

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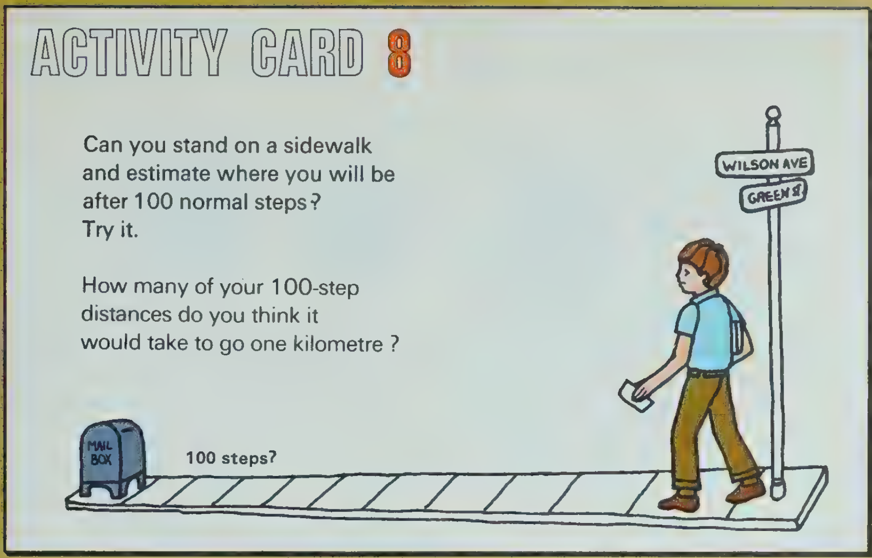

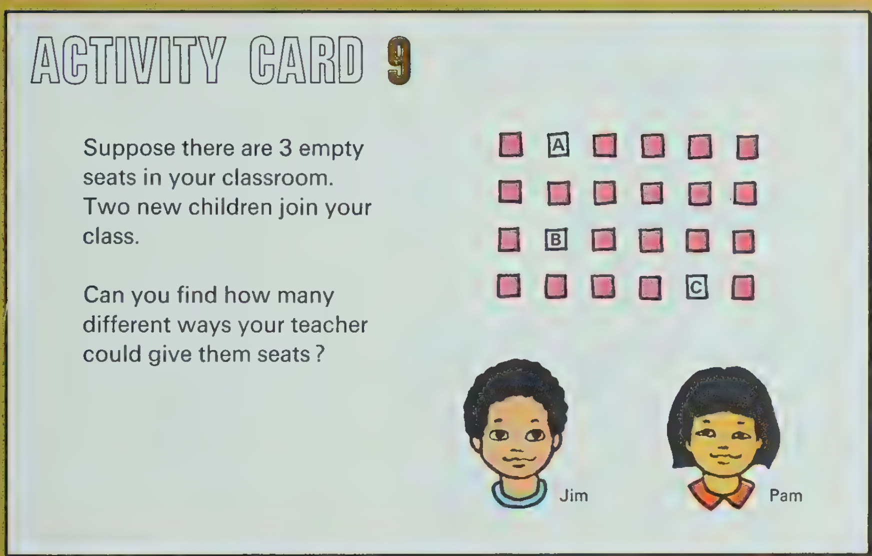

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Activity Card 8

This card is intended to provide the children with experiences in gaining a feeling for distances and working with larger numbers in a real situation. If it is inconvenient for children to go outside and check their estimates by actually walking along the sidewalk, you might suggest other activities involving 100 of some familiar objects. For example, you might have the children go into the hall and estimate how far away the hundredth tile would be, or you might have them estimate how full a given container would be with 100 marbles in it.

The second question on the card is intended to be open-ended, to encourage the child to seek his own methods to estimate how many 100-step distances would make a kilometre. (If his step is about 1 metre, a kilometre would take 10 of the units.) How the child finds the answer is less important than that he have freedom to seek his own method for making this determination.

Activity Card 9

One of this card's purposes is to encourage children to record results in an organized way. Some children may need your help in recording and reporting the answer, so you might suggest that they start by putting Jim in Seat A. Then ask, "Now, how many different seats could Pam sit in, once Jim is in A?" After the children have found that with Jim seated in A they could have arrangements AB and AC, encourage them to try to find the other arrangements (BA, BC, CA, CB). When the children have seen that 6 different arrangements are possible with 3 empty seats, you might challenge them further by asking how many arrangements would be possible if 4 seats were empty. In all, there would be 12, as shown below.

Jim	A	A	A	B	B	B	C	C	C	D	D	D
Pam	B	C	D	A	C	D	A	B	D	A	B	C

Activity Card 10

You will need to make clear to the children what problems would be involved in using these figures to tile a floor. The figures do not have to come out on a straight line on the edges of the floor, but one must be able to take a large number of one type of figure, fit them together, and completely cover a given surface. In the case of the figures given here, this can be done with the triangle, the quadrilateral, and the hexagon. It cannot be done with the pentagon. It might be interesting to have the children work together on this card, thereby producing more of the basic figures and enabling them to tile larger areas. They could use gummed colored paper if available, or make their own paper by using coloring crayons and creating various designs. The results of the tiling could be posted on the bulletin board. As an extension of this idea, the children might be encouraged to search for other figures that could be used to tile a floor. For example, you might pose such questions as, "Can you tile with isosceles triangles? With scalene triangles?"

Activity Card 11

Since children should have worked with magic squares previously, it should not be necessary to explain what makes the square "magic." However if some children are confused, remind them of what a magic square is by demonstrating a 3-by-3 square such as that below.

7	6	11
12	8	4
5	10	9

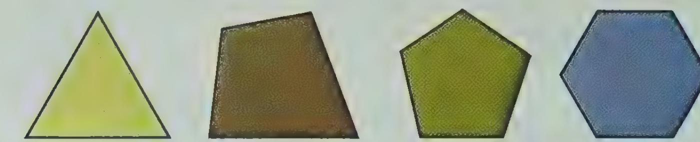
When they study square A, they must realize that the corners of the outside are exchanged along their diagonals and the corners of the inner square are exchanged similarly. If children begin the magic square with the number 5, their first square would be as shown in the top square; the magic square would be as shown beneath it.

5	6	7	8
9	10	11	12
13	14	15	16
17	18	19	20

20	6	7	17
9	15	14	12
13	11	10	16
8	18	19	5

ACTIVITY CARD 10

Trace these shapes and cut out ten of each.



Which of these shapes could be used to tile a floor? (The tiles must not overlap or have any space between them.)

Show each answer by pasting the ten shapes on a paper as if you were starting to tile the floor.

ACTIVITY CARD 11

Here is a way to make a 4-by-4 magic square.

Number a 4-by-4 square consecutively as in square A, starting with 3.

Exchange positions of the pairs of numerals connected by the arrows to get square B.

What is the magic sum in each row, column, and diagonal of square B?

Can you make your own magic square by starting with a different number?

A

3	4	5	6
7	8	9	10
11	12	13	14
15	16	17	18

B

18	4	5	15
7	13	12	10
11	9	8	14
6	16	17	3

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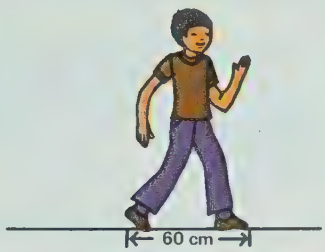
ACTIVITY CARD 12

How many letters are on the front page of your newspaper?

Can you find a way to estimate this number of letters without actually counting them all?

ACTIVITY CARD 13

How close can you come to finding length by counting your steps?



(Practice taking a 60-cm or 90-cm step. Then choose some distances to measure and make a table like the one shown.)

Distance to measure	By counting steps	By using a ruler or tape	Difference
Room length			
Room width			

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Activity Card 12

This is basically an estimating activity. Supply the children with some front pages of a daily newspaper. Suggest that they first guess how many letters appear on the page. Then encourage discussion of ways of checking their guess. If children do this activity in small groups, each member of a group might count the letters in a different column. Elicit from them that to do this quickly they might count the letters in one or two lines and then, after counting the number of lines in a column, find the total in the column by multiplication.

Activity Card 13

As was true of card 6, this card is unusually open-ended in that the child is given considerable freedom in determining those things which he will choose to measure and in determining how he is going to figure out and practice making either 60- or 90-cm steps. For example, the child might choose to attempt to practice 60-cm steps and then find out how many centimetres it is from the school to his home, or, perhaps, from the classroom to the office, etc. Encourage the children to be creative in attempting to decide what distances they will try to measure with their steps. Of course, if they measure very long distances, they will find it necessary to use instruments other than a ruler or tape to get the actual measure needed to complete their tables.

Activity Card 14

The activity suggested on this card is very simple. Children might enjoy doing this activity with their full name as well as their first. Also, the letters of the dial may serve as a code and children can try to figure out short words and phrases. You might give those who work this card the following numbers to decode:

4 3 5 5 6
(H E L L O)

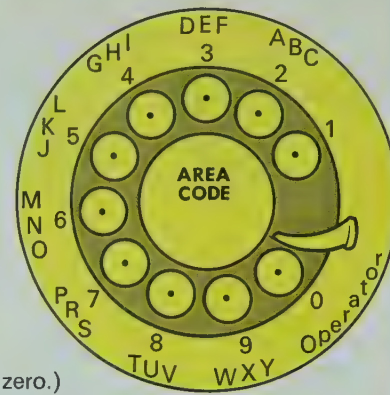
4 6 6 3 2 9 3
(G O O D B Y E)

Activity Card 15

Card 15 introduces the children to the word "symmetric." The children, by the cutting out of a piece of folded paper, establish a figure that already has a line of symmetry. As is indicated by the suggested activities for the children, this card is intended to be open-ended in that it invites the children to explore a variety of figures on their own. Encourage the children's creativity in attempting to cut out various kinds of symmetric figures.

ACTIVITY CARD 14

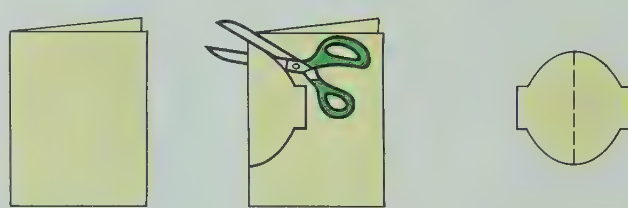
Here is the way Richard used the telephone dial to find the sum of the letters in his name.



R	I	C	H	A	R	D	
7 +	4 +	2 +	4 +	2 +	7 +	3	= 29

Can you find the sum for your name in this way?
Which one of your classmates has the largest sum for his name?
(If a name has a **Q** or **Z**, count it as zero.)

ACTIVITY CARD 15



Fold a piece of paper.

Make a cut that starts on the fold and ends on the fold.

Unfold the piece you cut out. It will be **symmetric** about the fold line.

Can you use this method to make a square? a rectangle? a heart? a triangle? a pumpkin? a letter of the alphabet? a rocket? a butterfly? a funny person?

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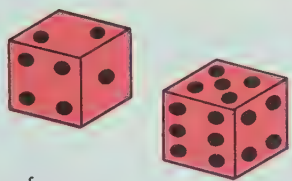
ACTIVITY CARD 16

Can you figure out a way to use a rectangular sheet of graph paper to make a picture of Merfel the Mule **just like this one only larger?**

ACTIVITY CARD 17

~~1~~ 2 3 4 5 ~~6~~ 7 8 9



Play this game with a classmate.
Write the numerals 1 through 9 on a piece of paper.
Toss two dice. Mark out the sum you toss. For example, if you toss a 5 and a 2, you may mark out any combination of digits that totals 7. Continue tossing the dice until you can no longer mark out the sum you toss from the remaining digits.
Your score is the sum of the digits that remain.
The person with the lower total wins.

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Activity Card 16

This card provides the child an opportunity to figure out how to use graph paper to enlarge figures. It embodies the notion of similarity, and the student should be encouraged to look at the points on the small picture, preferably naming them by ordered pairs, and to plot corresponding points on the larger sheet of graph paper. It would be helpful if graph paper rules with unit squares of various sizes were available. The ideas of similarity could be brought out by encouraging the children to use the different graph paper to make similar figures of different sizes. As an additional activity you might have the children trace cartoon characters using the smallest ruled graph paper and then enlarge these figures using the largest ruled paper, Artistically gifted students could choose pictures where some freehand drawing between the lattice points would be necessary. For example, some children might be encouraged to try to reproduce Snoopy or other favorite comic strip characters.

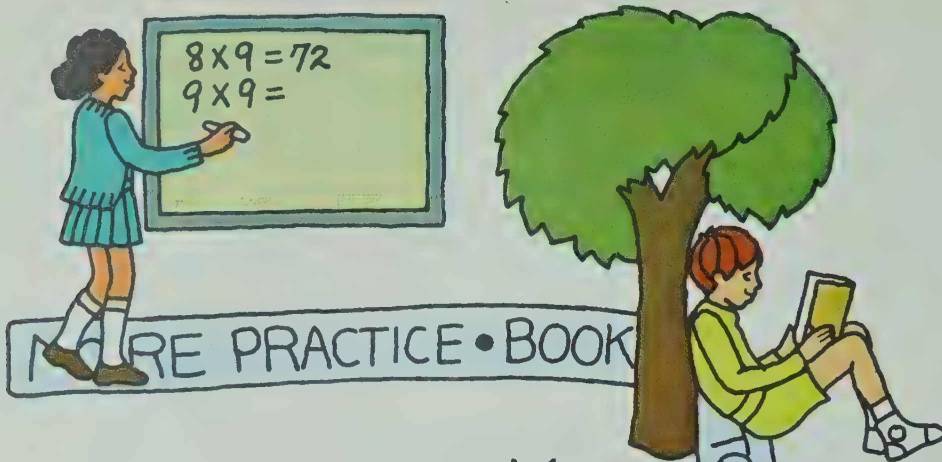
Activity Card 17

The game described on this card is most suitable for two or three players. The only materials required are a pair of dice and paper and pencil. Some children may need to read through the directions a few times and play a few trial games before they fully understand the procedure. You may wish to stress for them the idea that if they roll, say, a 5 and a 6 they are not required to cross out those digits (though they may if they choose to); rather, they may cross out *any* combination of digits whose sum is 11. (Among the possibilities in this case would be 9 and 2; 7 and 4; 8, 2, and 1; 6, 3, and 2; etc.) After the children play the game a few times they will probably realize that, since the winner is the player with the lowest score of remaining digits, they should try to cross out as many of the larger digits as their dice throws permit.

A sample record of one player's tosses might look like this:

Toss	*Record*
1st: 6, 3	1 2 3 4 5 6 7 8 ~~9~~
2nd: 5, 3	1 2 3 4 5 6 7 ~~8~~ ~~9~~
3rd: 5, 6	1 2 3 ~~4~~ 5 6 ~~7~~ ~~8~~ ~~9~~
4th: 4, 4	1 ~~2~~ 3 ~~4~~ 5 ~~6~~ ~~7~~ ~~8~~ ~~9~~
5th: 6, 6	(No more digits may be marked out since $1 + 3 + 5 < 6 + 6$.) Final score: $1 + 3 + 5 = 9$

Appendix

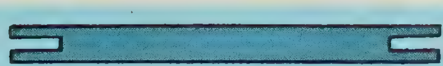
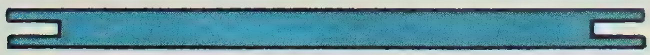
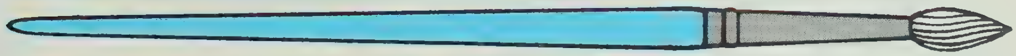
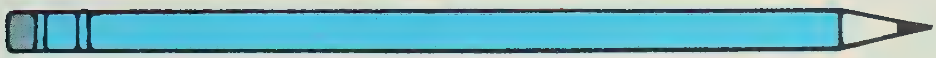

8 × 9 = 72
9 × 9 =
RE PRACTICE • BOOK


TO EXPLORE •
OSSARY • MEASURES

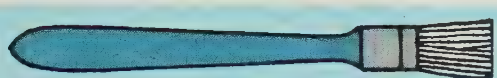
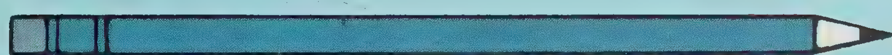
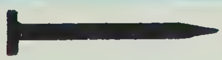

INDEX • TABL

Set 1	*For use with page 13*

Give the length of each object to the nearest centimetre.

1.  6cm
2.  4cm
3.  9cm
4.  8cm
5.  13cm
6.  6cm
7.  12 cm

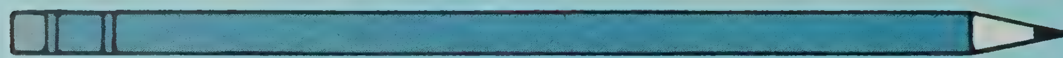
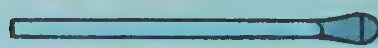
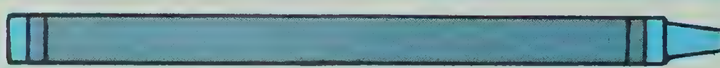
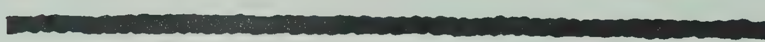
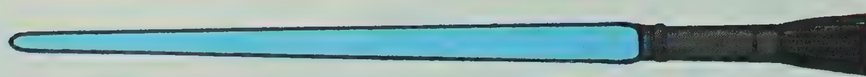
Give the length of each object to the nearest centimetre.

8.  6 cm
9.  9 cm
10.  11 cm (or 12 cm)
11.  3 cm
12.  1 cm
13.  5 cm
14.  14 cm

Reflected answers, Set 1: 1. 6, 2. 4, 3. 9, 8. 6, 9. 9, 10. 11

Set 2	*For use with page 15*

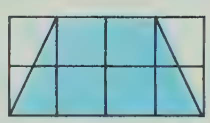
Measure each object to the nearest half centimetre.

1.  $5\frac{1}{2}$ cm
2.  $13\frac{1}{2}$ cm
3.  $4\frac{1}{2}$ cm
4.  9cm
5.  10cm
6.  11cm
7.  $1\frac{1}{2}$ cm

Reflected answers, Set 2:

Set 3	*For use with page 19*

Find the area of each shaded region. Use the square as the unit.

1. 2 
2. 4 
3. 6 
4. 6 
5. 6 
6. 7 

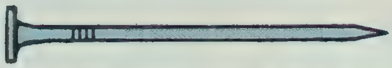
Reflected answers, Set 3:

Set 4 | *For use with page 23*

Find each length to the nearest half centimetre.

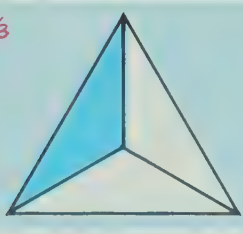
1.  7½ cm

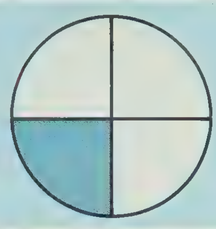
2.  13½ cm

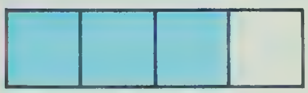
3.  5 cm

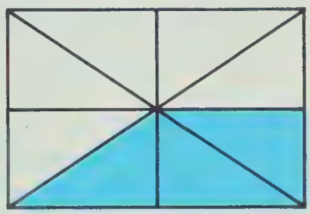
4.  7 cm

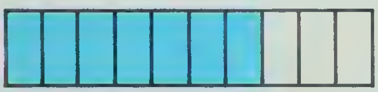
Give the fraction that tells what part of each region is shaded.

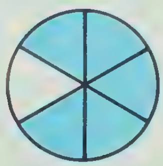
5. ⅓


6. ¼


7. ¾


8. ⅜


9. 7/10


10. ⅚


Solve each short story.

11. 4 candy bars.
Gave away $\frac{1}{2}$ of them.
How many left? 2

12. 6 children.
$\frac{1}{3}$ of them are girls.
How many are girls? 2

13. 12 cookies.
Dropped $\frac{1}{4}$ of them.
Dropped how many? 3

14. 12 dimes.
Spent $\frac{1}{3}$ of them.
Spent how many? 4

Reflected answers, Set 4: 1. [illegible], 2. [illegible], 5. [illegible], 6. [illegible], 11. 2, 12. 2

Set 5 *For use with page 31*

Write the 2-digit numeral for each of these.

1. 2 tens and 5 25
2. 3 tens and 6 36
3. 1 ten and 2 12
4. 4 tens and 0 40
5. 1 ten and 5 15
6. 7 tens and 3 73
7. 8 tens and 7 87
8. 4 tens and 9 49
9. 5 tens and 5 55
10. 9 tens and 0 90
11. 2 tens and 8 28
12. 8 tens and 1 81

Give the correct digit for each ||||.

13. 21 means 2 tens and ||||. 1
14. 37 means 3 tens and ||||. 7
15. 89 means |||| tens and 9. 8
16. 75 means |||| tens and 5. 7
17. 40 means |||| tens and 0. 4
18. 63 means 6 tens and ||||. 3
19. 92 means 9 tens and ||||. 2
20. 38 means |||| tens and 8. 3
21. 56 means |||| tens and 6. 5
22. 10 means 1 ten and ||||. 0

Reflected answers, Set 5: 1. 25, 2. 36, 5. 15, 6. 73, 9. 55, 10. 90, 13. 1, 14. 7, 18. 3, 19. 2

Set 6 *For use with page 33*

Write the word name for each of these.

1. 5 tens and 6 fifty-six
2. 3 tens and 5 thirty-five
3. 7 tens and 0 seventy
4. 8 tens and 1 eighty-one
5. 1 ten and 9 nineteen
6. 4 tens and 7 forty-seven
7. 2 tens and 8 twenty-eight
8. 6 tens and 2 sixty-two
9. 1 ten and 4 fourteen
10. 9 tens and 9 ninety-nine
11. 8 tens and 6 eighty-six
12. 2 tens and 9 twenty-nine

Copy each row. Give the missing numbers.

13. 7, 8, 9, 10, ? , ? , ? , ? , 15, 16 11, 12, 13, 14
14. 25, 26, 27, 28, ? , ? , ? , ? , 33, 34 29, 30, 31, 32
15. ? , ? , ? , ? , 40, 41, 42, 43 36, 37, 38, 39
16. 80, 81, 82, 83, ? , ? , ? , ? , 88, 89 84, 85, 86, 87
17. 2, 4, 6, 8, ? , ? , ? , ? , 18, 20 10, 12, 14, 16
18. 5, 10, ? , ? , ? , ? , 35, 40 15, 20, 25, 30

Reflected answers, Set 6: 1. fifty-six, 2. thirty-five, 5. nineteen, 6. forty-seven, 9. fourteen, 10. ninety-nine, 13. 11, 12, 13, 14, 14. 29, 30, 31, 32, 15. 36, 37, 38, 39

Set 7 — *For use with page 37*

Write the numeral. (*h* stands for *h*undreds and *t* for *t*ens.)

1. 5*h*, 3*t*, and 4 534
2. 6*h*, 5*t*, and 1 651
3. 4*h*, 1*t*, and 9 419
4. 1*h*, 2*t*, and 5 125
5. 3*h*, 8*t*, and 2 382
6. 7*h*, 9*t*, and 0 790
7. 9*h*, 0*t*, and 3 903
8. 8*h*, 0*t*, and 8 808
9. 2*h*, 5*t*, and 6 256
10. 3*h*, 8*t*, and 2 382
11. 5*h*, 7*t*, and 0 570
12. 1*h*, 3*t*, and 9 139
13. 7*h*, 9*t*, and 6 796
14. 4*h*, 0*t*, and 0 400
15. 9*h*, 1*t*, and 5 915
16. 8*h*, 4*t*, and 8 848
17. 2*h*, 2*t*, and 2 222
18. 6*h*, 5*t*, and 3 653

Give the missing digit.

19. 226 means 2 hundreds, _2_ tens, and 6 ones. 2
20. 384 means 3 hundreds, 8 tens, and _4_ ones. 4
21. 179 means _1_ hundreds, 7 tens, and 9 ones. 1
22. 838 means _8_ hundreds, 3 tens, and 8 ones. 8
23. 915 means 9 hundreds, _1_ tens, and 5 ones. 1
24. 475 means 4 hundreds, 7 tens, and _5_ ones. 5
25. 508 means 5 hundreds, _0_ tens, and 8 ones. 0
26. 657 means _6_ hundreds, 5 tens, and 7 ones. 6
27. 830 means 8 hundreds, 3 tens, and _0_ ones. 0
28. 100 means _1_ hundreds, 0 tens, and 0 ones. 1

Write the numeral for each part.

29. one hundred sixty-six 166
30. two hundred seventeen 217
31. five hundred ninety-nine 599
32. three hundred thirty-seven 337
33. eight hundred two 802
34. four hundred twenty-two 422
35. nine hundred fifty 950
36. seven hundred 700
37. one hundred ninety-three 193
38. nine hundred twelve 912
39. five hundred sixty-one 561
40. seven hundred forty-five 745
41. two hundred six 206
42. three hundred ninety-nine 399

Reflected answers, Set 7: 1. 534, 2. 651, 7. 903, 8. 808, 13. 796, 14. 400, 19. 2, 20. 4, 21. 1, 22. 8, 23. 1, 29. 166, 30. 217, 31. 599, 36. 700, 37. 193, 38. 912

Set 8 — *For use with page 41*

Write the 4-digit numeral for each of these. (*th* stands for *th*ousands, *h* stands for *h*undreds, and *t* stands for *t*ens.)

1. 5*th*, 3*h*, 8*t*, 5 5385
2. 7*th*, 8*h*, 2*t*, 4 7824
3. 9*th*, 6*h*, 1*t*, 3 9613
4. 1*th*, 0*h*, 3*t*, 9 1039
5. 6*th*, 4*h*, 6*t*, 7 6467
6. 2*th*, 4*h*, 8*t*, 8 2488
7. 3*th*, 5*h*, 9*t*, 1 3591
8. 8*th*, 9*h*, 0*t*, 6 8906
9. 4*th*, 9*h*, 0*t*, 0 4900
10. 8*th*, 1*h*, 7*t*, 2 8172
11. 2*th*, 8*h*, 1*t*, 3 2813
12. 9*th*, 9*h*, 9*t*, 9 9999

Find the missing digit for each of these.

13. 4872 means 4 thousands, _8_ hundreds, 7 tens, 2 ones. 8
14. 5396 means 3 hundreds, 6 ones, 9 tens, _5_ thousands. 5
15. 6003 means _0_ hundreds, 0 tens, 6 thousands, 3 ones. 0
16. 9218 means 1 ten, _9_ thousands, 2 hundreds, 8 ones. 9

9. 4900, 10. 8172, 13. 8, 14. 5
Reflected answers, Set 8: 1. 5385, 2. 7824, 5. 6467, 6. 2488,

Set 9 — *For use with page 43*

Which of the two numbers is greater?

1. 9 or 7
2. 12 or 20
3. 35 or 25
4. 126 or 226
5. 450 or 460
6. 796 or 790
7. 999 or 1999
8. 4921 or 4931
9. 8890 or 8889

Place the correct sign (< or >) between each pair of numbers.

10. 65 ▒ 55 >
11. 34 ▒ 64 <
12. 97 ▒ 79 >
13. 376 ▒ 476 <
14. 581 ▒ 571 >
15. 783 ▒ 873 <
16. 575 ▒ 585 <
17. 3092 ▒ 3192 <
18. 6426 ▒ 5426 >
19. 2361 ▒ 2351 >
20. 8805 ▒ 8905 <
21. 4223 ▒ 4333 <

16. 575 < 585, 17. 3092 < 3192, 19. 2361 > 2351, 20. 8805 < 8905
8. 4931, 10. 65 > 55, 11. 34 < 64, 13. 376 < 476, 14. 581 > 571,
Reflected answers, Set 9: 1. 9, 2. 20, 4. 226, 5. 460, 7. 1999,

Set 10 — *For use with page 55*

Find the sums.

1.	2.	3.	4.	5.	6.	7.	8.
0	3	0	1	0	1	4	6
+1	+5	+3	+2	+4	+5	+2	+2
1	8	3	3	4	6	6	8

9.	10.	11.	12.	13.	14.	15.	16.
2	0	7	6	0	1	2	1
+4	+2	+0	+1	+6	+7	+1	+3
6	2	7	7	6	8	3	4

17.	18.	19.	20.	21.	22.	23.	24.
2	2	4	1	2	0	5	2
+6	+5	+4	+8	+0	+5	+3	+2
8	7	8	9	2	5	8	4

25.	26.	27.	28.	29.	30.	31.	32.
3	1	0	1	3	5	8	3
+4	+1	+8	+6	+6	+2	+1	+0
7	2	8	7	9	7	9	3

33.	34.	35.	36.	37.	38.	39.	40.
6	7	8	1	3	4	6	2
+3	+2	+0	+4	+2	+5	+4	+3
9	9	8	5	5	9	10	5

Reflected answers, Set 10: 1. , 2. , 3. , 4. , 5. , 6. , 7. , 8. , 9. , 10. , 11. , 12. 7, 13. , 14. 8, 15. , 16.

Set 11 — *For use with page 57*

Find the missing addends.

1. 4 + 3 = 7
2. 4 + 5 = 9
3. 3 + 2 = 5
4. 7 + 1 = 8
5. 6 + 4 = 10
6. 3 + 5 = 8

Find the differences.

7.	8.	9.	10.	11.	12.	13.	14.
3	5	8	4	1	2	2	8
−0	−1	−7	−4	−0	−2	−1	−2
3	4	1	0	1	0	1	6

15.	16.	17.	18.	19.	20.	21.	22.
9	9	6	7	10	7	6	6
−4	−7	−5	−3	−8	−7	−4	−3
5	2	1	4	2	0	2	3

Reflected answers, Set 11: 1. 4, 3. 3, 5. 6, 7. , 8. , 9. 1, 10. 0, 11. , 12. , 13. , 14.

Set 12 *For use with page 59*

Find the differences.

1.	2.	3.	4.	5.	6.	7.	8.
9	6	9	9	9	8	8	7
−5	−5	−3	−6	−8	−3	−6	−5
4	1	6	3	1	5	2	2

9.	10.	11.	12.	13.	14.	15.	16.
9	8	7	7	10	10	10	10
−9	−0	−2	−4	−9	−7	−4	−5
0	8	5	3	1	3	6	5

Solve each story problem.

17. Bill had 10 marbles. He gave John 6 of them. How many marbles does Bill have left? 4
18. Betty is 9 years old. Her sister is 5 years old. How much older is Betty? 4 years
19. Ann has 8 cents. Dick has 3 cents less. How many cents does Dick have? 5¢

Reflected answers, Set 12: 1. 4, 2. 1, 3. 6, 4. 3, 5. 1, 6. 5, 7. 2, 8. 2

Set 13 *For use with page 63*

Find the sums.

1.	2.	3.	4.	5.	6.	7.	8.
4	2	1	2	3	6	5	3
1	4	5	5	1	2	1	1
+2	+3	+2	+3	+0	+2	+2	+6
7	9	8	10	4	10	8	10

9.	10.	11.	12.	13.	14.	15.	16.
6	6	9	2	5	6	1	8
4	7	8	8	4	8	5	9
+3	+3	+1	+7	+5	+2	+9	+2
13	16	18	17	14	16	15	19

17. 3 + 7 + 3 + 2 15
18. 7 + 5 + 3 + 2 17
19. 2 + 3 + 8 + 7 20
20. 1 + 7 + 9 + 5 22
21. 8 + 5 + 2 + 6 21
22. 4 + 9 + 6 + 8 27

Reflected answers, Set 13: 1. 7, 2. 9, 3. 8, 4. 10, 5. 4, 6. 10, 7. 8, 8. 10

Set 14 — *For use with page 65*

Find the sums.

1. 9	**2.** 3	**3.** 6	**4.** 8	**5.** 6	**6.** 4	**7.** 1	**8.** 6
+2	+7	+5	+4	+6	+9	+9	+4
11	10	11	12	12	13	10	10
9. 9	**10.** 8	**11.** 2	**12.** 5	**13.** 6	**14.** 8	**15.** 9	**16.** 8
+3	+2	+9	+7	+8	+7	+8	+3
12	10	11	12	14	15	17	11
17. 7	**18.** 9	**19.** 5	**20.** 3	**21.** 7	**22.** 8	**23.** 9	**24.** 4
+7	+5	+6	+9	+8	+9	+7	+7
14	14	11	12	15	17	16	11
25. 8	**26.** 7	**27.** 3	**28.** 5	**29.** 6	**30.** 4	**31.** 6	**32.** 9
+8	+6	+8	+8	+9	+8	+7	+6
16	13	11	13	15	12	13	15
33. 8	**34.** 9	**35.** 5	**36.** 9	**37.** 7	**38.** 8	**39.** 7	**40.** 7
+6	+4	+9	+9	+9	+5	+5	+4
14	13	14	18	16	13	12	11
41. 6	**42.** 7	**43.** 9	**44.** 2	**45.** 5	**46.** 6	**47.** 7	**48.** 1
4	3	1	8	5	8	2	5
+3	+6	+8	+7	+4	+2	+3	+9
13	16	18	17	14	16	12	15

49. 2 + 5 + 3 10
50. 6 + 2 + 4 12
51. 3 + 2 + 9 14
52. 3 + 4 + 6 13
53. 5 + 9 + 5 19

54. 6 + 7 + 4 17
55. 5 + 7 + 6 18
56. 9 + 6 + 3 18
57. 8 + 7 + 2 17
58. 9 + 4 + 1 + 3 17

59. 6 + 2 + 4 + 8 20
60. 2 + 7 + 8 + 1 18
61. 8 + 4 + 2 + 6 20
62. 5 + 3 + 4 + 3 15
63. 1 + 0 + 6 + 8 15

Solve each short story.

64. 9 boy's bikes. 5 girl's bikes.
How many bikes? 14

65. 8 robins. 9 blue jays.
How many birds? 17

Reflected answers, Set 14: 1. 11, 2. 10, 3. 11, 4. 12, 5. 12,
6. 13, 7. 10, 8. 10, 49. 10, 50. 12, 54. 17, 55. 18,
59. 20, 60. 18

Set 15 *For use with page 67*

Find the differences.

1. $\begin{array}{r}14\\-7\\\hline 7\end{array}$	**2.** $\begin{array}{r}10\\-6\\\hline 4\end{array}$	**3.** $\begin{array}{r}13\\-8\\\hline 5\end{array}$	**4.** $\begin{array}{r}11\\-8\\\hline 3\end{array}$	**5.** $\begin{array}{r}12\\-9\\\hline 3\end{array}$	**6.** $\begin{array}{r}15\\-7\\\hline 8\end{array}$	**7.** $\begin{array}{r}13\\-4\\\hline 9\end{array}$	**8.** $\begin{array}{r}11\\-2\\\hline 9\end{array}$
9. $\begin{array}{r}10\\-3\\\hline 7\end{array}$	**10.** $\begin{array}{r}11\\-7\\\hline 4\end{array}$	**11.** $\begin{array}{r}11\\-9\\\hline 2\end{array}$	**12.** $\begin{array}{r}10\\-8\\\hline 2\end{array}$	**13.** $\begin{array}{r}11\\-4\\\hline 7\end{array}$	**14.** $\begin{array}{r}12\\-3\\\hline 9\end{array}$	**15.** $\begin{array}{r}13\\-5\\\hline 8\end{array}$	**16.** $\begin{array}{r}14\\-6\\\hline 8\end{array}$
17. $\begin{array}{r}12\\-7\\\hline 5\end{array}$	**18.** $\begin{array}{r}12\\-8\\\hline 4\end{array}$	**19.** $\begin{array}{r}11\\-6\\\hline 5\end{array}$	**20.** $\begin{array}{r}12\\-5\\\hline 7\end{array}$	**21.** $\begin{array}{r}13\\-7\\\hline 6\end{array}$	**22.** $\begin{array}{r}13\\-9\\\hline 4\end{array}$	**23.** $\begin{array}{r}12\\-6\\\hline 6\end{array}$	**24.** $\begin{array}{r}14\\-5\\\hline 9\end{array}$
25. $\begin{array}{r}11\\-5\\\hline 6\end{array}$	**26.** $\begin{array}{r}15\\-6\\\hline 9\end{array}$	**27.** $\begin{array}{r}12\\-4\\\hline 8\end{array}$	**28.** $\begin{array}{r}16\\-8\\\hline 8\end{array}$	**29.** $\begin{array}{r}13\\-6\\\hline 7\end{array}$	**30.** $\begin{array}{r}16\\-7\\\hline 9\end{array}$	**31.** $\begin{array}{r}14\\-8\\\hline 6\end{array}$	**32.** $\begin{array}{r}16\\-9\\\hline 7\end{array}$
33. $\begin{array}{r}10\\-1\\\hline 9\end{array}$	**34.** $\begin{array}{r}11\\-3\\\hline 8\end{array}$	**35.** $\begin{array}{r}15\\-8\\\hline 7\end{array}$	**36.** $\begin{array}{r}17\\-9\\\hline 8\end{array}$	**37.** $\begin{array}{r}18\\-9\\\hline 9\end{array}$	**38.** $\begin{array}{r}14\\-9\\\hline 5\end{array}$	**39.** $\begin{array}{r}17\\-8\\\hline 9\end{array}$	**40.** $\begin{array}{r}15\\-9\\\hline 6\end{array}$

41. 10 − 6 4	**45.** 18 − 5 13	**49.** 17 − 2 15	**53.** 14 − 7 7
42. 15 − 9 6	**46.** 17 − 7 10	**50.** 19 − 8 11	**54.** 12 − 7 5
43. 17 − 8 9	**47.** 10 − 9 1	**51.** 16 − 9 7	**55.** 15 − 5 10
44. 15 − 6 9	**48.** 16 − 5 11	**52.** 10 − 7 3	**56.** 19 − 8 11

Solve each short story.

57. 12 cats and dogs. 5 cats.
How many dogs? 7

58. 15 candy bars. Ate 7.
How many left? 8

59. 8 girls. 8 shoes.
How many more shoes needed? 8

60. 14 boys. 6 hats.
How many more hats needed? 8

61. 19 cents. Lost 7 cents.
How much left? 12¢

62. 18 children. 9 boys.
How many girls? 9

Reflected answers, Set 15: 1. 7, 2. 4, 3. 5, 4. 3, 5. 3, 6. 8, 7. 9, 8. 9, 9. 7, 10. 4, 11. 2, 12. 2, 13. 7, 14. 9, 15. 8, 16. 8, 41. 4, 42. 6, 45. 13, 46. 10, 49. 15, 50. 11, 53. 7, 54. 5, 57. 7, 60. 8

Set 16 *For use with page 95*

Find the sums and differences.

1. 30 +46 76	2. 37 +22 59	3. 50 +27 77	4. 32 +56 88	5. 24 +63 87	6. 35 +54 89	7. 61 +18 79
8. 31 +24 55	9. 45 +31 76	10. 18 +51 69	11. 12 +86 98	12. 37 +61 98	13. 49 +40 89	14. 20 +68 88

15. 432 +264 696	16. 507 +462 969	17. 326 +453 779	18. 856 +133 989	19. 750 +108 858	20. 513 +264 777
21. 460 −150 310	22. 463 −322 141	23. 872 −541 331	24. 768 −306 462	25. 952 −340 612	26. 638 −414 224
27. 390 −240 150	28. 576 −215 361	29. 898 −344 554	30. 792 −410 382	31. 856 −313 543	32. 984 −753 231

Reflected answers, Set 16: 1. 76, 2. 59, 3. 77, 4. 88, 5. 87, 6. 89, 7. 79, 21. 310, 22. 141, 23. 331, 24. 462, 25. 612, 26. 224

Set 17 *For use with page 99*

Find the sums.

1. 36 +92 128	2. 27 +81 108	3. 13 +94 107	4. 45 +84 129	5. 34 +74 108	6. 72 +65 137	7. 24 +85 109
8. 27 +34 61	9. 32 +59 91	10. 46 +58 104	11. 83 +39 122	12. 56 +68 124	13. 47 +56 103	14. 91 +67 158
15. 92 +88 180	16. 77 +38 115	17. 19 +46 65	18. 53 +98 151	19. 42 +78 120	20. 37 +95 132	21. 72 +97 169

Reflected answers, Set 17: 1. 128, 2. 108, 3. 107, 4. 129, 5. 108, 6. 137, 7. 109

Set 18	*For use with page 101*

Find the sums.

1. 37 +8 45	**2.** 56 +26 82	**3.** 87 +36 123	**4.** 43 +9 52	**5.** 68 +24 92	**6.** 76 +76 152	**7.** 58 +4 62
8. 22 +9 31	**9.** 68 +27 95	**10.** 85 +79 164	**11.** 67 +5 72	**12.** 14 +69 83	**13.** 58 +69 127	**14.** 59 +4 63
15. 26 +6 32	**16.** 54 +29 83	**17.** 45 +98 143	**18.** 54 +6 60	**19.** 37 +38 75	**20.** 66 +78 144	**21.** 38 +4 42
22. 73 +8 81	**23.** 69 +25 94	**24.** 76 +74 150	**25.** 86 +5 91	**26.** 48 +46 94	**27.** 75 +99 174	**28.** 67 +8 75

29. 536 + 587 1123	**30.** 947 + 589 1536	**31.** 654 + 878 1532	**32.** 948 + 699 1647
33. 677 + 388 1065	**34.** 666 + 666 1332	**35.** 385 + 769 1154	**36.** 562 + 949 1511

37. 326 +468 794	**38.** 534 +259 793	**39.** 421 +309 730	**40.** 319 +146 465	**41.** 573 +152 725	**42.** 629 +152 781
43. 467 +368 835	**44.** 379 +123 502	**45.** 378 +147 525	**46.** 638 +287 925	**47.** 456 +269 725	**48.** 736 +199 935
49. 629 +153 782	**50.** 406 +279 685	**51.** 384 +455 839	**52.** 293 +563 856	**53.** 425 +395 820	**54.** 273 +588 861
55. 763 +578 1341	**56.** 637 +895 1532	**57.** 666 +444 1110	**58.** 858 +564 1422	**59.** 647 +886 1533	**60.** 792 +589 1381
61. 890 +469 1359	**62.** 876 +452 1328	**63.** 893 +486 1379	**64.** 989 +376 1365	**65.** 793 +537 1330	**66.** 927 +894 1821

Reflected answers, Set 18: 1. 45, 2. 82, 3. 123, 4. 52, 5. 92, 6. 152, 7. 62, 29. 1123, 30. 1536, 31. 1532, 32. 1647, 37. 794, 38. 793, 39. 730, 40. 465, 41. 725, 42. 781

Set 19 | *For use with page 107*

Find the differences.

1. 32 − 8 = 24	**2.** 51 − 9 = 42	**3.** 36 − 8 = 28	**4.** 22 − 5 = 17	**5.** 47 − 8 = 39	**6.** 83 − 9 = 74	**7.** 91 − 3 = 88
8. 73 − 27 = 48	**9.** 41 − 8 = 33	**10.** 66 − 7 = 59	**11.** 84 − 8 = 76	**12.** 40 − 2 = 38	**13.** 73 − 4 = 69	**14.** 57 − 8 = 49
15. 30 − 6 = 24	**16.** 42 − 15 = 27	**17.** 75 − 39 = 36	**18.** 60 − 54 = 6	**19.** 34 − 28 = 6	**20.** 26 − 17 = 9	**21.** 58 − 29 = 29
22. 53 − 18 = 35	**23.** 71 − 42 = 29	**24.** 84 − 29 = 55	**25.** 36 − 18 = 18	**26.** 97 − 49 = 48	**27.** 86 − 29 = 57	**28.** 57 − 28 = 29
29. 82 − 76 = 6	**30.** 92 − 66 = 26	**31.** 63 − 45 = 18	**32.** 40 − 29 = 11	**33.** 81 − 12 = 69	**34.** 65 − 39 = 26	**35.** 88 − 19 = 69
36. 123 − 29 = 94	**37.** 164 − 77 = 87	**38.** 186 − 98 = 88	**39.** 152 − 73 = 79	**40.** 285 − 37 = 248	**41.** 636 − 58 = 578	**42.** 347 − 59 = 288
43. 153 − 69 = 84	**44.** 174 − 89 = 85	**45.** 216 − 89 = 127	**46.** 741 − 53 = 688	**47.** 580 − 233 = 347	**48.** 634 − 126 = 508	**49.** 727 − 282 = 445

Find the missing numbers.

50. 56 − ▯ = 49 7	**55.** 713 − 9 = ▯ 704	**60.** 657 − 8 = ▯ 649
51. 42 − ▯ = 38 4	**56.** 675 − 7 = ▯ 668	**61.** 43 − 39 = ▯ 4
52. 73 − ▯ = 67 6	**57.** 416 − ▯ = 414 2	**62.** 254 − 48 = ▯ 206
53. 139 − 4 = ▯ 135	**58.** 537 − ▯ = 534 3	**63.** 163 − 159 = ▯ 4
54. 537 − 6 = ▯ 531	**59.** 879 − ▯ = 873 6	**64.** 442 − 39 = ▯ 403

Reflected answers, Set 19: 1. 24, 2. 42, 3. 28, 4. 17, 5. 39, 6. 74, 7. 88, 15. 24, 16. 27, 17. 36, 18. 6, 19. 6, 20. 9, 21. 29, 50. 7, 51. 4, 55. 704, 56. 668, 60. 649, 61. 4

Set 20 — *For use with page 111*

Find the sums and differences.

1.	2.	3.	4.	5.	6.	7.
72	90	63	57	96	453	613
+65	+47	−23	+84	−37	+236	−252
137	137	40	141	59	689	361

8.	9.	10.	11.	12.	13.	14.
513	416	780	32	653	716	876
+888	−126	+57	+499	−283	+24	+347
1401	290	837	531	370	740	1223

15.	16.	17.	18.	19.	20.	21.
64	79	83	117	356	572	629
+36	−48	+49	−48	+241	−148	−598
100	31	132	69	597	424	31

22.	23.	24.	25.	26.	27.	28.
823	962	430	651	365	888	777
+799	−897	+980	−279	+376	−499	+666
1622	65	1410	372	741	389	1443

29.	30.	31.	32.	33.	34.	35.
671	537	528	628	430	732	974
−346	+283	−141	+177	+398	−496	−876
325	820	387	805	828	236	98

36.	37.	38.	39.	40.	41.	42.
516	876	765	657	837	940	777
+694	+427	−379	−288	+999	−333	+567
1210	1303	386	369	1836	607	1344

Solve each short story problem.

43. 16 on the bus. 14 more get on. How many on bus? 30

44. 59 blue marbles. 87 red marbles. How many marbles? 146

45. Bill weighs 45 kilograms. Mary weighs 37. How much heavier is Bill? 8 kg

46. 188 math books. 275 books in all. How many are not math books? 87

47. There are 243 boys in Lozano School. There are 186 girls. How many more boys than girls are in the school? 57

Reflected answers, Set 20: 1. 137, 2. 137, 3. 40, 4. 141, 5. 59, 6. 689, 7. 361, 15. 100, 16. 31, 17. 132, 18. 69, 19. 597, 20. 424, 21. 31, 43. 30, 44. 146

Set 21	*For use with page 115*

Find the total amounts.

1. \$1.36	**2.** \$4.25	**3.** \$3.44	**4.** \$5.65	**5.** \$4.37	**6.** \$7.81
7.54	3.57	5.39	.16	2.91	1.46
\$8.90	\$7.82	\$8.83	\$5.81	\$7.28	\$9.27
7. \$8.34	**8.** \$4.53	**9.** \$5.64	**10.** \$3.75	**11.** \$1.18	**12.** \$9.26
6.79	.88	8.97	7.29	9.84	6.97
\$15.13	\$5.41	\$14.61	\$11.04	\$11.02	\$16.23
13. \$8.36	**14.** \$3.58	**15.** \$9.46	**16.** \$5.61	**17.** \$6.84	**18.** \$4.72
1.26	5.15	.37	3.09	2.41	4.92
\$9.62	\$8.73	\$9.83	\$8.70	\$9.25	\$9.64
19. \$9.67	**20.** \$10.83	**21.** \$14.47	**22.** \$11.63	**23.** \$8.64	**24.** \$16.27
.43	3.59	2.97	4.88	5.99	3.86
\$10.10	\$14.42	\$17.44	\$16.51	\$14.63	\$20.13

Find the difference in the amounts.

25. \$8.56	**26.** \$7.74	**27.** \$4.83	**28.** \$3.90	**29.** \$6.38	**30.** \$9.74
3.21	4.53	4.53	1.30	.24	4.34
\$5.35	\$3.21	\$ .30	\$2.60	\$6.14	\$5.40
31. \$6.72	**32.** \$8.65	**33.** \$9.44	**34.** \$4.36	**35.** \$7.29	**36.** \$6.14
1.37	2.26	7.19	.84	4.93	.39
\$5.35	\$6.39	\$2.25	\$3.52	\$2.36	\$5.75

Solve each story problem.

37. Jane had \$1.50. She spent 75 cents for some candy. How much did she have left? 75¢

38. George had \$7.43. He spent \$1.35 for a ball. He also spent \$3.59 for a bat. How much did he have left? \$2.49

Reflected answers, Set 21: 1. \$8.90, 2. \$7.82, 3. \$8.83, 4. \$5.81, 5. \$7.28, 6. \$9.27, 25. \$5.35, 26. \$3.21, 27. \$0.30, 28. \$2.60, 29. \$6.14, 30. \$5.40

Set 22	*For use with page 117*

Find the sums and differences.

1. 30	**2.** 25	**3.** 18	**4.** 63	**5.** 56	**6.** 64	**7.** 52
57	39	27	29	74	39	87
+14	+35	+43	+38	+58	+65	+36
101	99	88	130	188	168	175

8. 671	**9.** 362	**10.** 437	**11.** 612	**12.** 906	**13.** 751
−252	−133	−242	−136	−174	−227
419	229	195	476	732	524

14. 747	**15.** 535	**16.** 496	**17.** 368	**18.** 417	**19.** 925
−288	−237	−369	−179	−254	−287
459	298	127	189	163	638

Copy each problem. Write the missing digits.

20. 15□ 6	**21.** 377	**22.** 8 □6□ 0	**23.** 23□ 4	**24.** 5 □32	**25.** 282
+7□7 3	−26□ 8	−138	+6□9 4	−39□ 2	5 +□□4 9
893	1□9 0	722	883	140	876

26. 92□ 3	**27.** 94□ 5	**28.** 9 □39	**29.** 8 8□□ 0	**30.** 7□3 6	**31.** 7□7 3
1 +□78	3 +□□9 3	−46□ 2	6 +□94	−32□ 9	5 +□64
11□1 0	1284	477	1574	434	1301

Solve each short story problem.

32. 72 horses. Only 28 spotted. How many are not spotted? 44

33. \$9.49 for shoes. \$13.35 for dress. How much for both? \$22.84

34. 128 cherry trees. 237 peach trees. How many trees? 365

35. 432 children. 257 boys. How many girls? 175

36. Lakers score 134 points. Knicks score 97. How many more points for the Lakers? 37

37. 246 Fords. 317 Chevrolets. 196 Plymouths. How many cars in the lot? 759

Reflected answers, Set 22: 1. 101, 2. 99, 3. 88, 4. 130, 5. 188, 6. 168, 7. 175, 8. 419, 9. 229, 10. 195, 11. 476, 12. 732, 13. 524

Set 23 | *For use with page 145*

Solve each equation.

1. $6 \times 5 = (4 \times 5) + (n \times 5)$ 2
2. $7 \times 5 = (3 \times 5) + (n \times 5)$ 4
3. $8 \times 6 = (5 \times 6) + (n \times 6)$ 3
4. $8 \times 5 = (n \times 5) + (6 \times 5)$ 2
5. $6 \times 4 = (n \times 4) + (2 \times 4)$ 4
6. $8 \times 3 = (6 \times 3) + (n \times 3)$ 2
7. $7 \times 6 = (n \times 6) + (3 \times 6)$ 4
8. $7 \times 7 = (2 \times 7) + (n \times 7)$ 5
9. $8 \times 6 = (6 \times 6) + (n \times 6)$ 2
10. $7 \times 4 = (4 \times 4) + (n \times 4)$ 3
11. $9 \times 5 = (6 \times 5) + (n \times 5)$ 3
12. $9 \times 6 = (n \times 6) + (5 \times 6)$ 4
13. Since $(3 \times 5) + (2 \times 5) = 25$, we know that $5 \times 5 = n$. 25
14. Since $(3 \times 6) + (4 \times 6) = 42$, we know that $7 \times 6 = n$. 42

Reflected answers, Set 23: 1. 2, 2. 4, 7. 4, 8. 5, 13. 25

Set 24 | *For use with page 147*

Find the products.

1. 0×3 0
2. 2×1 2
3. 2×2 4
4. 2×3 6
5. 3×2 6
6. 3×4 12
7. 1×2 2
8. 3×0 0
9. 2×0 0
10. 5×2 10
11. 3×6 18
12. 1×0 0
13. 0×1 0
14. 2×5 10
15. 2×8 16
16. 0×2 0
17. 3×7 21
18. 6×3 18

Find the products.

19. 5	20. 3	21. 0	22. 4	23. 3	24. 2	25. 5	26. 2
×3	×8	×2	×3	×7	×8	×3	×6
15	24	0	12	21	16	15	12
27. 6	28. 3	29. 5	30. 0	31. 1	32. 8	33. 3	34. 9
×3	×9	×2	×0	×3	×2	×2	×1
18	27	10	0	3	16	6	9

Solve these equations.

35. $3 \times 0 = n$ 0
36. $8 \times 2 = n$ 16
37. $3 \times 5 = n$ 15
38. $7 \times 1 = n$ 7
39. $2 \times 9 = n$ 18
40. $3 \times 7 = n$ 21
41. $8 \times 1 = n$ 8
42. $4 \times 3 = n$ 12

Reflected answers, Set 24: 1. 0, 4. 6, 7. 2, 10. 10, 13. 0, 16. 0, 35. 0, 37. 15, 39. 18, 41. 8

Set 25 | *For use with page 149*

Find the products.

1. 4×5 20	4. 7×4 28	7. 8×2 16	10. 0×5 0	13. 2×7 14
2. 2×3 6	5. 2×8 16	8. 6×4 24	11. 8×5 40	14. 7×5 35
3. 0×5 0	6. 3×7 21	9. 2×5 10	12. 0×8 0	15. 0×0 0

16. $\begin{array}{r}9\\ \times 4\\ \hline 36\end{array}$ 17. $\begin{array}{r}2\\ \times 0\\ \hline 0\end{array}$ 18. $\begin{array}{r}8\\ \times 4\\ \hline 32\end{array}$ 19. $\begin{array}{r}0\\ \times 0\\ \hline 0\end{array}$ 20. $\begin{array}{r}4\\ \times 3\\ \hline 12\end{array}$ 21. $\begin{array}{r}8\\ \times 3\\ \hline 24\end{array}$ 22. $\begin{array}{r}5\\ \times 6\\ \hline 30\end{array}$ 23. $\begin{array}{r}9\\ \times 2\\ \hline 18\end{array}$

24. $\begin{array}{r}8\\ \times 5\\ \hline 40\end{array}$ 25. $\begin{array}{r}5\\ \times 3\\ \hline 15\end{array}$ 26. $\begin{array}{r}7\\ \times 1\\ \hline 7\end{array}$ 27. $\begin{array}{r}5\\ \times 9\\ \hline 45\end{array}$ 28. $\begin{array}{r}9\\ \times 4\\ \hline 36\end{array}$ 29. $\begin{array}{r}8\\ \times 1\\ \hline 8\end{array}$ 30. $\begin{array}{r}3\\ \times 9\\ \hline 27\end{array}$ 31. $\begin{array}{r}4\\ \times 6\\ \hline 24\end{array}$

Solve the equations.

32. $3 \times 5 = n$ 15	34. $7 \times 4 = n$ 28	36. $9 \times 4 = n$ 36	38. $9 \times 5 = n$ 45
33. $6 \times 5 = n$ 30	35. $7 \times 5 = n$ 35	37. $8 \times 5 = n$ 40	39. $1 \times 9 = n$ 9

Reflected answers, Set 25: 1. 20, 4. 28, 7. 16, 10. 0, 13. 14

Set 26 | *For use with page 151*

Find the products.

1. 5×4 20	6. 8×3 24	11. 1×5 5	16. 3×2 6	21. 5×7 35
2. 3×7 21	7. 3×4 12	12. 5×5 25	17. 1×3 3	22. 0×6 0
3. 4×8 32	8. 9×5 45	13. 6×4 24	18. 2×9 18	23. 2×7 14
4. 3×8 24	9. 4×0 0	14. 3×6 18	19. 0×5 0	24. 5×8 40
5. 7×4 28	10. 7×3 21	15. 5×1 5	20. 8×2 16	25. 4×4 16

Solve the equations.

26. $5 \times 8 = n$ 40	28. $4 \times 7 = n$ 28	30. $3 \times 9 = n$ 27	32. $4 \times 5 = n$ 20
27. $2 \times 0 = n$ 0	29. $6 \times 4 = n$ 24	31. $7 \times 1 = n$ 7	33. $1 \times 8 = n$ 8

12. 25, 16. 6, 17. 3, 21. 35, 22. 0
Reflected answers, Set 26: 1. 20, 2. 21, 6. 24, 7. 12, 11. 5

Set 27 | *For use with page 153*

Find the products.

1. 6 × 7 42	**6.** 2 × 7 14	**11.** 8 × 6 48	**16.** 3 × 6 18	**21.** 9 × 6 54
2. 7 × 6 42	**7.** 5 × 0 0	**12.** 4 × 3 12	**17.** 5 × 5 25	**22.** 8 × 5 40
3. 7 × 0 0	**8.** 2 × 9 18	**13.** 8 × 7 56	**18.** 0 × 5 0	**23.** 5 × 1 5
4. 6 × 3 18	**9.** 6 × 1 6	**14.** 3 × 7 21	**19.** 3 × 8 24	**24.** 5 × 8 40
5. 4 × 8 32	**10.** 2 × 4 8	**15.** 0 × 8 0	**20.** 5 × 6 30	**25.** 9 × 7 63

Solve the equations.

26. $6 \times 4 = n$ 24	**28.** $8 \times 7 = n$ 56	**30.** $9 \times 6 = n$ 54	**32.** $9 \times 7 = n$ 63
27. $3 \times 7 = n$ 21	**29.** $5 \times 6 = n$ 30	**31.** $8 \times 6 = n$ 48	**33.** $6 \times 5 = n$ 30

Reflected answers, Set 27: 1. 42, 6. 14, 11. 48, 16. 18, 21. 54

Set 28 | *For use with page 155*

Find the products.

1. 2 × 1 2	**14.** 5 × 3 15	**27.** 7 × 2 14	**40.** 0 × 6 0	**53.** 8 × 4 32
2. 2 × 2 4	**15.** 6 × 5 30	**28.** 8 × 1 8	**41.** 5 × 4 20	**54.** 0 × 8 0
3. 3 × 3 9	**16.** 4 × 6 24	**29.** 8 × 3 24	**42.** 5 × 1 5	**55.** 3 × 9 27
4. 3 × 4 12	**17.** 7 × 6 42	**30.** 0 × 3 0	**43.** 1 × 5 5	**56.** 5 × 8 40
5. 4 × 0 0	**18.** 9 × 1 9	**31.** 8 × 5 40	**44.** 8 × 2 16	**57.** 9 × 6 54
6. 4 × 3 12	**19.** 1 × 3 3	**32.** 4 × 8 32	**45.** 6 × 4 24	**58.** 1 × 7 7
7. 5 × 5 25	**20.** 4 × 2 8	**33.** 6 × 9 54	**46.** 2 × 6 12	**59.** 8 × 6 48
8. 4 × 4 16	**21.** 0 × 0 0	**34.** 1 × 9 9	**47.** 6 × 7 42	**60.** 1 × 6 6
9. 6 × 6 36	**22.** 2 × 9 18	**35.** 7 × 9 63	**48.** 5 × 6 30	**61.** 0 × 1 0
10. 8 × 7 56	**23.** 6 × 3 18	**36.** 9 × 8 72	**49.** 3 × 8 24	**62.** 8 × 8 64
11. 4 × 5 20	**24.** 7 × 3 21	**37.** 2 × 8 16	**50.** 3 × 1 3	**63.** 5 × 9 45
12. 9 × 0 0	**25.** 3 × 7 21	**38.** 5 × 7 35	**51.** 2 × 5 10	**64.** 2 × 4 8
13. 1 × 8 8	**26.** 7 × 5 35	**39.** 6 × 8 48	**52.** 9 × 2 18	**65.** 9 × 4 36

Reflected answers, Set 28: 1. 2, 14. 15, 27. 14, 40. 0, 53. 32

Set 29 *For use with page 159*

Solve the equations.

1. $5 \times 3 = n$ 15	9. $1 \times 6 = n$ 6	17. $3 \times 0 = n$ 0	25. $2 \times 6 = n$ 12
2. $2 \times 6 = n$ 12	10. $8 \times 6 = n$ 48	18. $7 \times 9 = n$ 63	26. $5 \times 9 = n$ 45
3. $4 \times 8 = n$ 32	11. $7 \times 7 = n$ 49	19. $4 \times 5 = n$ 20	27. $4 \times 4 = n$ 16
4. $6 \times 8 = n$ 48	12. $2 \times 9 = n$ 18	20. $6 \times 6 = n$ 36	28. $8 \times 0 = n$ 0
5. $9 \times 7 = n$ 63	13. $5 \times 8 = n$ 40	21. $8 \times 4 = n$ 32	29. $9 \times 5 = n$ 45
6. $8 \times 3 = n$ 24	14. $3 \times 4 = n$ 12	22. $9 \times 6 = n$ 54	30. $5 \times 5 = n$ 25
7. $2 \times 0 = n$ 0	15. $4 \times 9 = n$ 36	23. $3 \times 1 = n$ 3	31. $4 \times 0 = n$ 0
8. $5 \times 7 = n$ 35	16. $3 \times 3 = n$ 9	24. $9 \times 9 = n$ 81	32. $6 \times 8 = n$ 48

Solve each story problem.

33. 9 boys need shoes.
How many shoes? 18

34. 9 horses need shoes.
How many shoes? 36

35. Apples: 7 cents each.
9 apples.
How much money needed? 63 ¢

36. Each girl has 6 dolls.
7 girls.
How many dolls? 42

37. Stamps: 8 cents each.
6 letters.
How much money is needed? 48 ¢

38. 6 pencils in a set.
4 sets.
How many pencils? 24

39. Each package of gum has 5 sticks. Bill has 7 packages.
How many sticks of gum does Bill have? 35

40. A baseball team has 9 members. There are 8 teams playing.
How many members are playing? 72

41. A triangle has 3 angles. Bobbie drew 7 triangles on her paper.
How many angles did she form? 21

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18. 63, 25. 12, 26. 45, 33. 18, 34. 36
Reflected answers, Set 29: 1. 15, 2. 12, 9. 6, 10. 48, 17. 0,

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Set 30 *For use with page 161*

Solve the equations.

1. $9 \times 6 = n$ 54	**11.** $4 \times n = 20$ 5	**21.** $n \times 5 = 25$ 5	**31.** $n \times 9 = 45$ 5
2. $5 \times n = 35$ 7	**12.** $30 = 6 \times n$ 5	**22.** $21 = 7 \times n$ 3	**32.** $24 = 3 \times n$ 8
3. $n \times 4 = 28$ 7	**13.** $72 = 8 \times n$ 9	**23.** $24 = n \times 6$ 4	**33.** $n \times 9 = 45$ 5
4. $n \times 5 = 0$ 0	**14.** $56 = n \times 8$ 7	**24.** $n \times 8 = 64$ 8	**34.** $n \times 2 = 14$ 7
5. $56 = 7 \times n$ 8	**15.** $n \times 9 = 63$ 7	**25.** $9 \times n = 72$ 8	**35.** $48 = 6 \times n$ 8
6. $6 \times n = 24$ 4	**16.** $7 \times n = 49$ 7	**26.** $n \times 9 = 54$ 6	**36.** $n \times 3 = 27$ 9
7. $n \times 8 = 32$ 4	**17.** $8 \times n = 40$ 5	**27.** $48 = n \times 6$ 8	**37.** $4 \times n = 24$ 6
8. $9 \times n = 36$ 4	**18.** $7 \times 6 = n$ 42	**28.** $7 \times n = 42$ 6	**38.** $32 = n \times 8$ 4
9. $8 \times n = 8$ 1	**19.** $30 = 5 \times n$ 6	**29.** $35 = 7 \times n$ 5	**39.** $0 = 5 \times n$ 0
10. $n \times 3 = 21$ 7	**20.** $n \times 7 = 28$ 4	**30.** $8 \times n = 0$ 0	**40.** $35 = 7 \times n$ 5

Solve each story problem.

41. Candy bars are packed in bags of 6. Janet bought 8 bags. How many bars did she buy? 48

42. Children are seated in groups of 8. There are 9 groups in the library. How many children are in the library? 72

43. Pete colored 7 designs. Each design has 6 parts. How many parts did Pete color? 42

44. There are 4 litres in each jug. Mother bought 3 jugs of milk. How many litres of milk did she buy? 12

45. Betty put pictures in a book. She put 5 on each page. She used 9 pages. How many pictures did she put in her book? 45

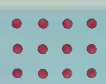
46. Bill has 8 nickels and 7 pennies. How much does he have? 47¢

Reflected answers, Set 30: 1. 54, 2. 7, 11. 5, 12. 5, 21. 5, 22. 3, 31. 5, 32. 8, 41. 48

Set 31 | *For use with page 177*

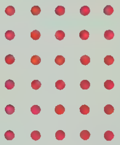
Make sets of dots to help answer each of the following.
Then solve each equation.

1. How many sets of 4 are in a set of 12?
   $12 \div 4 = n$ 3

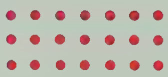


2. How many sets of 3 are in a set of 6?
   $6 \div 3 = n$ 2

3. How many sets of 5 are in a set of 30?
   $30 \div 5 = n$ 6



4. How many sets of 7 are in a set of 21?
   $21 \div 7 = n$ 3



Solve the equations. Use number lines to help.

5. $24 \div 6 = n$ 4	9. $15 \div 3 = n$ 5	13. $27 \div 9 = n$ 3	17. $30 \div 10 = n$ 3
6. $12 \div 2 = n$ 6	10. $48 \div 8 = n$ 6	14. $14 \div 7 = n$ 2	18. $20 \div 2 = n$ 10
7. $40 \div 5 = n$ 8	11. $48 \div 6 = n$ 8	15. $36 \div 9 = n$ 4	19. $54 \div 9 = n$ 6
8. $16 \div 4 = n$ 4	12. $18 \div 3 = n$ 6	16. $24 \div 8 = n$ 3	20. $56 \div 7 = n$ 8

Solve each story problem.

21. 24 children.
    4 in each group.
    How many groups? 6

22. 32 marbles.
    4 in each circle.
    How many circles? 8

23. 35 cents.
    5 cents for each boy.
    How many boys? 7

24. 28 dolls.
    7 in each box.
    How many boxes? 4

25. There are 48 cars in the parking lot. The cars are parked in rows of 8. How many rows in the lot? 6

Reflected answers, Set 31: 1. 3, 2. 2, 5. 4, 6. 6, 13. 3, 17. 3

Set 32 — *For use with page 179*

Find the quotients. Use multiplication to check your answers.

1. $8 \div 4 = n$ 2	**11.** $0 \div 5 = n$ 0	**21.** $16 \div 8 = n$ 2	**31.** $45 \div 5 = n$ 9
2. $12 \div 4 = n$ 3	**12.** $9 \div 1 = n$ 9	**22.** $14 \div 2 = n$ 7	**32.** $24 \div 8 = n$ 3
3. $36 \div 6 = n$ 6	**13.** $64 \div 8 = n$ 8	**23.** $10 \div 2 = n$ 5	**33.** $12 \div 6 = n$ 2
4. $27 \div 9 = n$ 3	**14.** $18 \div 3 = n$ 6	**24.** $54 \div 6 = n$ 9	**34.** $4 \div 4 = n$ 1
5. $7 \div 7 = n$ 1	**15.** $36 \div 4 = n$ 9	**25.** $63 \div 7 = n$ 9	**35.** $35 \div 7 = n$ 5
6. $30 \div 5 = n$ 6	**16.** $10 \div 5 = n$ 2	**26.** $56 \div 8 = n$ 7	**36.** $16 \div 2 = n$ 8
7. $24 \div 6 = n$ 4	**17.** $54 \div 9 = n$ 6	**27.** $81 \div 9 = n$ 9	**37.** $49 \div 7 = n$ 7
8. $12 \div 2 = n$ 6	**18.** $32 \div 4 = n$ 8	**28.** $9 \div 3 = n$ 3	**38.** $42 \div 6 = n$ 7
9. $27 \div 3 = n$ 9	**19.** $28 \div 7 = n$ 4	**29.** $0 \div 3 = n$ 0	**39.** $64 \div 8 = n$ 8
10. $56 \div 8 = n$ 7	**20.** $24 \div 4 = n$ 6	**30.** $36 \div 6 = n$ 6	**40.** $15 \div 3 = n$ 5

Reflected answers, Set 32: 1. 2, 2. 3, 11. 0, 12. 9, 21. 2, 22. 7, 31. 9, 32. 3

Set 33 — *For use with page 185*

Find the products.

1. $6 \times 5 = n$ 30	**4.** $4 \times 6 = n$ 24	**7.** $9 \times 8 = n$ 72	**10.** $3 \times 3 = n$ 9
2. $8 \times 3 = n$ 24	**5.** $1 \times 8 = n$ 8	**8.** $2 \times 1 = n$ 2	**11.** $4 \times 8 = n$ 32
3. $2 \times 7 = n$ 14	**6.** $9 \times 7 = n$ 63	**9.** $0 \times 6 = n$ 0	**12.** $7 \times 6 = n$ 42

Find the missing factors.

13. $5 \times n = 20$ 4	**16.** $9 \times n = 72$ 8	**19.** $7 \times n = 7$ 1	**22.** $n \times 8 = 56$ 7
14. $n \times 3 = 18$ 6	**17.** $4 \times n = 32$ 8	**20.** $n \times 3 = 21$ 7	**23.** $1 \times n = 12$ 12
15. $n \times 0 = 0$ Any number	**18.** $n \times 5 = 40$ 8	**21.** $n \times 9 = 54$ 6	**24.** $3 \times n = 12$ 4

Find the quotients.

25. $16 \div 8 = n$ 2	**29.** $0 \div 2 = n$ 0	**33.** $24 \div 3 = n$ 8	**37.** $81 \div 9 = n$ 9
26. $12 \div 6 = n$ 2	**30.** $4 \div 1 = n$ 4	**34.** $8 \div 2 = n$ 4	**38.** $18 \div 2 = n$ 9
27. $72 \div 8 = n$ 9	**31.** $63 \div 7 = n$ 9	**35.** $14 \div 7 = n$ 2	**39.** $6 \div 6 = n$ 1
28. $21 \div 7 = n$ 3	**32.** $48 \div 6 = n$ 8	**36.** $16 \div 4 = n$ 4	**40.** $64 \div 8 = n$ 8

Reflected answers, Set 33: 1. 30, 4. 24, 7. 72, 10. 9, 13. 4, 16. 8, 19. 1, 22. 7, 25. 2, 29. 0, 33. 8, 37. 9

Set 34 — *For use with page 187*

Find the missing factors and quotients.

1. $27 \div 3 = n$ 9	**11.** $7 = n \div 3$ 21	**21.** $n = 36 \div 4$ 9	**31.** $28 \div n = 4$ 7
2. $48 \div n = 6$ 8	**12.** $72 \div n = 8$ 9	**22.** $n \div 3 = 8$ 24	**32.** $n \div 3 = 4$ 12
3. $n \div 4 = 4$ 16	**13.** $3 = 21 \div n$ 7	**23.** $30 \div n = 6$ 5	**33.** $48 \div n = 8$ 6
4. $72 \div 8 = n$ 9	**14.** $81 \div n = 9$ 9	**24.** $40 \div n = 5$ 8	**34.** $81 \div n = 9$ 9
5. $n = 63 \div 7$ 9	**15.** $40 \div 8 = n$ 5	**25.** $5 = 15 \div n$ 3	**35.** $n = 63 \div 9$ 7
6. $48 \div 8 = n$ 6	**16.** $n \div 7 = 3$ 21	**26.** $n \div 8 = 0$ 0	**36.** $n \div 6 = 9$ 54
7. $32 \div n = 4$ 8	**17.** $20 \div 4 = n$ 5	**27.** $42 \div 7 = n$ 6	**37.** $72 \div n = 8$ 9
8. $n \div 9 = 5$ 45	**18.** $n = 42 \div 6$ 7	**28.** $9 \div n = 9$ 1	**38.** $7 = 35 \div n$ 5
9. $n \div 7 = 7$ 49	**19.** $n \div 6 = 4$ 24	**29.** $56 \div 8 = n$ 7	**39.** $n = 12 \div 2$ 6
10. $54 \div n = 6$ 9	**20.** $72 \div n = 9$ 8	**30.** $n = 24 \div 4$ 6	**40.** $64 \div n = 8$ 8

Solve each story problem.

41. 18 coins.
9 in each set.
How many sets? 2

42. 24 coins.
8 sets.
How many in each set? 3

43. 48 cookies.
8 trays.
How many on each tray? 6

44. 42 Girl Guides.
7 cars.
How many in each car? 6

45. 72 books.
8 for each boy.
How many boys? 9

46. 32 cents.
Carton: 4 cents.
How many cartons? 8

47. Bill put his hockey cards into stacks of 4.
Bill has 36 cards. How many stacks does he have? 9

Reflected answers, Set 34: 1. 9, 2. 8, 11. 21, 12. 9, 21. 9, 22. 24, 31. 7, 32. 12, 41. 2, 42. 3

Set 35 *For use with page 189*

Find the quotients.

1. 2 ÷ 2 = 1	**18.** 3 ÷ 3 = 1	**35.** 7 ÷ 7 = 1	**52.** 3 ÷ 1 = 3	**69.** 5 ÷ 1 = 5
2. 6 ÷ 3 = 2	**19.** 64 ÷ 8 = 8	**36.** 4 ÷ 2 = 2	**53.** 48 ÷ 8 = 6	**70.** 8 ÷ 1 = 8
3. 16 ÷ 4 = 4	**20.** 35 ÷ 7 = 5	**37.** 9 ÷ 3 = 3	**54.** 5 ÷ 5 = 1	**71.** 4 ÷ 4 = 1
4. 9 ÷ 1 = 9	**21.** 12 ÷ 2 = 6	**38.** 1 ÷ 1 = 1	**55.** 7 ÷ 1 = 7	**72.** 36 ÷ 4 = 9
5. 14 ÷ 2 = 7	**22.** 49 ÷ 7 = 7	**39.** 21 ÷ 7 = 3	**56.** 24 ÷ 8 = 3	**73.** 40 ÷ 5 = 8
6. 6 ÷ 2 = 3	**23.** 48 ÷ 6 = 8	**40.** 63 ÷ 7 = 9	**57.** 42 ÷ 7 = 6	**74.** 16 ÷ 2 = 8
7. 2 ÷ 1 = 2	**24.** 15 ÷ 5 = 3	**41.** 12 ÷ 3 = 4	**58.** 21 ÷ 3 = 7	**75.** 0 ÷ 5 = 0
8. 42 ÷ 6 = 7	**25.** 72 ÷ 8 = 9	**42.** 25 ÷ 5 = 5	**59.** 30 ÷ 6 = 5	**76.** 40 ÷ 8 = 5
9. 63 ÷ 9 = 7	**26.** 4 ÷ 1 = 4	**43.** 20 ÷ 5 = 4	**60.** 27 ÷ 3 = 9	**77.** 36 ÷ 9 = 4
10. 32 ÷ 4 = 8	**27.** 24 ÷ 3 = 8	**44.** 56 ÷ 8 = 7	**61.** 24 ÷ 4 = 6	**78.** 14 ÷ 7 = 2
11. 24 ÷ 6 = 4	**28.** 32 ÷ 8 = 4	**45.** 16 ÷ 8 = 2	**62.** 18 ÷ 9 = 2	**79.** 12 ÷ 6 = 2
12. 36 ÷ 6 = 6	**29.** 30 ÷ 6 = 5	**46.** 20 ÷ 4 = 5	**63.** 28 ÷ 7 = 4	**80.** 72 ÷ 8 = 9
13. 8 ÷ 2 = 4	**30.** 6 ÷ 1 = 6	**47.** 15 ÷ 3 = 5	**64.** 10 ÷ 5 = 2	**81.** 72 ÷ 9 = 8
14. 18 ÷ 2 = 9	**31.** 56 ÷ 7 = 8	**48.** 0 ÷ 1 = 0	**65.** 9 ÷ 9 = 1	**82.** 30 ÷ 5 = 6
15. 6 ÷ 6 = 1	**32.** 28 ÷ 4 = 7	**49.** 18 ÷ 6 = 3	**66.** 35 ÷ 5 = 7	**83.** 81 ÷ 9 = 9
16. 18 ÷ 3 = 6	**33.** 8 ÷ 8 = 1	**50.** 45 ÷ 9 = 5	**67.** 54 ÷ 6 = 9	**84.** 8 ÷ 4 = 2
17. 27 ÷ 9 = 3	**34.** 12 ÷ 4 = 3	**51.** 10 ÷ 2 = 5	**68.** 0 ÷ 9 = 0	**85.** 48 ÷ 8 = 6

Solve each story problem.

86. How many pennies equal 7 nickels? 35

87. How many weeks are 49 days? 7

88. Alice drew several triangles. She counted 27 angles. How many triangles did she draw? 9

Reflected answers, Set 35: 1. 1, 2. 2, 3. 4, 18. 1, 19. 8, 20. 5, 35. 1, 36. 2, 37. 3, 52. 3, 53. 6, 54. 1, 69. 5, 70. 8, 71. 1

Set 36 | *For use with page 191*

Solve each equation.

1. $28 \div 4 = n$ 7	**10.** $32 \div n = 8$ 4	**19.** $n \div 3 = 4$ 12	**28.** $48 \div n = 6$ 8
2. $n = 54 \div 6$ 9	**11.** $49 \div 7 = n$ 7	**20.** $27 \div n = 3$ 9	**29.** $n \div 8 = 5$ 40
3. $n \div 6 = 6$ 36	**12.** $63 \div n = 9$ 7	**21.** $6 = n \div 6$ 36	**30.** $16 \div n = 4$ 4
4. $5 = 40 \div n$ 8	**13.** $54 \div n = 6$ 9	**22.** $48 \div 6 = n$ 8	**31.** $n \div 2 = 10$ 20
5. $25 \div 5 = n$ 5	**14.** $n = 7 \div 7$ 1	**23.** $n \div 8 = 1$ 8	**32.** $42 \div n = 7$ 6
6. $n = 72 \div 9$ 8	**15.** $n \div 6 = 5$ 30	**24.** $28 \div n = 4$ 7	**33.** $63 \div n = 7$ 9
7. $7 = 49 \div n$ 7	**16.** $n \div 4 = 7$ 28	**25.** $54 \div 9 = n$ 6	**34.** $15 \div n = 5$ 3
8. $35 \div 5 = n$ 7	**17.** $n = 36 \div 4$ 9	**26.** $40 \div 8 = n$ 5	**35.** $n \div 2 = 4$ 8
9. $16 \div n = 2$ 8	**18.** $56 \div n = 7$ 8	**27.** $n = 30 \div 5$ 6	**36.** $n \div 3 = 9$ 27

Solve each story problem.

37. 42 dots.
6 sets.
How many in each set? 7

38. 24 dots.
6 in each set.
How many sets? 4

39. 27 plants.
3 rows.
How many plants per row? 9

40. 16 dogs.
2 dogs per kennel.
How many kennels? 8

41. Passengers flying in a 707 airplane sit in rows of 6.
48 passengers can sit in the "no smoking" section.
How many rows have "no smoking"? 6

42. The book store is having an 8¢ sale on 10¢ tablets.
Shelly has 56¢. How many tablets can she buy? 7

43. How many 6-bottle cartons are needed to pack 54 bottles? 9

Reflected answers, Set 36: 1. 7, 2. 9, 10. 4, 11. 7, 19. 12, 20. 9, 28. 8, 29. 40, 37. 7, 38. 4

Set 37 *For use with page 223*

Tell whether the sentence is true (T) or false (F).

T **1.** 16 is a multiple of 8.

T **2.** 9 is a factor of 18.

F **3.** 4 is a factor of 6.

T **4.** 3 is a factor of 12.

T **5.** 7 is a prime number.

T **6.** 12 is a multiple of 6.

F **7.** 10 is a multiple of 3.

F **8.** 15 is a prime number.

T **9.** 0 is a multiple of 6.

F **10.** 0 is a factor of 6.

F **11.** 35 is a prime number.

F **12.** 54 is a multiple of 8.

T **13.** 9 is a factor of both 36 and 72.

F **14.** 42 is a multiple of both 7 and 5.

F **15.** A prime number has more than two factors.

T **16.** 18 has exactly 6 factors.

T **17.** 24 is a multiple of 3, 6, and 8.

F **18.** 40 is a multiple of 0, 10, 20, and 40.

Complete the sentence.

19. Since $5 \times 8 = 40$, 5 and 8 are factors of 40.

20. Since $6 \times 9 = 54$, 54 is a multiple of 6 and 9.

21. The first 5 multiples of 9 are 0, 9, 18, 27, 36.

22. The factors of 4 and 8 are 1, 2, 4.

23. The first 3 multiples of both 2 and 3 are 0, 6, 12.

24. The factors of 37 are 1, 37.

25. The multiples of 6 between 20 and 40 are 24, 30, 36.

26. The prime numbers between 20 and 30 are 23, 29.

27. The factors of 36 that are between 10 and 20 are 12, 18.

Reflected answers, Set 37: 1. T, 2. T, 7. F, 8. F, 19. factors, 20. 9

Set 38 | *For use with page 229*

Find the value in cents.

1. 3 dimes 30¢	**4.** 18 dimes 180¢	**7.** 32 dimes 320¢	**10.** 1 dime 10¢	**13.** 67 dimes 670¢
2. 7 dimes 70¢	**5.** 25 dimes 250¢	**8.** 85 dimes 850¢	**11.** 75 dimes 750¢	**14.** 51 dimes 510¢
3. 12 dimes 120¢	**6.** 11 dimes 110¢	**9.** 91 dimes 910¢	**12.** 0 dimes 0¢	**15.** 87 dimes 870¢

Solve the equations.

16. $(20 \times 10) + (3 \times 10) = n$ 230	**20.** $(70 \times 10) + (5 \times 10) = n$ 750
17. $(60 \times 10) + (4 \times 10) = n$ 640	**21.** $(40 \times 10) + (6 \times 10) = n$ 460
18. $(30 \times 10) + (7 \times 10) = n$ 370	**22.** $(10 \times 10) + (9 \times 10) = n$ 190
19. $(80 \times 10) + (1 \times 10) = n$ 810	**23.** $(90 \times 10) + (2 \times 10) = n$ 920

24. $18 \times 10 = n$ 180	**31.** $77 \times 10 = n$ 770	**38.** $10 \times n = 120$ 12
25. $24 \times 10 = n$ 240	**32.** $31 \times 10 = n$ 310	**39.** $50 \times n = 500$ 10
26. $37 \times 10 = n$ 370	**33.** $10 \times 11 = n$ 110	**40.** $n \times 22 = 220$ 10
27. $7 \times 10 = n$ 70	**34.** $10 \times 89 = n$ 890	**41.** $36 \times n = 360$ 10
28. $97 \times 10 = n$ 970	**35.** $10 \times 63 = n$ 630	**42.** $10 \times n = 270$ 27
29. $48 \times 10 = n$ 480	**36.** $10 \times n = 90$ 9	**43.** $n \times 10 = 570$ 57
30. $21 \times 10 = n$ 210	**37.** $n \times 10 = 40$ 4	**44.** $n \times 10 = 970$ 97

Find the products.

45. 9×10 90	**54.** 8×10 80	**63.** 34×10 340	**72.** 65×100 6500
46. 7×10 70	**55.** 10×7 70	**64.** 52×10 520	**73.** 100×73 7300
47. 10×8 80	**56.** 6×10 60	**65.** 65×10 650	**74.** 32×10 320
48. 10×0 0	**57.** 10×4 40	**66.** 20×10 200	**75.** 10×58 580
49. 2×10 20	**58.** 0×10 0	**67.** 53×10 530	**76.** 36×10 360
50. 10×6 60	**59.** 3×10 30	**68.** 10×81 810	**77.** 58×10 580
51. 1×10 10	**60.** 12×10 120	**69.** 40×10 400	**78.** 76×10 760
52. 14×10 140	**61.** 15×10 150	**70.** 10×80 800	**79.** 10×40 400
53. 10×16 160	**62.** 10×18 180	**71.** 38×100 3800	**80.** 62×10 620

Reflected answers, Set 38: 1. 30¢, 4. 180¢, 7. 320¢, 10. 10¢,
13. 670¢, 16. 230, 17. 640, 20. 750, 21. 460, 24. 180,
31. 770, 38. 12, 45. 90, 46. 70, 54. 80, 55. 70, 63. 340,
64. 520, 72. 6500, 73. 7300

Set 39 *For use with page 231*

Find the products.

1. 3 × 40 120
2. 60 × 4 240
3. 50 × 9 450
4. 8 × 70 560
5. 2 × 30 60
6. 8 × 50 400
7. 20 × 9 180
8. 7 × 30 210
9. 4 × 80 320
10. 50 × 6 300
11. 9 × 90 810
12. 4 × 70 280
13. 80 × 6 480
14. 60 × 6 360
15. 8 × 60 480
16. 7 × 70 490
17. 80 × 8 640
18. 90 × 6 540
19. 8 × 20 160
20. 90 × 7 630
21. 4 × 50 200
22. 70 × 3 210
23. 60 × 8 480
24. 7 × 90 630
25. 3 × 20 60
26. 7 × 60 420
27. 30 × 9 270
28. 6 × 30 180
29. 5 × 80 400
30. 6 × 60 360
31. 8 × 90 720
32. 5 × 60 300
33. 70 × 8 560
34. 80 × 7 560
35. 4 × 90 360
36. 4 × 50 200
37. 90 × 5 450
38. 60 × 9 540
39. 70 × 5 350
40. 90 × 9 810
41. 275 × 10 2750
42. 10 × 832 8320
43. 653 × 10 6530
44. 10 × 976 9760
45. 8345 × 10 83450
46. 8 × 100 800
47. 100 × 9 900
48. 18 × 100 1800

Reflected answers, Set 39: 1. 120, 2. 240, 13. 480, 14. 360, 25. 60, 26. 420, 37. 450, 38. 540

Set 40 *For use with page 239*

Solve the equations.

1. 5 × 18 = (5 × 10) + (5 × *n*) 8
2. 7 × 16 = (7 × *n*) + (7 × 6) 10
3. 4 × 22 = (4 × 20) + (4 × *n*) 2
4. 8 × 27 = (8 × *n*) + (8 × 7) 20
5. 6 × 35 = (6 × 30) + (6 × *n*) 5
6. 9 × 42 = (9 × *n*) + (9 × 2) 40
7. 7 × 53 = (7 × 50) + (7 × *n*) 3
8. 2 × 83 = (2 × *n*) + (2 × 3) 80
9. 3 × 75 = (3 × 70) + (3 × *n*) 5
10. 9 × 62 = (9 × *n*) + (9 × 2) 60
11. (5 × 10) + (5 × 3) = *n* 65
12. (6 × 10) + (6 × 8) = *n* 108
13. (4 × 20) + (4 × 2) = *n* 88
14. (5 × 20) + (5 × 9) = *n* 145
15. (3 × 30) + (3 × 8) = *n* 114
16. (8 × 30) + (8 × 2) = *n* 256
17. (7 × 60) + (7 × 1) = *n* 427
18. (6 × 50) + (6 × 7) = *n* 342
19. (4 × 70) + (4 × 6) = *n* 304
20. (9 × 40) + (9 × 7) = *n* 423
21. (2 × 90) + (2 × 7) = *n* 194
22. (7 × 80) + (7 × 8) = *n* 616
23. (8 × 60) + (8 × 7) = *n* 536
24. (9 × 90) + (9 × 3) = *n* 837

Reflected answers, Set 40: 1. 8, 2. 10, 3. 2, 13. 88, 14. 145, 15. 114

Set 41 | *For use with page 241*

Find the products.

1. 32×2 64	**2.** 16×4 64	**3.** 45×5 225	**4.** 41×4 164	**5.** 28×5 140	**6.** 37×3 111
7. 27×4 108	**8.** 81×5 405	**9.** 90×3 270	**10.** 65×8 520	**11.** 56×2 112	**12.** 43×9 387
13. 84×4 336	**14.** 52×7 364	**15.** 63×8 504	**16.** 98×3 294	**17.** 21×9 189	**18.** 79×2 158
19. 11×3 33	**20.** 22×5 110	**21.** 53×4 212	**22.** 43×6 258	**23.** 51×3 153	**24.** 24×7 168
25. 45×2 90	**26.** 67×7 469	**27.** 81×6 486	**28.** 60×5 300	**29.** 79×8 632	**30.** 85×9 765

Solve each story problem.

31. Mr. Williams is planting pear trees. He plants 16 rows. Each row has 8 trees. How many trees did he plant? 128

32. Model airplane kits have 24 pieces in them. Mark buys 7 kits. How many airplane pieces does Mark have? 168

33. A building has 38 floors. Each floor has 9 offices. How many offices are in the building? 342

34. Candy is 79 cents a kilogram. How much will Nancy have to pay for 6 kilograms of candy? $4.74

35. Mike collects coins. Each card contains 64 coins. Mike has 5 cards filled. How many coins does he have? 320

31. 128, 32. 168
6. 111, 7. 108, 8. 405, 9. 270, 10. 520, 11. 112, 12. 387,
Reflected answers, Set 41: 1. 64, 2. 64, 3. 225, 4. 164, 5. 140,

Set 42	*For use with page 243*

Find the products.

1. $\begin{array}{r}19\\ \times 2\\ \hline 38\end{array}$	**2.** $\begin{array}{r}62\\ \times 4\\ \hline 248\end{array}$	**3.** $\begin{array}{r}35\\ \times 3\\ \hline 105\end{array}$	**4.** $\begin{array}{r}13\\ \times 4\\ \hline 52\end{array}$	**5.** $\begin{array}{r}16\\ \times 2\\ \hline 32\end{array}$	**6.** $\begin{array}{r}22\\ \times 4\\ \hline 88\end{array}$
7. $\begin{array}{r}29\\ \times 6\\ \hline 174\end{array}$	**8.** $\begin{array}{r}43\\ \times 7\\ \hline 301\end{array}$	**9.** $\begin{array}{r}78\\ \times 3\\ \hline 234\end{array}$	**10.** $\begin{array}{r}13\\ \times 3\\ \hline 39\end{array}$	**11.** $\begin{array}{r}32\\ \times 5\\ \hline 160\end{array}$	**12.** $\begin{array}{r}34\\ \times 4\\ \hline 136\end{array}$
13. $\begin{array}{r}13\\ \times 7\\ \hline 91\end{array}$	**14.** $\begin{array}{r}24\\ \times 5\\ \hline 120\end{array}$	**15.** $\begin{array}{r}32\\ \times 4\\ \hline 128\end{array}$	**16.** $\begin{array}{r}63\\ \times 8\\ \hline 504\end{array}$	**17.** $\begin{array}{r}99\\ \times 9\\ \hline 891\end{array}$	**18.** $\begin{array}{r}54\\ \times 3\\ \hline 162\end{array}$
19. $\begin{array}{r}63\\ \times 8\\ \hline 504\end{array}$	**20.** $\begin{array}{r}76\\ \times 6\\ \hline 456\end{array}$	**21.** $\begin{array}{r}67\\ \times 4\\ \hline 268\end{array}$	**22.** $\begin{array}{r}46\\ \times 2\\ \hline 92\end{array}$	**23.** $\begin{array}{r}53\\ \times 9\\ \hline 477\end{array}$	**24.** $\begin{array}{r}77\\ \times 8\\ \hline 616\end{array}$
25. $\begin{array}{r}98\\ \times 7\\ \hline 686\end{array}$	**26.** $\begin{array}{r}39\\ \times 2\\ \hline 78\end{array}$	**27.** $\begin{array}{r}55\\ \times 6\\ \hline 330\end{array}$	**28.** $\begin{array}{r}80\\ \times 5\\ \hline 400\end{array}$	**29.** $\begin{array}{r}39\\ \times 3\\ \hline 117\end{array}$	**30.** $\begin{array}{r}79\\ \times 9\\ \hline 711\end{array}$

Solve each story problem.

31. Nancy delivers 78 papers a day. How many papers does she deliver in 7 days? 546

32. There are 24 hours in one day. How many hours are in 6 days? 144

33. There are 48 jars of baby food in a case. Mrs. Fong bought 9 cases. How many jars did she buy? 432

34. A school bus can carry 68 children. How many children can ride in 8 buses this size? 544

35. During his vacation trip, Mr. Raymond drove his car an average of 92 kilometres per hour. He drove for 4 hours on Friday and 5 hours on Saturday. How far did Mr. Raymond drive? 828 Km

Reflected answers, Set 42: 1. 38, 2. [illegible], 3. 105, 4. 52, 5. 32,
6. [illegible], 7. 174, 8. 301, 9. 234, 10. [illegible], 11. 160, 12. 136,
31. 546, 32. 144

Set 43	*For use with page 247*

Find each product.

1. $\begin{array}{r}47\\ \times 2\\ \hline 94\end{array}$	**2.** $\begin{array}{r}56\\ \times 3\\ \hline 168\end{array}$	**3.** $\begin{array}{r}75\\ \times 5\\ \hline 375\end{array}$	**4.** $\begin{array}{r}86\\ \times 4\\ \hline 344\end{array}$	**5.** $\begin{array}{r}93\\ \times 4\\ \hline 372\end{array}$	**6.** $\begin{array}{r}76\\ \times 3\\ \hline 228\end{array}$
7. $\begin{array}{r}88\\ \times 7\\ \hline 616\end{array}$	**8.** $\begin{array}{r}96\\ \times 8\\ \hline 768\end{array}$	**9.** $\begin{array}{r}47\\ \times 6\\ \hline 282\end{array}$	**10.** $\begin{array}{r}52\\ \times 9\\ \hline 468\end{array}$	**11.** $\begin{array}{r}61\\ \times 8\\ \hline 488\end{array}$	**12.** $\begin{array}{r}78\\ \times 7\\ \hline 546\end{array}$
13. $\begin{array}{r}42\\ \times 5\\ \hline 210\end{array}$	**14.** $\begin{array}{r}65\\ \times 8\\ \hline 520\end{array}$	**15.** $\begin{array}{r}82\\ \times 3\\ \hline 246\end{array}$	**16.** $\begin{array}{r}97\\ \times 2\\ \hline 194\end{array}$	**17.** $\begin{array}{r}94\\ \times 7\\ \hline 658\end{array}$	**18.** $\begin{array}{r}58\\ \times 4\\ \hline 232\end{array}$
19. $\begin{array}{r}74\\ \times 6\\ \hline 444\end{array}$	**20.** $\begin{array}{r}83\\ \times 4\\ \hline 332\end{array}$	**21.** $\begin{array}{r}86\\ \times 9\\ \hline 774\end{array}$	**22.** $\begin{array}{r}71\\ \times 6\\ \hline 426\end{array}$	**23.** $\begin{array}{r}88\\ \times 8\\ \hline 704\end{array}$	**24.** $\begin{array}{r}49\\ \times 7\\ \hline 343\end{array}$
25. $\begin{array}{r}38\\ \times 8\\ \hline 304\end{array}$	**26.** $\begin{array}{r}92\\ \times 6\\ \hline 552\end{array}$	**27.** $\begin{array}{r}79\\ \times 9\\ \hline 711\end{array}$	**28.** $\begin{array}{r}97\\ \times 2\\ \hline 194\end{array}$	**29.** $\begin{array}{r}65\\ \times 4\\ \hline 260\end{array}$	**30.** $\begin{array}{r}87\\ \times 5\\ \hline 435\end{array}$

Solve each story problem.

31. The assembly hall has 78 rows of seats. There are 9 seats in each row. How many persons can sit in the hall? 702

32. There are 8 rows of bricks in a wall. Each row contains 98 bricks. How many bricks are in the wall? 784

33. Paper costs 89 cents a tablet. Pens cost 29 cents each. How much do 7 tablets and 4 pens cost? $7.39

34. The basketball team practices 85 minutes on Monday and Wednesday. They practice 75 minutes on Tuesday, Thursday and Friday. How many minutes does the team practice each week? 395 min

35. Mike builds bookcases with 6 shelves. Each shelf holds 16 books. How many books can be put in 8 bookcases? 768

Reflected answers, Set 43: 1. 94, 2. 168, 3. 375, 4. 344, 5. 372, 6. 228, 7. 616, 8. 768, 9. 282, 10. 468, 11. 488, 12. 546, 31. 702, 32. 784

Set 44 *For use with page 249*

Find the products.

1. 413	**2.** 525	**3.** 334	**4.** 242	**5.** 453	**6.** 264
×3	×2	×4	×5	×3	×2
1239	1050	1336	1210	1359	528
7. 563	**8.** 652	**9.** 835	**10.** 496	**11.** 741	**12.** 416
×9	×3	×8	×7	×6	×4
5067	1956	6680	3472	4446	1664
13. 123	**14.** 564	**15.** 345	**16.** 153	**17.** 254	**18.** 347
×2	×3	×4	×5	×6	×4
246	1692	1380	765	1524	1388
19. 243	**20.** 563	**21.** 674	**22.** 755	**23.** 486	**24.** 327
×3	×5	×7	×2	×9	×8
729	2815	4718	1510	4374	2616

25. 1542	**26.** 3361	**27.** 5782	**28.** 7954
×2	×3	×5	×3
3084	10 083	28 910	23 862
29. 1738	**30.** 2138	**31.** 3279	**32.** 4960
×3	×5	×6	×9
5214	10 690	19 674	44 640
33. 4189	**34.** 7498	**35.** 5276	**36.** 1556
×5	×6	×7	×8
20 945	44 988	36 932	12 448

Solve each story problem.

37. A certain truck company allows their truckers to drive 575 kilometres per day. How far can one trucker drive in 6 days? 3450 km

38. The average weight of each lineman on a certain professional football team is 119 kilograms. There are 7 men in the line. How much does the line weigh? 833 kg

39. The 9 children in the Tiger Club collected paper for a paper drive. Each child collected 108 kilograms of paper per week. The drive lasted 6 weeks. How many kilograms of paper were collected by the Tiger Club? 5832 kg

Reflected answers, Set 44: 1. 1239, 2. 1050, 3. 1336, 4. 1210, 5. 1359, 6. 528, 25. 3084, 26. 10,083, 27. 28,910, 28. 23,862,

Set 45 *For use with page 279*

Find the products.

1. 7×9 63	**5.** 7×4 28	**9.** 3×40 120	**13.** 90×2 180	**17.** 200×3 600
2. 8×6 48	**6.** 20×6 120	**10.** 7×50 350	**14.** 30×7 210	**18.** 100×4 400
3. 3×5 15	**7.** 40×8 320	**11.** 8×10 80	**15.** 70×6 420	**19.** 700×6 4200
4. 2×9 18	**8.** 7×70 490	**12.** 9×80 720	**16.** 800×7 5600	**20.** 8×900 7200

Find the quotients.

21. $72 \div 8$ 9	**25.** $30 \div 5$ 6	**29.** $180 \div 2$ 90	**33.** $300 \div 6$ 50	**37.** $5400 \div 9$ 600
22. $64 \div 8$ 8	**26.** $560 \div 7$ 80	**30.** $210 \div 7$ 30	**34.** $400 \div 5$ 80	**38.** $1000 \div 5$ 200
23. $63 \div 7$ 9	**27.** $270 \div 3$ 90	**31.** $480 \div 6$ 80	**35.** $60 \div 3$ 20	**39.** $2800 \div 7$ 400
24. $24 \div 6$ 4	**28.** $180 \div 3$ 60	**32.** $360 \div 4$ 90	**36.** $2100 \div 3$ 700	**40.** $6300 \div 9$ 700

Find the missing factors.

41. $5 \times n = 35$ 7	**46.** $8 \times n = 320$ 40	**51.** $2 \times n = 1000$ 500	**56.** $n \times 5 = 1500$ 300
42. $n \times 2 = 16$ 8	**47.** $n \times 5 = 450$ 90	**52.** $n \times 6 = 5400$ 900	**57.** $n \times 7 = 4200$ 600
43. $6 \times n = 18$ 3	**48.** $7 \times n = 70$ 10	**53.** $9 \times n = 6300$ 700	**58.** $6 \times n = 4200$ 700
44. $n \times 9 = 36$ 4	**49.** $n \times 8 = 640$ 80	**54.** $n \times 4 = 2800$ 700	**59.** $n \times 2 = 1400$ 700
45. $n \times 7 = 280$ 40	**50.** $3 \times n = 150$ 50	**55.** $n \times 9 = 3600$ 400	**60.** $9 \times n = 7200$ 800

Find the missing quotients.

61. $48 \div 6 = n$ 8	**66.** $270 \div 3 = n$ 90	**71.** $60 \div 2 = n$ 30	**76.** $3200 \div 4 = n$ 800
62. $20 \div 4 = n$ 5	**67.** $210 \div 7 = n$ 30	**72.** $120 \div 3 = n$ 40	**77.** $1500 \div 5 = n$ 300
63. $16 \div 8 = n$ 2	**68.** $360 \div 9 = n$ 40	**73.** $4800 \div 6 = n$ 800	**78.** $2400 \div 3 = n$ 800
64. $56 \div 7 = n$ 8	**69.** $300 \div 5 = n$ 60	**74.** $4900 \div 7 = n$ 700	**79.** $5600 \div 8 = n$ 700
65. $120 \div 2 = n$ 60	**70.** $320 \div 8 = n$ 40	**75.** $4500 \div 9 = n$ 500	**80.** $8100 \div 9 = n$ 900

51. 500, 56. 300, 61. 8, 66. 90, 71. 30, 76. 800
21. 9, 25. 6, 29. 90, 33. 50, 37. 600, 41. 7, 46. 40,
Reflected answers, Set 45: 1. 63, 5. 28, 9. 120, 13. 180, 17. 600,

Set 46 *For use with page 289*

Find the quotients.

1. 36 ÷ 2 18	**10.** 168 ÷ 7 24	**19.** 117 ÷ 3 39	**28.** 296 ÷ 8 37	**37.** 336 ÷ 8 42
2. 48 ÷ 4 12	**11.** 39 ÷ 3 13	**20.** 162 ÷ 6 27	**29.** 153 ÷ 3 51	**38.** 279 ÷ 9 31
3. 96 ÷ 8 12	**12.** 74 ÷ 2 37	**21.** 272 ÷ 8 34	**30.** 232 ÷ 4 58	**39.** 364 ÷ 7 52
4. 72 ÷ 6 12	**13.** 189 ÷ 9 21	**22.** 140 ÷ 5 28	**31.** 265 ÷ 5 53	**40.** 504 ÷ 7 72
5. 65 ÷ 5 13	**14.** 108 ÷ 4 27	**23.** 210 ÷ 5 42	**32.** 180 ÷ 4 45	**41.** 336 ÷ 4 84
6. 77 ÷ 7 11	**15.** 174 ÷ 6 29	**24.** 287 ÷ 7 41	**33.** 252 ÷ 6 42	**42.** 456 ÷ 6 76
7. 52 ÷ 4 13	**16.** 175 ÷ 7 25	**25.** 387 ÷ 9 43	**34.** 203 ÷ 7 29	**43.** 364 ÷ 4 91
8. 88 ÷ 4 22	**17.** 258 ÷ 6 43	**26.** 198 ÷ 9 22	**35.** 333 ÷ 9 37	**44.** 376 ÷ 8 47
9. 110 ÷ 5 22	**18.** 132 ÷ 4 33	**27.** 368 ÷ 8 46	**36.** 112 ÷ 2 56	**45.** 469 ÷ 7 67

Reflected answers, Set 46: 1. , 2. , 3. , 10. , 11. , 12. , 19. , 20. , 21. , 28. , 29. , 30. , 37. , 38. , 39.

Set 47 *For use with page 291*

Solve each story problem.

1. 112 players. 8 on each team. How many teams? 14
2. 252 books. 6 shelves. How many on each shelf? 42
3. 85 cents. All nickels. How many nickels? 17
4. 364 children. 7 per car. How many cars? 52
5. 184 truck tires sold. 4 tires for each truck. How many trucks? 46
6. 243 balloons. 9 packs. How many in each pack? 27
7. 228 bottles. 6 per carton. How many cartons? 38
8. 336 seats. 8 rows. How many in each row? 42
9. 336 days. How many weeks? 48
10. 639 kilograms of shrimp. 9 kilograms per carton. How many cartons? 71

Reflected answers, Set 47: 1. , 2. , 6. , 7.

Set 48 — *For use with page 295*

Find the quotients.

1. 2)24 12	2. 3)63 21	3. 4)48 12	4. 6)96 16	5. 7)84 12	6. 5)80 16
7. 3)78 26	8. 5)75 15	9. 7)98 14	10. 4)144 36	11. 2)88 44	12. 3)138 46
13. 4)52 13	14. 2)32 16	15. 3)48 16	16. 5)60 12	17. 6)72 12	18. 7)91 13
19. 7)112 16	20. 8)176 22	21. 9)207 23	22. 6)78 13	23. 8)96 12	24. 9)108 12
25. 2)108 54	26. 4)92 23	27. 6)204 34	28. 3)171 57	29. 4)272 68	30. 5)235 47
31. 5)470 94	32. 3)222 74	33. 2)134 67	34. 7)385 55	35. 8)752 94	36. 6)588 98
37. 7)301 43	38. 9)585 65	39. 5)415 83	40. 6)564 94	41. 5)445 89	42. 4)308 77
43. 8)368 46	44. 6)342 57	45. 4)384 96	46. 7)483 69	47. 8)312 39	48. 9)666 74

Reflected answers, Set 48: 1. 12, 2. 21, 3. 12, 4. 16, 5. 12, 6. 16, 25. 54, 26. 23, 27. 34, 28. 57, 29. 68, 30. 47

Set 49 — *For use with page 301*

Find the quotients and remainders.

1. 2)57 28 R1	2. 3)46 15 R1	3. 5)82 16 R2	4. 4)77 19 R1	5. 7)98 14 R0	6. 6)86 14 R2
7. 2)69 34 R1	8. 3)92 30 R2	9. 4)58 14 R2	10. 5)76 15 R1	11. 6)69 11 R3	12. 7)87 12 R3
13. 5)235 47 R0	14. 2)146 73 R0	15. 5)478 95 R3	16. 6)379 63 R1	17. 7)598 85 R3	18. 4)192 48 R0
19. 3)163 54 R1	20. 6)475 79 R1	21. 3)286 95 R1	22. 4)379 94 R3	23. 4)281 70 R1	24. 2)192 96 R0
25. 9)654 72 R6	26. 8)355 44 R3	27. 7)463 66 R1	28. 3)264 88 R0	29. 8)789 98 R5	30. 9)237 26 R3
31. 5)142 28 R2	32. 2)173 86 R1	33. 3)274 91 R1	34. 8)274 34 R2	35. 5)385 77 R0	36. 7)496 70 R6
37. 8)572 71 R4	38. 7)351 50 R1	39. 2)138 69 R0	40. 3)249 83 R0	41. 4)354 88 R2	42. 9)665 73 R8
43. 2)187 93 R1	44. 6)496 82 R4	45. 6)576 96 R0	46. 8)791 98 R7	47. 4)284 71 R0	48. 9)823 91 R4

Reflected answers, Set 49: 1. 28 R1, 2. 15 R1, 3. 16 R2, 4. 19 R1, 5. 14 R0, 6. 14 R2, 25. 72 R6, 26. 44 R3, 27. 66 R1, 28. 88 R0, 29. 98 R5, 30. 26 R3

Set 50	*For use with page 303*

Find the quotients.

1. 102 ÷ 6 17   5. 243 ÷ 9 27   9. 273 ÷ 3 91   13. 396 ÷ 4 99   17. 260 ÷ 5 52
2. 92 ÷ 4 23   6. 140 ÷ 5 28   10. 301 ÷ 7 43   14. 434 ÷ 7 62   18. 837 ÷ 9 93
3. 195 ÷ 3 65   7. 217 ÷ 7 31   11. 423 ÷ 9 47   15. 608 ÷ 8 76   19. 784 ÷ 8 98
4. 256 ÷ 8 32   8. 384 ÷ 8 48   12. 402 ÷ 6 67   16. 170 ÷ 2 85   20. 320 ÷ 5 64

Find the quotients and remainders.

21. 77 ÷ 3 25R2   24. 91 ÷ 2 45R1   27. 270 ÷ 4 67R2   30. 715 ÷ 9 79R4   33. 699 ÷ 9 77R6
22. 87 ÷ 6 14R3   25. 275 ÷ 8 34R3   28. 428 ÷ 8 53R4   31. 517 ÷ 7 73R6   34. 775 ÷ 8 96R7
23. 190 ÷ 7 27R1   26. 192 ÷ 5 38R2   29. 259 ÷ 3 86R1   32. 538 ÷ 6 89R4   35. 438 ÷ 5 87R3

Find the sums, differences, products, and quotients.

36. 654 − 276 = 378   37. 9)477 53   38. 364 × 7 = 2548   39. 7)448 64   40. 719 − 263 = 456

41. 923 − 357 = 566   42. 8)596 74 R4   43. 539 + 885 = 1424   44. 9)396 44   45. 396 × 6 = 2376

46. 466 × 8 = 3728   47. 6)564 94   48. 927 + 878 = 1805   49. 4)373 93 R1   50. 832 − 578 = 254

51. 394 + 934 = 1328   52. 8)440 55   53. 971 − 796 = 175   54. 3)261 87   55. 356 × 8 = 2848

Solve each story problem.

56. Nancy has 185 stamps. She places 8 on each page of her book. How many pages are full? How many more stamps are needed to fill the next page? 23 full pages; 7 more stamps needed

36. 378, 37. 53, 38. 2548, 39. 64, 40. 456,
21. 25 R2, 24. 45 R1, 27. 67 R2, 30. 79 R4, 33. 77 R6,
Reflected answers, Set 50: 1. 17, 5. 27, 9. 91, 13. 99, 17. 52,

Books to Explore

Adler, Irving. ***The Giant Golden Book of Mathematics.***
New York, Golden Press, 1960.
(Available from Whitman Golden Ltd., Cambridge, Ontario)

Have you ever wondered how a tree grows or why a volcano is shaped as it is or what makes a card trick work? This colorful book answers these and many other questions, through exploring the world of mathematics. You'll find all kinds of exciting ideas about numbers and what they mean in our daily lives. Here are just a few of the interesting topics:

How the Mayan Indians wrote 100	13
The puzzle of the King's reward	21
Bridges, planets and whispering galleries	54
Why a navigator needs a clock	62

Brindze, Ruth. ***The Story of Our Calendar.***
New York, Vanguard, 1949.
(Available from The Copp Clark Publishing Co., Toronto)

Men did not always have calendars to tell what day of the month it was. In fact, they did not always know about months. It took thousands of years to develop the calendar as we know it, and each version presented new problems. This book tells you:

The difference between sun years and moon years	8
Who decreed the first leap year	36
The year the calendar was set back 11 whole days	47

Carona, Philip. ***Things That Measure.***
Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1962.
(Available from Prentice-Hall of Canada Ltd., Scarborough, Ontario)

Many pictures and diagrams help the author tell you about different ways to measure things. He covers a wide range of subjects, including:

Using tobacco for money	15
Who invented the first watch	47
Weather instruments	50-55

Smith, George O. ***Mathematics: The Language of Science.***
New York, G. P. Putnam's Sons, 1961.
(Available from Longman Canada Ltd., Don Mills, Ontario)

Interesting stories show how mathematical language is a handy tool for scientists. You will learn how the first men kept track of their possessions, how Roman businessmen struggled with Roman numerals, how the Babylonians discovered place value, and how Copernicus had trouble convincing people that the sun was the centre of the universe.

Other stories you might enjoy reading in this book are:

The discovery of nothing	23
Mr. Newton's apple	47-49
Investigating a Mobius strip	63-64

Some shorter books to look for in your library are listed below.

Adler, Irving and Ruth. ***Numbers Old and New.***
New York, John Day, 1960.
(Longman Canada Ltd., Don Mills, Ont.)

This book explains the counting methods of Australian natives and Mayan Indians, and the fractions used by Egyptians and Greeks. For fun, read the chapters about lucky and unlucky numbers, number tricks, and numbers in nature.

Bendick, Jeanne. ***First Book of Time.***
New York, Watts, 1963.
(Grolier Limited, Toronto)

Excellent pictures help tell the story of time and how to measure it. All kinds of clocks are described—from sun dials and water clocks to the atomic clock and clocks in your body.

Bendick, Jeanne, and **Levin, Marcia O.** ***Take a Number.***
New York, Whittlesey House, 1961.
(McGraw-Hill Ryerson, Scarborough, Ontario)

Stories and puzzles explain different ways of counting with numbers. Some are very easy, like counting on your fingers; others are more complicated, like using a computer.

Charosh, Mannis. ***Straight Lines, Parallel Lines, Perpendicular Lines.***
New York, Thomas Y. Crowell, 1970.
(Fitzhenry & Whiteside Ltd., Don Mills, Ont.)

By using a piece of string and a checkerboard, you can explore the world of straight, parallel and perpendicular lines.

Clarke, Mollie. ***Beads. A Group of Children. Numbers. Dominoes. What is Missing? 20 Sticks. Houses. The Calendar. The Piggy Bank. A Box of Crayons. Sweets. Cakes and Candles. What Is Inside? Shapes. Buttons. A Dozen Eggs. Symmetrical Shapes.***
Newton, Massachusetts, Selective Educational Equipment, Inc.

There should be something here to interest you.

Friskey, Margaret. ***The Mystery of the Farmer's Three Fives.***
Chicago, Children's Press, 1963.
(Scholars Choice Limited, Stratford, Ont.)

A book about animals that compares groups in the barnyard. You'll enjoy the mystery.

Hine, Al. ***Money Round the World.***
New York, Harcourt, Brace & World, Inc., 1963.
(Longman Canada Ltd., Don Mills, Ont.)

All about trading and money, from stones to metal coins and paper money.

Hoban, Tana. ***Shapes and Things.***
New York, Macmillan Co., 1970.
(Collier-Macmillan Canada Ltd., Don Mills, Ont.)

The author shows simple, everyday objects in photograms—pictures made without a camera. This is a real treat for your eyes.

Hutchins, Pat. ***Clocks and More Clocks.***
New York, Macmillan, 1970.
(Collier-Macmillan Canada Ltd., Don Mills, Ont.)

Have you ever tried to check the time on three different clocks and found three different answers? Look through this picture book for fun with clocks.

Jacobs, Leland B. ***Delight in Number.***
New York, Holt, Rinehart and Winston, Inc., 1964.
(Holt, Rinehart and Winston of Canada Ltd., Toronto)

These happy poems about groups of objects use many number words.

Kettlekamp, Larry. ***Spirals.***
Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1964.
(Prentice-Hall of Canada Ltd., Scarborough, Ontario)

An enjoyable look at spirals—some are in nature, some man-made.

Linn, Charles. ***Estimation.***
New York, Thomas Y. Crowell, 1970.
(Fitzhenry & Whiteside, Don Mills, Ont.)

Interesting experiments and activities help you improve your estimating skills.

Massoglia, Elinor. ***Fun-Time Paper Folding.***
Chicago, Children's Press, 1959.
(Scholars Choice Limited, Stratford, Ont.)

You would be surprised how many different things you can make by folding a single piece of paper. This book gives directions, and there is no cutting or pasting.

Rhodes, Dorothy. ***How To Read a City Map.***
Chicago, Elk Grove Press, Inc., 1967.
(Griffin House, Toronto)

Through pictures you can learn to read a map. How do you locate rivers, railroads, airports, historical landmarks, hospitals, roads, and government buildings on a map?

Russell, Solveig P. ***One, Two, Three and Many: A First Look At Numbers.***
New York, Walck, 1970.
(Oxford University Press, Don Mills, Ont.)

This book tells you all about the history of numbers and counting.

Sitomer, Mindel and Harry. ***What Is Symmetry?***
New York, Thomas Y. Crowell, 1970.
(Fitzhenry & Whiteside Ltd., Don Mills, Ontario)

With the help of the alligators, you can locate symmetries in nature and in man-made objects.

Savastava, Jane. ***Weighing and Balancing.***
New York, Thomas Y. Crowell, 1970.
(Fitzhenry & Whiteside, Don Mills, Ont.)

This book shows you how to make a simple balance.

Whitney, David C. ***The Easy Book of Multiplication.***
New York, Franklin Watts, Inc., 1969.
(Grolier Limited, Toronto)

This book will help you understand the process of multiplication with examples to show the facts.

Glossary

addend Any one of a set of numbers to be added. In the equation $4 + 5 = 9$, the numbers 4 and 5 are addends.

addition An operation that combines a first number and a second number to give exactly one number. The two numbers are called addends, and the one number which is the result of combining the two numbers is called the sum of the addends.

angle Two rays from a single point.

approximation One number is an approximation of another number if the first number is suitably "close" (according to context) to the other number.

area The area of a closed figure or region is the measure of that region as compared to a given selected region called the unit, usually a square region in the case of area.

borrow A commonly used term for the regrouping process involved in certain types of subtraction.

Example:

$$\begin{array}{r} {\scriptstyle 3\ 13} \\ \cancel{4}\,\cancel{3} \\ -1\ 7 \\ \hline \end{array} \rightarrow \begin{array}{r} 3\ 0 \\ -1\ 0 \\ \hline 2\ 0 \end{array} \quad \begin{array}{r} 1\ 3 \\ -\ \ 7 \\ \hline 6 \end{array} \rightarrow \begin{array}{r} 4\ 3 \\ -1\ 7 \\ \hline 2\ 6 \end{array}$$

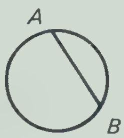
carry A commonly used term for the regrouping that is involved in addition.

Example:

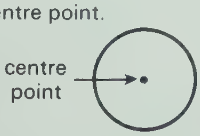
$$\begin{array}{r} 57 \\ +26 \\ \hline 83 \end{array} \longrightarrow \begin{array}{r} 50 + \ 7 \\ 20 + \ 6 \\ \hline 70 + 13 = 83 \end{array}$$

centimetre A unit of length. One centimetre is $\frac{1}{100}$ metre.

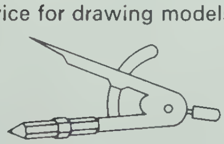
chord A line segment that has its endpoints on a given circle.



circle A set of all points in a plane which are a specified distance from a given point called the centre or centre point.



compass A device for drawing models of a circle.



co-ordinate Number pair used in graphing.

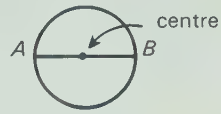
co-ordinate axes Two number lines intersecting at right angles at 0.

count To name numbers in regular succession.

cube A rectangular prism (box) such that all faces are squares.

diagonal A segment joining two nonadjacent vertices of a polygon. In the figure, the diagonal is segment *AB*.

diameter A chord that passes through the centre point of the circle.



difference The number resulting from the subtraction operation.

digits The basic Hindu-Arabic symbols used to write numerals. In the base-ten system, these are the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

division An operation related to multiplication as illustrated:

$$3 \times 4 = 12 \begin{array}{l} \nearrow 12 \div 3 = 4 \\ \searrow 12 \div 4 = 3 \end{array}$$

divisor In the problem $33 \div 7$, 7 is called the divisor.

Example:

$$\text{divisor} \rightarrow 7\overline{)33}^{\,4} \qquad \begin{array}{r} 4 \\ 7)\overline{33} \\ 28 \\ \hline 5 \end{array}$$

empty set A set that has no objects in it.

equality (equals; or =) A mathematical relation of being exactly the same.

equation A mathematical sentence involving the use of the equality symbol. Examples: $5 + 4 = 9$; $7 + \square = 8$; $n + 3 = 7$.

equivalent fractions Two fractions are equivalent when it can be shown that they each can be used to represent the same amount of a given object. Also, two fractions are equivalent if these two products are the same:

$$\frac{3}{4} \times \frac{6}{8} \qquad \begin{array}{l} 6 \longrightarrow 4 \times 6 \longrightarrow 24 \\ 8 \longrightarrow 3 \times 8 \longrightarrow 24 \end{array}$$

equivalent sets Two sets that may be placed in a one-to-one correspondence.

estimate To find an approximation for a given number. (Sometimes a sum, a product, etc.)

even numbers The whole-number multiples of 2 (0, 2, 4, 6, 8, 10, 12, . . .).

factor See multiplication. The equation $6 \times 7 = 42$ illustrates that both 6 and 7 are factors of 42.

fraction A symbol for a rational number, usually written $\frac{2}{3}, \frac{3}{4}, \frac{1}{2}$, and so on.

graph (1) A set of points associated with a given set of numbers or set of number pairs. (2) A picture used to illustrate a given collection of data. The data might be pictured in the form of a bar graph, a circle graph, a line graph, or a pictograph. (3) To draw the graph of.

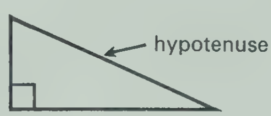
greater than (>) One of the two basic inequality relations. Examples: $8 > 5$, $28 > 25$, $80 > 50$.

grouping principle (associative principle) When adding (or multiplying) three numbers, you can change the grouping and the sum (or product) is the same.

Examples: $2 + (8 + 6) = (2 + 8) + 6$
$3 \times (4 \times 2) = (3 \times 4) \times 2$

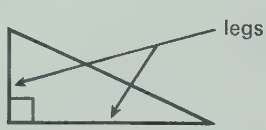
hexagon A six-sided polygon.

hypotenuse The side opposite the right angle in a right triangle.



inequality (<, ≠, >) In arithmetic, a relation indicating that two numbers are not the same.

legs of a right triangle The two sides of a right triangle other than the hypotenuse.



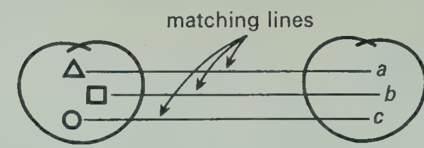
length (1) A number indicating the measure of one segment with respect to another segment, called the unit. (2) Sometimes used to denote one dimension (usually the greater) of a rectangle.

less than (<) One of the two basic inequality relations. Examples: $5 < 8$, $25 < 28$, $50 < 80$.

line A line is a set of points that "goes on and on" in both directions. There is only one line through any two points.

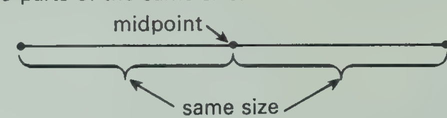
line segment See segment.

matching lines Lines used to indicate the correspondence between the objects in two sets.



measure (1) A number indicating the relation between a given object and a suitable unit. (2) The process of finding the number described in (1).

midpoint A point that divides a line segment into two parts of the same size.



minus (—) Used to indicate the subtraction operation, as in $7 - 3 = 4$ (read, "7 minus 3 equals 4").

multiple A first number is a multiple of a second number if there is a whole number that multiplies by the second number to give the first number. Example: 24 is a multiple of 6 since $4 \times 6 = 24$.

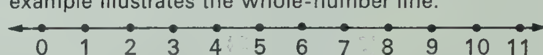
multiplication An operation that combines a first number and a second number to give exactly one number. The two numbers are called factors, and the one number which is a result of combining the two numbers is called the product of the two numbers.

multiplication-addition principle (distributive principle) This principle is sometimes described in terms of "breaking apart" a number before multiplying. Example: $6 \times (20 + 4) = (6 \times 20) + (6 \times 4)$

negative number If a number adds to a whole number to give 0, it is a negative number.

For example: $5 + {}^{-}5 = 0$
$19 + {}^{-}19 = 0$

number line A line on which specified points are given number labels or names. The following example illustrates the whole-number line.



number pair Any pair of numbers. In this book, usually a pair of whole numbers.

numeral A symbol for a number.

odd number Any whole number that is not even.

one principle (for multiplication) Any number multiplied by 1 is that same number.

one-to-one correspondence A one-to-one correspondence exists between two sets when the elements of one can be matched with the elements of the other in such a way that each element of the first set is matched with exactly one element of the second set and each element of the second set is matched with exactly one element of the first set.

order principle (commutative principle) When adding (or multiplying) two numbers, the order of the addends (or factors) does not affect the sum (or product). Examples: $4 + 5 = 5 + 4$, $2 \times 3 = 3 \times 2$.

parallel lines Two lines which lie in the same plane and do not intersect.

parallelogram A quadrilateral with its opposite sides parallel.

parentheses A pair of curved symbols, (), used to indicate grouping or order of performing operations. Examples:

$$(5 \times 4) - 2 = 18; \qquad 5 \times (4 - 2) = 10.$$

pentagon A five-sided polygon.

place value A system used for writing numerals for numbers, using only a definite number of symbols or digits. In the numeral 3257 the 5 stands for 50; in the numeral 36 289 the 6 stands for 6000.

plus (+) Used to indicate the addition operation, as in $4 + 3 = 7$ (read, "4 plus 3 equals 7").

polygon A closed geometric figure made up of line segments.

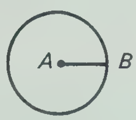
prime number A number greater than 1 whose only factors are itself and 1.

product The result of the multiplication operation. In $6 \times 7 = 42$, 42 is the product of 6 and 7.

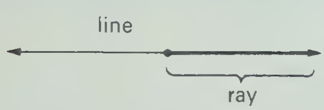
quadrilateral A four-sided polygon.

quotient The number (other than the remainder) that is the result of the division operation. It may be thought of as a factor in a multiplication equation.

radius (1) Any segment from the centre point to a point on the circle. (2) The distance from the centre point to any point on the circle.



ray The heavy part of the line shows a ray.



rectangle A quadrilateral that has four right angles.

regrouping A method of handling place value symbols in adding or subtracting numbers.

remainder

Example:
$$\begin{array}{r} 6 \\ 7\overline{)47} \\ \underline{42} \\ 5 \end{array} \longleftarrow \text{remainder}$$

repeated addition Finding the sum of a set of numbers, each of which is the same.

Example: $5 + 5 + 5 + 5$

repeated subtraction Starting with a number and repeatedly subtracting the same given number from each difference that is obtained.

rhombus A parallelogram with 4 sides of the same size.

right angle An angle that has the measure of 90 degrees.

A right angle

right triangle A triangle that has one right angle.

Roman numerals Numerals used by the Romans. Used primarily to record numbers rather than for computing. Examples: IV, IX, XIV.

segment Two points on a line and all the points on that line that are between the two points.

sequence A cllection or set of numbers given in a specific order. Such numbers are commonly given according to some rule or pattern.

set A group or collection of objects.

skip count To count by multiples of a given number. Example: Counting by fives – 0, 5, 10, 15, 20, • • •.

solution The number or numbers which result from solving an equation or a given problem.

solve To find the number or numbers which, when substituted for the variable or placeholder, make the given equation true.

square A quadrilateral that has four right angles and four sides that are the same length.

subtraction An operation related to addition as illustrated:

$$7 + 8 = 15 \begin{array}{l} \nearrow 15 - 8 = 7 \\ \searrow 15 - 7 = 8 \end{array}$$

sum A result obtained by adding any set of numbers is referred to as the sum of the numbers.

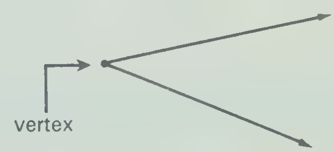
symmetric figure A figure that can be folded in half so the two halves match.

times (×) Used to indicate the multiplication operation, as in $3 \times 4 = 12$ (read, "3 times 4 equals 12").

triangle A three-sided polygon.

unit An amount or quantity adopted as a standard of measurement.

vertex The point that the two rays of an angle have in common.



volume The measure, obtained using an appropriate unit (usually a cube), of the interior region of a space figure.

whole number Any number in the set. {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14,• • •}.

zero principle (for addition) Any number added to zero is that same number.

Tables of Measures

LENGTH	
10 millimetres (mm) = 1 centimetre (cm)	1000 millimetres = 1 metre
10 centimetres = 1 decimetre (dm)	100 centimetres = 1 metre
10 decimetres = 1 metre (m)	10 decimetres = 1 metre
1000 metres = 1 kilometre (km)	1/1000 kilometres = 1 metre

TIME	
60 seconds (s) = 1 minute (min)	52 weeks = 1 year (yr)
60 minutes = 1 hour (h)	12 months (mo) = 1 year
24 hours = 1 day	365 days = 1 year
7 days = 1 week (wk)	366 days = 1 leap year

CAPACITY
10 millilitres (ml) = 1 centilitre (cl)
10 centilitres = 1 decilitre (dl)
10 decilitres = 1 litre (*l*)
1000 litres = 1 kilolitre (kl)

WEIGHT
1000 grams (g) = 1 kilogram (kg)
1000 kilograms = 1 tonne

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A Text for Teachers

Investigating Mathematics Learning

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INVESTIGATING MATHEMATICS LEARNING

I. Some Thoughts About Learning

Almost everyone has some observations on teaching and learning. A recent quote that has been making the rounds is: "If we tried to teach children to speak, they would never learn." However, in *The Process of Education* (Harvard University Press, 1960), Jerome Bruner observes, "Any subject can be taught effectively in some intellectually honest form to any child at any stage of development." But Linus (of *Peanuts* cartoon fame), considerably less optimistic, laments: "How can I learn 'New Math' with an 'Old Math' mind?"

In a more critical vein, John Holt in *How Children Fail* (Pitman Publishing Co., 1964) asserts: "In our classes, we begin with words, carry on with words, and often fail to get beyond words." He also says, "All too often the mathematics classroom becomes a temple of worship for the right answers, and the way to get ahead is to lay plenty of them on the altar." We know, of course, that many teachers for many years have been doing an excellent job helping elementary school children learn mathematics. Yet, it is worthwhile for us to reevaluate our approaches and, if possible, find even better ways to create situations where children learn more effectively.

The implications of the research of Piaget and others in how children learn mathematics and the observations of countless classroom teachers concerning the directions we should take are well summarized by a familiar Chinese proverb:

I hear and I forget.
I see and I remember.
I do and I understand.

The message of this proverb is that hearing and seeing are not enough: to learn with understanding, the child should experience *active involvement* with mathematical ideas. In order for the child to become actively involved, it has been found that the use of *physical materials* which contain the seeds of the mathematical ideas are valuable and often necessary. Coupled with the idea of active involvement with physical materials is the idea that teachers should encourage *student responsibility* and create conditions in which the student is not always encouraged to rely solely on the teacher but rather to take initiative for figuring out some things for himself.

Z. P. Dienes summarized a multitude of suggestions from researchers and teachers when he said: "It is suggested that we shift the emphasis from teaching to learning, from our experience to the children's, in fact, from our world to their world."

Teachers vary considerably in their views of how best to help children become actively involved with mathematics. While one teacher desires to convert his classroom immediately into a mathematics laboratory, another teacher may prefer a very modest beginning with a limited amount of active student involvement with physical materials inserted into his usual classroom approach. In this text we suggest a number of approaches for modest beginnings and indicate ways in which these approaches might be expanded to provide for a total laboratory approach and a more extensive individualized program.

To introduce one possible approach, let us simulate a teaching strategy by outlining one way to organize a specific lesson. Thus, suppose a teacher wanted to devise a lesson which would help children understand the idea of congruent segments in geometry. First the teacher provides each child with a geoboard and a sheet containing several 5-by-5 arrays of dots. Then he reviews, very briefly, the idea of a segment and the endpoints of a segment. Next, after helping the children see that they can use a rubber-band around two nails to represent a segment on the geoboard, he passes out the investigation suggested in Figure 1.

INVESTIGATION

How many different segments with endpoints just as far apart as these can you find on your geoboard? Record your results on dot paper.

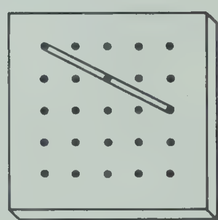


Figure 1

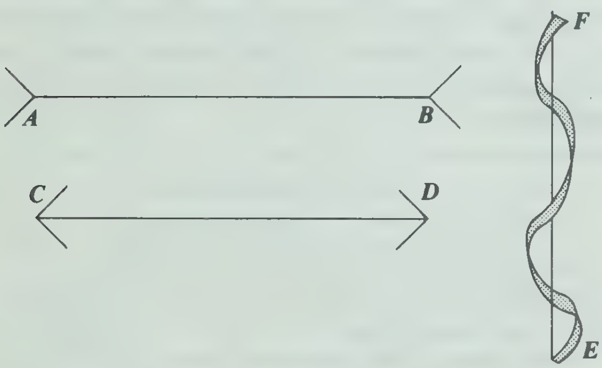
The teacher may choose to have chairs or desks rearranged so that children can communicate with each other as they become involved in the investigation. The teacher will check to be sure that everyone understands the investigation question; then he should encourage the children to find their own way to answer the question and record their findings. (To gain a fuller appreciation of an investigation situation, play the role of the child and complete the investigation yourself.)

Brief discussions among children or between teacher and children may occur during investigations, but the main discussion is most effective after the investigation has been completed. At this time, the teacher might ask such questions as: "How many different

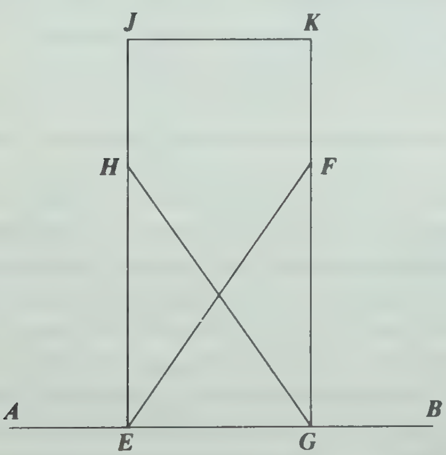
segments did you find?" "How can you be sure that you have found all such segments?" "How could you convince someone that each of your segments has endpoints just as far apart as all the others?" Such questions could then be followed up with a definition of congruent segments: When the endpoints of one segment are just as far apart as the endpoints of another segment, we say the segments are *congruent*. Then ask, "Can you think of some other ways to tell when two segments are congruent?" This question might lead into a discussion of how tracing paper, compasses, or marks on the edge of a piece of paper can be used to determine whether or not two segments are congruent.

After the children have discussed the ideas, the teacher may provide them with some problems which *utilize* these ideas. The child would probably be encouraged to use the ideas for testing congruence of segments that were developed in the discussion. The following are examples of possible exercises.

1. Find 2 segments below that are congruent to each other.



2. Name each pair of congruent segments in this picture.



One way to individualize a lesson is through an *extension* of the exercises. Extending the exercises can provide for remediation, reinforcement, or enrichment. As an extension to individualize this lesson, the teacher might give certain students the follow-up investigation below. (For a fuller appreciation of this lesson, complete the exercises and the investigation yourself.)

INVESTIGATION

Segment AB is not congruent to segment CD.
How many different segments (no two congruent) can you find on your geoboard? Record your results on dot paper.

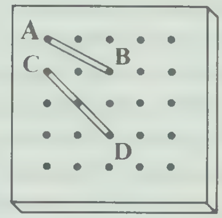


Figure 2

This abbreviated "lesson" provides a preview of one possible technique for encouraging children to become actively involved with physical materials in situations where they take more responsibility in the learning of mathematical ideas.

In the next section of this text, the parts of lessons such as the one described above will be analyzed and discussed. An outline for planning such lessons will be given, and various suggestions for carrying out each part of such a lesson will be proposed.

Since the investigation phase of the lesson provides the encouragement for active involvement by the child and since the kind of investigation used depends upon the type of learning involved, Section III in this text will focus on specific types of learning in elementary school mathematics. For example, the "lesson" described above helped children learn the *concept* of congruent segments; other lessons might be concerned with developing a *skill,* forming a *generalization,* learning a *fact,* or developing an *attitude*. Each type of learning will be analyzed and related to activity-oriented lessons that provide modest beginnings toward an active approach to mathematics learning.

Edith Biggs and James MacLean, in their book *Freedom to Learn* (Addison-Wesley, 1969), state: "A few schools scattered throughout the world are responding with some speed to a message which has been repeated with increasing urgency for some three hundred years. It is a simple message: Schools should be organized, not for teachers to teach, but for children to learn." In the same book, there appears an extensive list of "homemade" materials and devices that can be easily acquired for use in the mathematics classroom. Many materials, from newsprint and drinking straws, to string, popsicle sticks, beans, and homemade geoboards, can be made available to children at minimal cost. Rather than dismiss the possibility of actively involving children with materials in the classroom because no funds are available, a teacher should study this list carefully; he may be amazed by how much can be done with minimum expense.

Teachers sometimes feel that to involve children with physical materials and allow them to communicate with other children in the classroom is to invite

chaos. On the contrary, it has been found that, when children really become involved in using materials to investigate a situation, there may be a bit more low-keyed noise about the room but the usual discipline problems are almost nonexistent. It is helpful if there are tables available in the classroom so that children can work in small groups. If tables are not available, desks could be moved to assist in small-group work. On occasion, an investigation might call for children to leave their desks and to engage in other activity in the room. A simple set of "ground rules" should suffice to make the situation quite manageable.

It is interesting to consider the number of elementary school teachers who prefer to say that they are "helping children learn mathematics" rather than that they are "teaching mathematics." What one says, of course, does not always describe accurately what one does. It does seem important, however, in the light of recent studies and observations about how children learn mathematics, to focus on the child and try to create an environment in which the child has a greater opportunity to make decisions and to become really interested in his study. It is hoped that the following sections of this teachers' text will provide some ideas which may help you improve your ability to "help children learn mathematics."

EXERCISE SET 1

1. What was your reaction to the investigations in this section? **A** Did you become involved in the activity? **B** Were you interested? **C** Did you watch the clock? **D** Did you talk to anyone else while completing the investigation? (If so, was it helpful?) **E** Did the investigation situation help you better understand the idea involved? **F** What other feelings did you have?
2. Which quotation in this section seemed most significant to you? Why?
3. **A** Do you think most teachers teach the same way they were taught as elementary school children? **B** What do they do differently? **C** What are some ways you think our teaching of elementary school mathematics might be improved?
4. Look through the *Investigating School Mathematics* text at your grade level. How do the comments in this first section of the text relate to the approach taken in the child's text?

II. A Plan for a Learning Experience

First consider the practical matter of how the teacher proceeds in the daily task of helping children learn mathematics. A structured outline (inherently flexible) around which daily learning experiences may be planned can be a valuable organizational aid for the teacher and can give him a fresh insight into the role of new approaches to instruction.

Here is the outline that was used in planning the "lesson" in Section I. It has proven to be quite useful, especially for those teachers who have desired to make a beginning toward providing children more opportunities for active involvement with mathematical materials and ideas.

Preparation and Investigation
Discussion
Utilization and Extension

Since this outline offers a variety of possibilities for a teacher to reevaluate his approach to classroom instruction, the following sections provide an examination of its individual elements.

Preparation and Investigation

The investigation phase (often called simply "the investigation") is central to the learning experience. In this phase, the children are encouraged to become actively involved, individually or in groups, in the investigation of a situation that contains the seed for the central idea of the lesson. The investigation should be the "main event" in terms of pupil activity and involvement. The teacher should think of the investigation as a child-centred activity. Completion of the investigation in Figure 3 will help clarify the ideas of investigation.

INVESTIGATION

Can you find an investigation in a text from the *Investigating School Mathematics* series that

(a) uses centimetre strips?
(b) utilizes paper folding?
(c) has a question like "How many can you find?"
(d) involves the geoboard?
(e) encourages children to use graph paper?
(f) asks the children to record their findings?
(g) directs the children to use reference material?

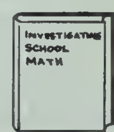


Figure 3

Homemade or commercially produced manipulative materials often provide the stimulus for the situation to be investigated. At other times, even more simple teacher-devised activities provide this stimulus. For example, the suggested investigation in Figure 4 might have been made by a teacher to initiate an investigation in a lesson designed to help children form the generalization, "You can rearrange three addends any way you please, and the sum will always be the same."

Sometimes by asking appropriate questions about a situation of interest to the children the teacher may involve them in an exploration of a central idea to be developed.

Regardless of how an investigation is initiated, a teacher should remember that the investigation situation is specifically designed to encourage children to

INVESTIGATION

Make three slips of paper like these.

2 3 4

Then turn them over and mix them up. Pick any two slips and add the numbers on them. Then add the number on the other slip.

If you do this five times, will you get the same number each time?

Figure 4

take responsibility for the thinking and exploring. Too much "teacher help" can hinder the achievement of these aims.

In an investigation, it is not uncommon to see children deeply involved and assuming full responsibility for completing the task at hand. The teacher, who plays a key role in initiating the investigation, may appear not to be needed as he moves about the room. Occasionally, a brief discussion between teacher and child occurs, but most of the larger-group discussion occurs after the investigation. The investigation itself should embody an attitude toward learning that could be easily stifled by too many words from the teacher. Perhaps, in an investigation, a new adage should replace the old: the teacher, rather than the children, should be "seen but not heard."

The investigation is predicated on the assumption that the best way to minimize the need for words is to substitute an appropriate question for a wordy explanation at a time when a child's interest in a mathematical situation is beginning to ripen.

For example, suppose a certain group of children understand the concept of a triangle and are ready to consider characteristics that distinguish one type of triangle from another. An appropriate question to initiate an investigation might be the one shown in Figure 5. (Try this investigation yourself.)

INVESTIGATION

How many triangles with different shapes can you make that have no nails inside?

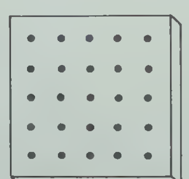


Figure 5

This question is both activity-stimulating and activity-sustaining. It helps involve the child in a search which he will continue with little further motivation. Notice also that the answer is not as important as the experiences the child will have as he responds to the question. Further, the question is sufficiently clear that the child immediately becomes involved with the challenge of the investigation rather than dissipating energy in efforts to understand the question. Another characteristic of this type of question is that it provides for individual differences: when the child is asked "How many can you find?" he can feel successful even if he finds only one. Of course, not all investigations can or should be introduced by this type of question, but it is important for the teacher to recognize that as the children respond to these questions, they will achieve in widely differing ways. In an investigation, the teacher should give recognition for all levels of achievement.

It should be noted that the amounts of time used for the investigations may vary considerably. One investigation may involve a very brief "happening" which sparks a simple idea within the child. Another investigation may utilize a large part of the period of time available for the mathematics lesson and might involve the child in a sustained exploration of a game or a set of manipulative materials.

To set the stage for an investigation of any duration, a preliminary *preparation* phase is sometimes needed. This phase provides for a brief review of key ideas needed for the investigation and for any motivational activity helpful in introducing it. This phase should be kept fairly short and care should be taken to see that this preliminary work does not preempt the central idea or activities involved in the investigation or the work that follows it.

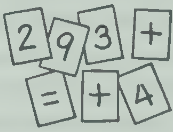
In summary, the investigation phase is the child-involvement phase. It often requires materials, and is usually motivated by a carefully selected question which focusses the student's attention on the central idea of the lesson. Proper consideration of this phase in your lesson planning can be highly rewarding.

EXERCISE SET 2

1. Find some investigations in the *Investigating School Mathematics* text that contain features not mentioned in Figure 4.
2. Choose a lesson from an *Investigating School Mathematics* text and write a description of the role you think the teacher would play in using the investigation phase of the lesson.
3. Choose an idea to be taught and prepare an investigation situation which has the potential of involving the child in working with this idea.
4. Two investigations follow. Give the central idea of a possible lesson based on the use of each one.

A

INVESTIGATION

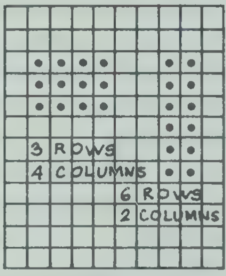


Cut out 7 slips of paper. Put one of these numerals or one of these signs on each one.

How many different equations with 3 addends can you write with your slips of paper? Record each equation you find.

B

INVESTIGATION



The graph paper shows two different ways to arrange 12 counters in a rectangular array.

How many different ways can you arrange 24 counters in a rectangular array? Record your findings by drawing pictures on graph paper.

5. Here is an interesting investigation you may like to try. Through it, you will be introduced to a basic idea of mathematics. Be sure to record your findings and be ready to discuss them further in the next section.

Copy and continue this array of numbers until you reach 52.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16
17	18	19	20
	. . .		

Then circle all the prime numbers in the array.

Notice that the numbers in the right-hand column can be written as $4 \times$ (a whole number).
For example: $8 = 4 \times 2$, $12 = 4 \times 3$, and $20 = 4 \times 5$.

Can you make a statement about prime numbers that is suggested by this activity?

Discussion

Following the investigation, a *discussion* phase allows teacher and children to further share ideas in a discussion of what they found in the investigation. The teacher has an excellent opportunity in this phase to ask questions and to supply examples to help children further develop their understanding of the ideas germinated by the investigation.

Another valuable aspect of the discussion phase is that it provides additional opportunities for children to communicate with other children as a means of shaping their ideas. In a good discussion, it is not unusual for children, having reached an impasse in *their* thinking and communication about an idea, to ask the teacher if he can clarify the point. This is when the teacher as a resource person emerges. At other times, when ideas new to the teacher arise, the teacher participates in the discussion, not as a resource person, but as a fellow-learner. Both of these situations can contribute to a comfortable, meaningful discussion, but its potential benefits may never be realized if the teacher monopolizes the discussion to the extent that the children are denied the opportunity to draw their own inferences and make their own decisions. Since it is the child who is involved in the investigation, the child's ideas about the findings should be of primary importance, and the child should supply as many details leading to the understanding of the idea as possible.

By listening to the child and asking appropriate questions, the teacher can build on the child's initial ideas and help him develop a deeper understanding in preparation for further work. This understanding cannot be developed, however, by always asking questions which require simply that a child remember a fact correctly or perform a practical skill. Nor is it sufficient to ask questions to which a child can respond with a guess of "Yes" or "No." Rather, the questions that should be asked often are those that require a deeper thinking on the part of the child.

For examples of the more effective type of question, consider again the investigation described in Figure 7. This investigation, designed to set the stage for the development of the concepts of isosceles triangle, right triangle, scalene triangle, and equilateral triangle, might be followed by a discussion in which the teacher would ask questions such as the following:

1. Can you choose a pair of triangles you found and describe ways in which one is different from another?
2. In what ways are some triangles you found alike? (Note: Children may respond, "Some have a square corner," "Some have two sides the same," "Some have no sides the same," "Some are large," and so on.)
3. How would you describe a triangle that is different from any of the triangles you formed on the geoboard?

As the teacher asks thought-provoking questions and listens to the children's responses, he will be able to find ways to clarify the basic idea of the lesson and to prepare the children for the independent work which is to follow. It is in the latter stages of the discussion that the teacher may want to explain more carefully, show additional examples, and, in general, lead the child to a deeper mastery of the ideas involved.

EXERCISE SET 3

1. Can you find a question in the "Discussing the Ideas" section of an *Investigating School Mathematics* text which **A** asks the children to recall something previously learned? **B** asks the children to restate or explain an idea in their own words? **C** asks the children to interpret a diagram, picture, or explanation? **D** asks the children to analyze a given situation? **E** asks the children to evaluate a given situation?
2. What do you think about the effectiveness of the investigation described in Figure 5 as a means of meeting the goals indicated?
3. Write five questions you might ask while conducting a discussion in a mathematics lesson of your choice.
4. The following discussion exercises refer to the investigation presented in exercise 5 of Exercise Set 2. **A** What statement did you make about prime numbers? **B** Can you find a prime number that does not appear in the first or the third column? Can you find more than one? **C** $4 \times n$ is an algebraic expression. What algebraic expression can you devise to describe the prime numbers in the third column? in the first column? **D** Of the prime numbers less than 100, which type of prime occurs more often? **E** 113 is a prime number. Which type of prime is it?
5. Investigation questions may be open ("In how many ways can you measure a ball?") or closed ("Can you find the circumference and diameter of this ball?"). Discuss the merits of open and closed questions.

Utilization and Extension

The *utilization* phase allows each child to work on his own and to use the ideas developed in the investigation and discussion phases.

Often children need to practice recalling facts that have been developed or introduced in the lesson. Appropriate exercises requiring written answers are often valuable in providing this practice.

In another lesson, a child may have learned an algorithm or a skill. In order to refine this skill, he may need considerable practice using it. Appropriately designed written exercises which children complete independently can be quite helpful in polishing these skills.

In another lesson, a new idea may have been presented. In order to become more familiar with this idea and to understand how it relates to other ideas, the child may need thought-provoking problems which involve the idea. The *utilization* phase presents an opportunity for the child to solve problems which involve ideas that have been presented previously or to look at an idea that is different but closely related to one he has already encountered.

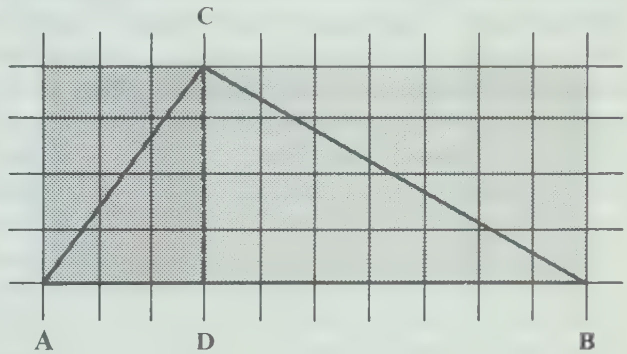
Creative activities for independent work can do much to extend the learnings developed in the investigation and discussion phases. The utilization exercises in examples A and B below are sequenced in such a way that the child has an opportunity to discover a new procedure or new ideas as a result of his work.

EXAMPLE A

Find the differences.

75	75	75	75	75	75	75
−32	−33	−34	−35	−36	−37	−38
43	42	41				

EXAMPLE B

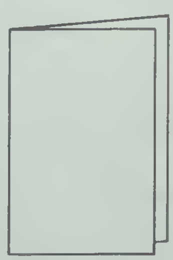


What is the area of the region shaded dark gray?
What is the area of the region shaded light gray?
What is the area of the two regions together?
What is the area of triangle *ADC*?
What is the area of triangle *BDC*?
What is the area of triangle *ABC*?
The area of triangle *ABC* is what part of the entire shaded region?

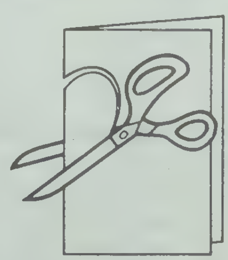
The teacher should appreciate the great potential value of discovery-sequenced exercises such as these, and should look for opportunities to make his own exercise sets using such sequences. Another set of utilization exercises might encourage the child to independently delve more deeply into the idea initiated in the investigation. Further activities with mathematical materials often provide opportunities for the child to use and extend the idea of the investigation. Example C provides an opportunity for the child to reinforce his concept of symmetry.

EXAMPLE C

Do this to make symmetrical figures.



Fold a piece of paper.



Make a cut that starts and ends on the fold.

Unfold the piece you cut out. It will be symmetrical.

Make cuts so that the unfolded shape will be:

A a rectangle	**D** a square	**G** a rocket
B a leaf	**E** a house	**H** a hexagon
C a triangle	**F** a pumpkin	**I** a butterfly

Regarding the utilization phase, it should be noted that on occasion it may be more valuable to have pairs or small groups of children work the exercises together.

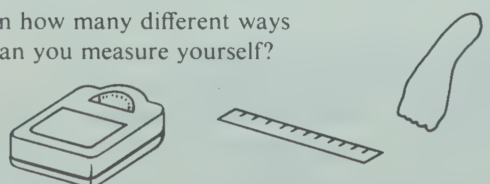
Finally, the *extension* phase provides for use of remedial, maintenance, or enrichment activities to further individualize the learning opportunities. This individualization offers numerous advantages. The slower children can avoid the frustration of having to proceed to new ideas before the previously presented ideas are understood, and the more capable children are spared the tedium of completing long lists of drill problems involving ideas they already understand.

The teacher might look for creative ways to meet individual differences in the ability to learn mathematics. For example, the slower child might profit from additional drill on certain facts and skills. Drill tapes or audio cassettes made by the teacher might provide a novel way to present the necessary practice. Duplicator masters and commercial workbooks are also available to provide extra work for those who need it. For other situations, an appropriate programmed instructional unit might serve the needs of the slower child. Single-concept film loops, which the child can play again and again, often are useful in helping him grasp an important concept. Appropriately conceived tutorial situations, in which classmates who understand the ideas work with children who do not, can be quite effective. Simple investigations utilizing physical objects which clarify more abstract ideas can also provide remedial work for certain children.

The teacher must also be concerned with those children who understand the basic ideas of the lesson and who can quickly work all the utilization exercises provided. These children can often become quite interested in activity cards which contain "open-ended" questions, such as the card shown below. (You are encouraged to try the suggested activity yourself.)

ACTIVITY CARD 10

In how many different ways can you measure yourself?



Make as many different measurements of **you** as you can and make a chart to show the information. Here are just a few suggestions:

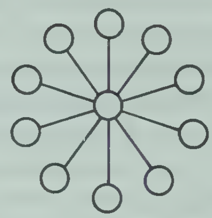
Pulse	Length of step
Height	Number of calories used
Weight	Area of bottom of foot
Arm span	Distance you can jump

Activities such as these give the child an opportunity to make his own decisions about which ideas he uses from the lesson and how he uses them.

Puzzles or riddles can also provide a useful extension of ideas for your children. Consider, for example, those shown in Figure 6.

Think

Draw a figure like this one on your paper. Place the numbers 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 in the circles so that the sum along any line is 21.



Think

I can be found halfway between Twenty-seven and seventeen.

WHO AM I?

27 ? 17

Figure 6

Conceptually fertile games can also provide valuable experiences to supplement the basic lesson. For example, the game "Sleuth" (3M Company) is fun for children and gives them valuable experience in classification and drawing logical inferences.

The methods suggested for extending the ideas for slower children are often suitable for use in certain situations with more capable children. Similarly, the more exciting modes of extension suggested for faster children can often be quite stimulating and valuable if used appropriately for the slower children.

It is to be hoped that the teacher will share a sense of excitement in providing extra stimulation to broaden the mathematical perspective of the children. Perhaps, he will also see that much of the extension activity can truly be fun for children while at the same time inspiring new interest and involvement in mathematical ideas. In using this suggested lesson outline, if the teacher chooses to maximize the investigation phase while deemphasizing the others, it might justly be said he is using the laboratory approach. On the other hand, should he maximize the discussion phase, he may find increased options for a guided discovery approach to mathematics learning. Also, it is possible that maximization of the utilization phase accompanied by appropriate student materials would allow the teacher to embark on a course of individually prescribed instruction.

EXERCISE SET 4

1. Find an example of an exercise set in which a learning sequence occurs in an *Investigating School Mathematics* text.

2. Choose a mathematics topic and write a set of exercises which might lead the student to discovery of a central idea.
3. Can you find a lesson in an *Investigating School Mathematics* text in which the "Using the Ideas" section provides for varying degrees of student ability.
4. Choose a learning experience appropriate for your children and list some possible specific activities for use in the extension phase of this learning experience.
5. Describe your views concerning the role of drill for slow, average, and bright children.
6. Select and play a game that could be used to extend a lesson with children.
7. In Exercise Set 2, you investigated an idea of mathematics. In Exercise Set 3, you had an opportunity to discuss this idea. The exercises below enable you to use the idea you learned, and suggest an extension of the idea.

Complete each exercise.

A List five prime numbers of the "$4n + 1$" type that are greater than 50.

B List five prime numbers of the "$4n - 1$" type that are greater than 50.

C 997 is the largest prime number less than 1000. Is it a "$4n + 1$" or a "$4n - 1$" prime?

D Suppose you used a continuation of the array of numbers shown below and circled all the prime numbers. What does this suggest about another way to classify the primes?

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
...					

III. A Focus on Specific Types of Learning

In considering the more specific aspects of mathematics learning it is helpful to categorize the general types of things children learn. A simplified categorization is given below.

Concepts
Skills
Generalizations
Facts
Attitudes

It is important to recognize that each of these types of learnings has unique characteristics. Because of this, the approaches and children's activities chosen to promote these learnings may often be quite different. In the sections that follow, we will consider each of these types of learning and suggest possible approaches and activities.

Concepts

Suppose that a child is having difficulty and comes to the teacher for assistance. When the teacher asks what the difficulty is, the child points to the multiplication 9×6 and says, "I can't do this because we haven't had it yet." This reflects a common attitude among children who have been in school for a few years. Somehow they learn to feel that they are incapable of figuring out anything new in mathematics. Literally, they can do nothing that they "haven't had yet."

If this child had confidence in his ability to "figure something out" and had a clear understanding of the *concept* of multiplication, he could have found the product by perhaps adding sixes, using sets, or making jumps on the number line. Another child who knew no division "facts" but who had a clear concept of division (as illustrated below) could use his knowledge of multiplication to find any of the basic quotients desired.

$$\overset{\textbf{P}}{72} \div \overset{\textbf{F}}{8} = \overset{\textbf{F}}{n} \quad \leftarrow \quad \text{You find this quotient,}$$

$$\text{when you find this factor.} \quad \rightarrow \quad \overset{\textbf{F}}{n} \times \overset{\textbf{F}}{8} = \overset{\textbf{P}}{72}$$

A *concept*, then, may be thought of as an idea which, when properly understood, will help the child to solve problems he "hasn't had yet," to figure something out for himself. As another example, consider the concept of prime number. Once a child understands that a prime number is a whole number with exactly two factors, he has the power, providing he understands how to find the factors of a number, to seek out and list those numbers that are prime. Of course, the task of deciding whether or not a given number is prime may be quite laborious, but understanding the concept does give the child the power to succeed.

To look more carefully at what concepts are and how they are taught, consider a model in which concept learning is relatively easy, namely, that of a set of attribute pieces. Suppose there are pieces of four different shapes (triangles, squares, circles, and rectangles), of three different colors (red, blue, and yellow), and of two different sizes (large and small), as pictured in Figure 7. (In the figures the colors red, blue, and yellow are denoted by the initials R, B, and Y.)

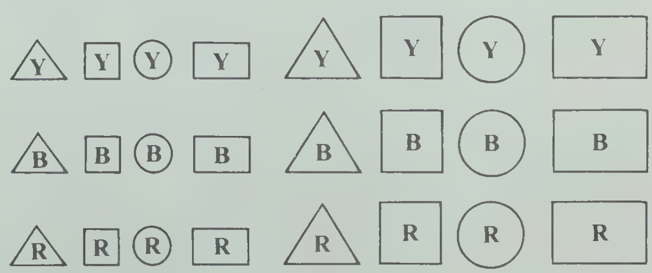


Figure 7

Now consider the Concept Card in Figure 8.

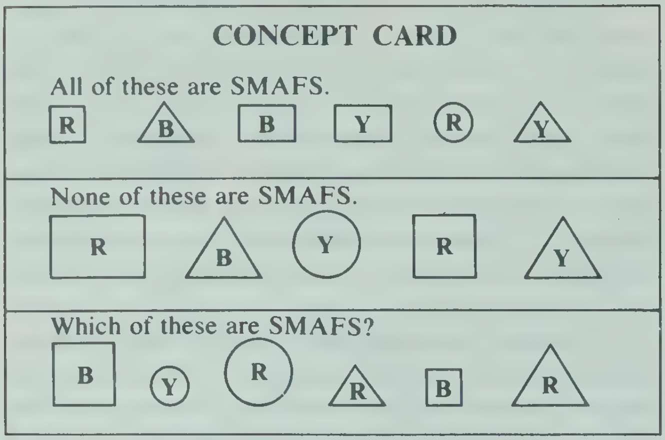


Figure 8

If you study the preceding Concept Card carefully, you will develop the simple concept of a SMAF. Notice that the key means used to teach this concept is by examples, along with *non-examples*. Both examples and non-examples play important roles in teaching many concepts in mathematics. The concept of a triangle may be taught to young children by using the Concept Card in Figure 9.

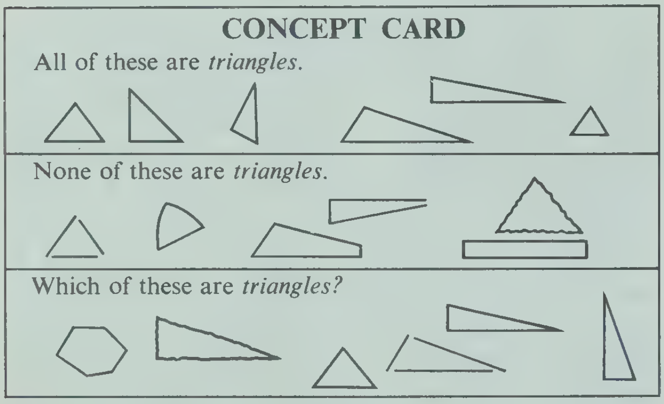


Figure 9

Clearly, the child would need further experiences in order to develop fully the concept of triangle, but the beginnings are embodied in the Concept Card shown in Figure 9.

One of the important ideas to remember when considering concepts is that concepts, unlike some other things that children learn, are developed over a period of time. Simple concepts may be developed very quickly, but other, more complicated concepts must be germinated when the child is very young and broadened through a spiralling return to the concept at various stages throughout the child's development. Many concepts are not fully developed until the child becomes an adult and encounters the idea in a variety of situations. For example, the concept of a fraction or fractional number may be introduced in grade 1 or grade 2, but a full understanding of this concept may not come until many years later. The child may acquire only an embryonic idea of a concept the first time it is presented, so it is important for the teacher to recognize the true nature of concepts and be willing to return often to the idea and carefully nurture its growth within the child. If he does not expect complete mastery after the initial presentation, he will spare himself considerable frustration when he recognizes later that the child needs further development of the basic idea.

Another key feature of concept learning suggested by the experiments of Piaget and supported and extended by the theories developed by Z. P. Dienes concerns the role of physical manipulative materials in young children's concept learning. In general, the implication of these authors' works is that it is through child involvement with physical environment that a firm basis for the development of more abstract concepts is laid. In fact, it is suggested that concept learning is facilitated by exposing children to as many different physical situations which embody the concept as possible.

It should be recognized that there are different levels of concept development and different types of concepts within these levels. For example, in the very earliest stages of mathematical learning, most concept learning involves the *concept of physical objects* such as balls, blocks, and circular or triangular objects. Very soon, the concept of certain *relations between objects* is developed: above, below, taller, shorter, larger, wider, longer, behind, and so on. A subsequent stage involves the concept of a *set of objects* such as a set of golf clubs, a set of dishes, a box of crayons, a set of blocks, a collection of stamps, or the children in a classroom. A slightly higher level of concept learning involves *relations between sets of objects:* equivalent, equal, has more than, has less than, and so on. It is at this stage that the important concept of *number* arises. For, in a sense, the concept of number involves a consideration of a set of equivalent sets. At a higher level of abstraction, the concept of certain *relations between numbers* (is less than, is greater than, is equal to, and so on) is developed. Ascending the ladder of abstraction, another level of development might involve the concept of *sets of numbers,* such as odds, evens, primes, composites, and perfects.

Clearly, the realm of concepts is vast, and the elementary teacher need not concern himself directly with many of the types of higher-level concepts. He must recognize, however, that the beginning stages in the development of many important concepts occur in the elementary school and that, through utilization of a variety of manipulative materials and appropriate strategies, he can do much to help the children learn concepts appropriate for their level.

EXERCISE SET 5

1. Use the attribute pieces shown in Figure 13. Invent a concept, name it, and make an appropriate concept card for it.

2. Choose at least two *Investigating School Mathematics* concepts from the list given below and develop concept cards which illustrate the use of examples and non-examples to teach the concepts you have chosen.
   A quadrilateral
   B simple closed curve
   C odd number
   D greater than (the relation)
   E right triangle
   F is congruent to (the relation)
   G lowest-terms fraction
   H parallelogram
   I diagonal of a polygon
   J parallel lines
   K one half
   L isosceles triangle
   M equivalent sets
   N symmetrical figure
3. Answer the questions on the sets of Creature cards from the set of attribute materials published by the Webster Division of McGraw-Hill Book Company (if available).
4. Choose an unusual concept of your own invention and make a concept card from which a person might discover your concept.
5. The investigation in Figure 1 was used to teach the concept of congruent segments. Make a card to teach this concept using examples and non-examples.
6. Complete "Learning a Concept" on pages I-18 and I-19; then answer the following questions.
   A What are some examples of the concept you learned?
   B Give some characteristics of the concept you learned.
   C What were your feelings about the lesson? How could the lesson be improved to make the learning of the concept easier?

Skills

Broadly speaking, there are several types of skills that children develop in the elementary school. Hopefully, many children will develop a skill in estimating distance, weight, capacity, and time. Some teachers may wish to help children develop skill in drawing geometric figures. Some teachers set goals for upper-grade children which include developing skills in reasoning and even in "proof" of simple ideas. In elementary mathematics the most fundamental skill, by far, is that of computation with whole and rational numbers. It is these specific computational skills involving addition, subtraction, multiplication, and division and the processes related to these operations with which we are particularly concerned in the discussion that follows.

Two types of skills, power skills and speed skills, are available for completing each arithmetic process. A *power skill* is any effective way to find an answer. A *speed skill* is the most efficient way to find an answer. A power skill is a process through which a given problem is attacked by means of some technique which, though possibly quite inefficient, can produce a correct solution. This power skill may involve a long, tedious process, one which may be totally unrelated to the most efficient method for arriving at the solution. On the other hand, when a speed skill is employed, the problem is attacked with the most efficient technique available, and the problem is solved relatively quickly, usually in a mechanical fashion.

For example, suppose a child wants to find the sum of 27 and 48. If he simply starts at 48 and counts on 27 more, he is using a power skill. If, however, he finds the answer by using the usual algorithm for addition, then a speed skill is being employed.

Two additional points are worth noting about the previous example. First, in order to utilize the power

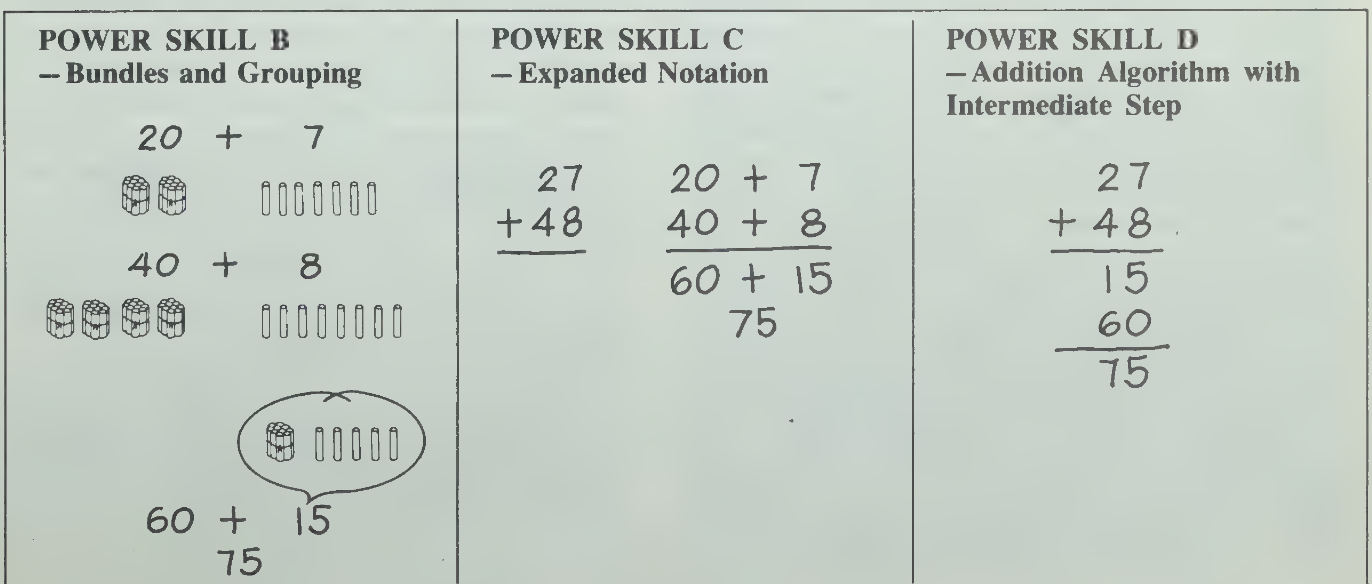


Figure 10

skill, the child needed a clear concept of addition as it relates to counting. Thus, a power skill relies on a previously learned concept. As the child uses the concept in a power-skill situation, he gains new confidence in his ability to do something he "hasn't had yet." Secondly, the teacher should observe the evolution from power to speed. In finding the sum of 27 and 48, the initial power skill involved a basic concept of addition and the counting process. In practice, the child may continue the evolutionary trek from power to speed by next utilizing power skills B, C, and D as shown in figure 10.

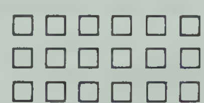
Note that each of these power skills represents a small step toward the ultimate, more efficient speed skill. When considering this process of evolution, it should also be noted that the earlier stages in a power-skill sequence often involve manipulative materials with subsequent power skills exhibiting a transition from the concrete to the more abstract. This physical beginning, which utilizes bundles and grouping, is illustrated as Power Skill B in Figure 10.

The use of power skill is available to all children. The slower child may well attempt the problem by the only means he knows, one which may often be quite laborious. For example, in finding the quotient 5863 ÷ 72, the slower child might subtract 1 seventy-two at a time until he has reduced the dividend to some number less than seventy-two. The more able and creative child might tire of this method and attempt to subtract some multiple of seventy-two, such as 10 seventy-twos. Since each child is working on his own for a period of time, the development of power skill is extremely helpful in working with individual differences.

One decision that the teacher must make in relation to each child is the extent to which he should be encouraged to develop an efficient speed skill for a given algorithm. Obviously, skills are important and should be taught in elementary mathematics, yet it is the good judgment of the teacher that plays the crucial role in guiding a given child from power to speed. For certain processes, children should probably never be forced to attain a speed skill, but should be allowed to operate at the power-skill level. Other children should be directed toward the speed skill as quickly as possible in order that they may proceed to more interesting aspects of mathematics. In rare instances, a child might profit from an initial consideration of a speed skill with no previous power-skill development of a given process. The emphasis on the role of conceptual power in the performance of a skill is a key feature of the so-called "new" mathematics. It is quite probable that we cannot predict the future mathematical needs of children in our classes today, but we can help them develop the confidence, even in the area of learning skills, to utilize concepts previously learned to discover some of the basic processes for themselves.

EXERCISE SET 6

1. Write *power* or *speed* depending on the type of skill you think is being employed.

Specific Skill	Example
A Using sets to find sums	 $3 + 2 = \square$
B Using number line to find sums	 $3 + 2 = \square$
C Counting fingers to find sums	 $3 + 2 = \square$
D Memorizing that $3 + 2 = 5$	Think: 3, 2 → 5 $3 + 2 = 5$
E Thinking about "take away" to find differences	 $5 - 2 = \square$
F Using the inverse relation (missing addend) to find differences	Think: ? + 2 = 5 $5 - 2 = \square$
G Memorizing that $5 - 2 = 3$	Think: 5, 2 → 3 $5 - 2 = 3$
H Using sets to find products	 $3 \times 6 = \square$
I Using the number line to find products	 $3 \times 5 = \square$
J Using logic (basic principles) to find products	Since $5 \times 5 = 25, 6 \times 5 = \square$ or Since $3 \times 5 = 15, 6 \times 5 = \square$

2. Four different power skills are shown for finding 91 ÷ 7. These skills would lead up to finding this quotient by "ordinary short division."

$$\begin{array}{r} 13 \\ 7\overline{)91} \end{array}$$

In what order should these be presented?

A

```
   7)91
     70    (10)
     21    (3)
     21
      0
```

B Subtract 1 seven at a time.

```
   91
 −  7
   84
 −  7
    ⋮
```

C

```
     13
   7)91
     70
     21
     21
```

D Group 91 objects into sets of 7.

3. Complete the "Learning a Skill" lesson on pages I-19 and I-20; then do these exercises.
   A Discuss the skill you learned and the way you learned in terms of power skills and speed skills.
   B What part of the lesson helped you evolve a speed skill?
   C What were your feelings about the lesson? How could it be improved?

Generalizations

Imagine that one of your students is engaged in an investigation in which he was asked to cut out a large quadrilateral and draw colored lines connecting the midpoints of each side of the quadrilateral. The question stimulating the investigation was, "Can you make an odd-shaped quadrilateral so that when you connect the midpoints you do not form a parallelogram?" As a result of this investigation and the subsequent discussion of his findings, the child was led to form a generalization: "The segments connecting the midpoints of any quadrilateral form a parallelogram."

In another lesson, a child might be responding to an investigation question which asked: "If you cut off the corners of a triangle and place the tips at the centre of a circle, what part of the circle can you cover? Can you find a triangle for which this is not true?"

As the child completes the investigation and engages in the discussion which follows, he forms this tentative, unproved *generalization:* "If a compass is used to draw arcs on the corners of any triangle and these corners are cut off along the arcs, then these corners will cover exactly one half of a circle drawn with the same compass opening." This tentative generalization, of course, is the forerunner of the familiar generalization that the sum of the degree measures of the three angles of any triangle is 180.

A generalization provides the economy of moving from consideration of isolated, specific cases to a general statement which holds true for a complete set of numbers or geometric figures. For example, the generalizations stated above deal with the set of all quadrilaterals and the set of all triangles. The regular occurrence of the word "any" in the generalization statements implies that the observation is true for *every* such geometric figure.

The key to teaching a generalization effectively is to provide children with appropriately chosen examples (or instances) which lead them to the generalization. An approach often used by teachers to help children learn generalizations is that of *guided discovery*. In this approach the teacher uses carefully sequenced questions and carefully chosen examples to focus the child's thought on the generalization to be discovered.

It is instructive for children in the upper elementary grades to have experiences in forming generalizations which seem obvious from a set of examples, but which, in fact, do not hold true. For example, consider the equations below.

$$\boxed{1} \times \boxed{1} - \boxed{1} + 11 = 11$$
$$\boxed{2} \times \boxed{2} - \boxed{2} + 11 = 13$$
$$\square \times \square - \square + 11 = ?$$

Figure 11

If 1 is written in the box and the operations are performed, the result is 11, which is a prime number. If 2 is written in the box, the result is 13, also a prime number. Upper-grade children are likely to conjecture that the sum is always a prime number. When they try 3, the sum is 17, also a prime. Similarly, the child finds that the numbers 4, 5, 6, 7, 8, 9, and 10, when written in the box produce a prime number. A child accustomed to forming generalizations from even fewer examples than this will likely conclude that this formula will always produce a prime number. It is instructive to note that when the next number, 11, is written in the box, the result is 121, which, being divisible by 11, is not a prime. This example illustrates the important idea that, even though the generalizations the child might make seem quite plausible and are most often true, it is only by means of a mathematical proof of a generalization that one can be completely sure that it is correct. These proofs, of course, are often not accessible to elementary school children. Thus, a healthy attitude might be characterized by references to generalizations which include phrases such as, "appears to be true," "is probably true," or "could most likely be proven."

Often a search for a generalization is initiated by a question such as, "Do you see any patterns?" For example, several simple generalizations might be formulated about the multiplication table in Figure 12. One child might observe that every number on the main diagonal of the table is a square number. Another student might observe that for every number on one side of this main diagonal, such as 10, there is a matching number symmetrically placed on the other side of the main diagonal.

×	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

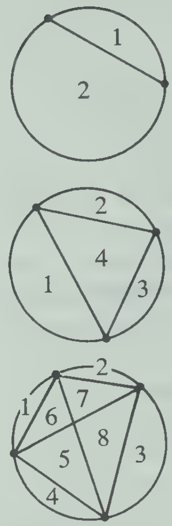
Figure 12

This last generalization is the table counterpart of the commutative principle for multiplication; that is, in the case of 12, $6 \times 2 = 2 \times 6$ or $4 \times 3 = 3 \times 4$. Another generalization that might be reached by careful consideration of the table is that the only primes in the table occur in the one-row or one-column of that table. Still another interesting generalization suggested by the table is that the sum of any number in the two-row and a number below it in the five-row will equal the number below these numbers in the seven-row. Of course, there are many other generalizations ranging from the very simple to the more complex that could be made about this multiplication table.

Perhaps the illustrations above will suggest that the mathematics available to the elementary school child is replete with possibilities for discovery of generalizations. The teacher's task is to create a learning environment in the classroom, not only in terms of physical materials and situations, but in terms of attitude toward learning and toward children, which provides opportunities for discoveries of generalizations and an atmosphere in which it is rewarding to make these discoveries. The teacher should be ever aware of the possibility that the habit of seeking generalizations may well be one of the most valuable things the child learns from his experiences in mathematics.

EXERCISE SET 7

1. Choose a text from the *Investigating School Mathematics* series and list some generalizations which the students who study this text might discover.
2. Investigate the Madison Project shoe boxes and complete the activities for at least two boxes.
3. The illustrations and the table which follow show that if you connect two points on a circle, you divide the interior of the circle into two regions; if you connect three points on a circle, you divide its interior into four regions; if you connect four points on a circle, you divide its interior into eight regions. Note that the points chosen should not be evenly spaced on the circle.



Number of points on a circle	Number of regions formed inside circle
2	2
3	4
4	8
5	
6	

   A Fill in the table to show how many regions are formed if five points on a circle are connected.
   B Form a generalization about the right-hand column of the table.
   C Test your generalization by finding out how many regions are formed inside when six points on a circle are connected.
4. Devise an investigation which might enable a student to discover this generalization: "The sum of the degree measures of the angles of a quadrilateral is 360."
5. Write some questions you would ask and show some examples you would use in guiding a child to discover one of the following generalizations.
   A The commutative principle for multiplication
   B The volume of a "box" is found by multiplying length times width times height.
   C In measuring length, the shorter the unit, the greater the measure.
   D Any angle inscribed in a semicircle is a right angle.
   E Every even number ends in 0, 2, 4, 6, or 8.
6. Complete the "Learning a Generalization" lesson on page I-20; then answer the following questions.
   A What generalization did you learn from the lesson?
   B How many specific examples did you consider before you understood the generalization?
   C In what way did you use the generalization after you discovered it?

Facts

In elementary mathematics, there are certain bits of information that are used so frequently that it is

beneficial for the child to be able to recall them quickly when they are needed. These items are ordinarily called *facts*. There are three main types of facts that are of major concern. The first type of fact is one which evolves from a concept. It might be an example of a specific concept ("Two is a prime number," "25 is a square number," "A parallelogram is a quadrilateral"), or it might be a characteristic of a specific concept, possibly even a part of the definition for the concept ("An isosceles triangle has two congruent sides," "An even number is a number divisible by two," "A pentagon has five sides"). Examples of, or characteristics of, concepts are not always considered as facts; only if such an example or characteristic is deemed important enough to be remembered for immediate recall, is it considered to be a fact and committed to memory.

A second type of fact is a fact derived from a generalization; that is, if a generalization is simple, or deemed important enough to remember for immediate recall, it might often be considered a fact. For example: "The sum of the squares of the lengths of the legs of a right triangle is equal to the square of the length of the hypotenuse of the right triangle"; or "The length of the segment joining the midpoints of two sides of a triangle is one half the length of the third side." Each of these statements might be considered facts since they are sometimes useful for immediate recall. A third type of fact—one that is given a great deal of attention in the elementary school mathematics program—is the type of fact derived from a power skill. For example, the child may have utilized a sequence of power skills for finding sums such as $4 + 3$. He may have used sets of counters, centimetre strips, jumps on the number line, or reasoning from facts such as $3 + 3 = 6$. These power skills, based on certain important concepts, provided the evolutionary progression toward the final speed skill used in finding sums. In this particular case, however, the speed skill used is simply that of memorizing the sum. Whenever the speed-skill stage involves memorization, the particular learning which was classified as a skill or a process during the power-skill stage is reclassified as a fact. The basic addition and multiplication facts fall into this category, and they are given major attention in the elementary school. It is these facts to which primary attention will be given in this section.

A first important point to be made in discussing the teaching of facts is that extensive power-skill work preceding the memorization stage can pay valuable dividends. The broad base of understanding provided by the power-skill work removes the aura of magic from this aspect of mathematics and not only makes the task of memorization of the facts easier, but helps the child view it as a "reasonable thing to do." Figure 13, for example, shows some of the power skills that might be utilized in the initial development of procedures for finding products. Careful development using some or all of these power skills can give the child a basic feeling for a procedure by which products may be found.

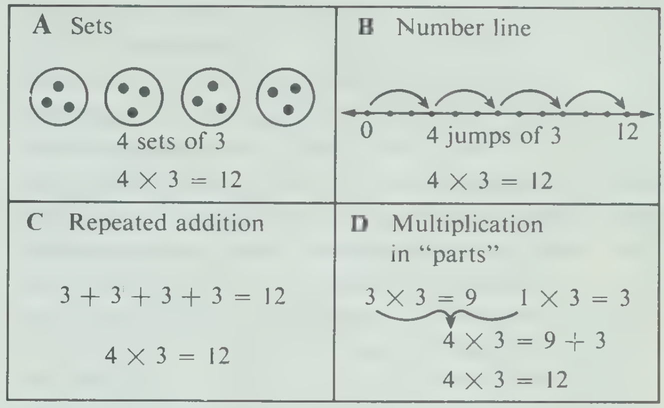
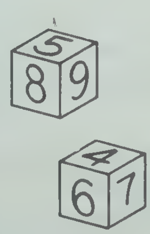


Figure 13

The teacher must use good judgment in deciding when a given child should move from this power-skill stage to memorization of the facts. The appropriate time could vary extensively depending upon the ability and experiences of the child. If the power-skill work is started early in the elementary grades, the child will have ample time to reap the benefits of this basic experience with materials and concepts before the transition to speed skill is made.

When the time has come to memorize the facts, it is important for the child to have a clear idea of the nature of this goal and the reasons it is appropriate. The teacher should even take time to help the child see the very clear difference between "figuring out the fact" and "memorizing the fact." Hopefully, he could help the child develop a feeling for situations in which the facts will be used and in which immediate recall would be quite valuable and time-saving for the child.

After the addition or multiplication facts to be memorized have been placed in perspective, the teacher should seek interesting situations and creative ways in which to practice recalling the facts. For example, the children might make their own flash cards and use a timer to see how long it takes them to give these facts. If desired, two children could work together and see which of them could go through the flash cards most quickly. Another game utilizes a pair of homemade colored dice and an empty multiplication table (Figure 14) for each child. As the game



×	4	5	6	7	8	9
4						
5						
6						
7						
8						
9						

Figure 14

proceeds, a child rolls his dice and writes the product of the numbers on the dice in the appropriate space on his multiplication table. His partner then does the same thing when it is his turn. If a child arrives at an incorrect product or writes the product in the wrong space in the table, he is penalized by missing a turn. The object of the game is to see who can complete the table first. Various modifications of this game are possible, including one in which each child works independently and keeps a tally of the number of times he rolls the dice and also keeps track of the time it takes. The basic objective, of course, is to provide an interesting situation in which the child is motivated to recall multiplication facts rapidly.

Some children may need to spend considerable time in the power-skill stage before they begin to memorize. If there are children who have attempted to memorize the facts and find the job more difficult than anticipated, the teacher may want to consider allowing them to prepare a fact card on which they write the facts that they still do not know. Perhaps it would be realistic and beneficial to let some children use this fact card during the year whenever they desire, thus relieving the tension that could result from difficulties they encounter in memorizing the facts at one specific time. As the school year progresses, the teacher may want to suggest from time to time that a particular child concentrate on one of the troublesome facts and attempt to memorize it so that he can remove it from his fact card. The accomplishment of this goal, of course, would merit recognition and reward. After one fact is removed, the child might start working on removing another fact. The ultimate goal would be to remove all of the facts by the end of the year. Teachers who are interested in helping children learn mathematics in a comfortable way may find that a more realistic, less pressured approach to learning facts may enable the child to find greater enjoyment and success in his mathematical experience.

EXERCISE SET 8

1. Invent a game that could be used to help children practice recalling addition or multiplication facts.
2. Find a commercially produced game that is designed to help children practice recalling facts.
3. Complete the "Learning Some Facts" lesson on pages I-20 and I-21 of this text; then answer the following questions.
   A How many of the facts did you know?
   B What techniques did you use to help you memorize the remaining facts? Did you find this lesson difficult?
   C Can you imagine some of the difficulties your children might have in learning facts?
   D Did you find any mnemonic devices which were helpful in remembering the facts?

Attitudes

In his poem "Arithmetic," Carl Sandburg wrote:

Arithmetic is numbers you squeeze from your head
  to your hand to your pencil to your paper
  till you get the answer.
Arithmetic is where the answer is right
  and everything is nice
  and you can look out of the window
  and see the blue sky—
  or the answer is wrong
  and you have to start all over and try again
  and see how it comes out this time.

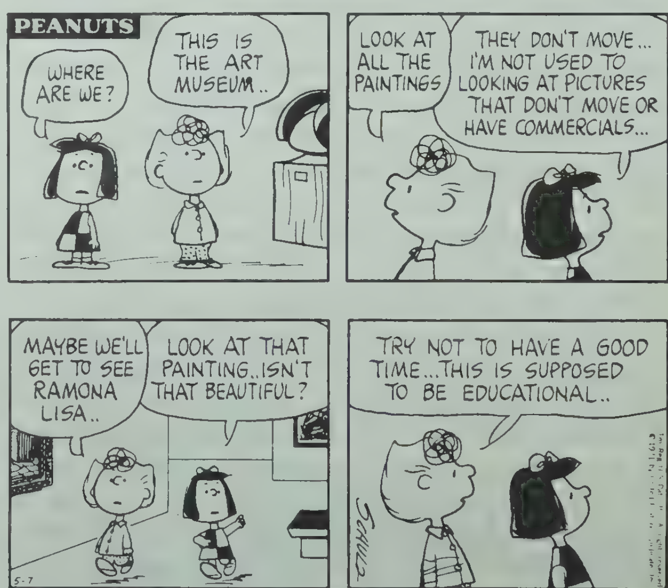
. . . . .

Arithmetic is where you have to multiply—
  and you carry the multiplication table in your head
  and hope you won't lose it.*

The attitude toward mathematics, school, one's ability, and learning in general that one senses on reading this part of Sandburg's poem is surely typical of many children in classrooms today. Perhaps, it was a feeling similar to this that caused Huckleberry Finn to say:

> I had been to school 'most all the time and could spell and read and write just a little and could say the multiplication table up to six times seven is thirty-five, and I don't reckon I could get any further if I was to live forever. I don't take no stock in math, anyway.

There are many different kinds of attitudes exhibited by children who have been exposed to classroom mathematical experiences in different parts of the world. There are, of course, the more general attitudes that a child has toward his teacher, toward his school, toward his fellow students, and toward the process of education. All too often the child's attitude toward education in general is that suggested by Charles Schulz in this *Peanuts* cartoon.



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*From *Complete Poems*, copyright, 1950, by Carl Sandburg. Reprinted by permission of Harcourt Brace Jovanovich, Inc.

Two of the attitudes to be considered here, however, are the child's attitude toward mathematics and the child's attitude toward himself as he relates to mathematics. It has been said that the mathematical experiences of a child before the age of 11, and the responses he has been encouraged to make to those experiences, largely determine his potential mathematical development. If this is so, then a child's attitude toward mathematics and his feelings about how he relates to mathematics are extremely important considerations for the classroom teacher.

A moment's reflection on the number of people who are willing to say that they hate mathematics and on the multitude of others who seem to harbor a fear regarding their inability to cope with ideas of mathematics leads the teacher to realize that he does indeed teach attitudes, whether he tries to or not. Clearly, the teacher who conducts a classroom in which children's achievements are evaluated almost exclusively on the basis of how many right answers they can come up with must surely engender attitudes in children which differ greatly from those engendered by the sensitive teacher who recognizes the child's need to think his own thoughts and to become involved in an exciting exploration of ideas that interest him. Or, consider the difference between the teacher who teaches only speed skills and facts and the teacher who recognizes the central importance of concepts and generalizations, as well as the facts and skills. The child exposed to the first teacher must surely have a feeling toward mathematics, and his ability to interact with it, that is far different from that of the child who learned with the second teacher.

If what happens in the classroom is of such importance in developing attitudes within the child, then the teacher may want to reevaluate his approaches to instruction by reconsidering certain fundamental questions. What subject matter and methods most effectively instill within the child the feeling that mathematics is interesting, fun, and a source of adventure? Will these means provide an opportunity for the child to exercise his freedom of choice and to make decisions about what he does with mathematics? Aldous Huxley said: "A child is a genius until the age of ten." Could it be that our classroom approaches squelch this genius? Can we select mathematical experiences and materials that enable the children to experience success and thus maintain that sense of worthiness and prestige with peers that is of such importance? Can we structure these experiences in such a way that the child maintains within this atmosphere of freedom a sense of security and safety, thus avoiding the fear that can erode his ability to approach mathematical situations with confidence? Can we help children see the usefulness and importance of mathematics without boring them?

Clearly, the questions just raised are difficult to answer and specific techniques for developing healthy attitudes are hard to come by. But even though prescriptions for developing attitudes are scarce, many of the ideas about teaching suggested in earlier sections of this text can provide assistance for the teacher. The investigation, for example, provides the child with an opportunity to make independent decisions and to interact with mathematics and materials and encourages him to take responsibility for his own learning. As difficult as it may seem at times, a child's acceptance of responsibility for his own learning inculcates an attitude that is ultimately invaluable. Also, the manipulative materials or activities that are made available to the child in the investigation situation provide an interaction with the physical world that is often extremely valuable in making mathematics real to a student. Unless a child is ready for more abstract thinking, he cannot be induced to sense the adventure in mathematics without a physical environment to explore. Opportunities for attitude development are implicit not only in the investigation phase of a lesson but in the discussion as well. If a teacher can convince the child that his ideas are important, then the child finds himself in a situation, albeit a mathematical one, in which *he* feels important. His prestige with his peers increases and he feels successful. Exercises in the utilization phase of a lesson that begin simply and gradually increase in difficulty can also help the child feel that he can do mathematics on his own; and, of course, carefully selected extension activities can provide the child with a variety of opportunities to experience the fun of mathematics.

Not only do the phases of the learning experience provide unique opportunities of attitude development, but the particular types of learnings involved within these phases also have their effect. The teaching of concepts and generalizations provides the child with a feeling of power regarding mathematics, for when he experiences the thrill of discovering a concept or a generalization, or when he uses these to solve a problem, he is also developing a useful and wholesome attitude toward mathematics learning. He is developing a habit of reacting to a mathematical situation which will be invaluable when he later encounters mathematical situations possibly undreamed of today. Also, careful teaching of skills and facts can provide the child with that basic sense of security that comes simply from being able to do something or to remember something.

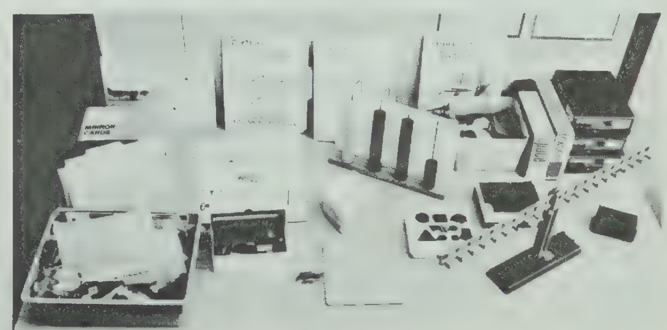


Figure 16

Regarding the child's level of confidence in his ability to cope with mathematical problems, one of the child's paramount needs is to experience success, and as mentioned previously, having entertaining experiences with mathematics might decrease the fear that can erode his confidence. To provide these experiences, the teacher might create in the classroom a "Fun with Mathematics" centre (see Figure 16) that contains mazes, puzzles, design materials, and so on. This centre represents an extra effort to encourage the child to successfully play with mathematics. Some of the materials that might be in such a centre are as follows: the soma cube, the tangram pieces, 2-cm cubes, materials for curve stitching, a kaleidoscope, pattern blocks, Cuisenaire rods, multi-base arithmetic blocks, geoboards, a wide variety of counters, attribute blocks, scales and balances, timers, calendars, measuring tapes and rulers, yarn and string, an assortment of boxes and cans, magazines and catalogues, mirrors, dice, play money, graph paper, assorted plane and solid shapes, abacus, pegboard, compass, mathematical balance, etc.

Perhaps, as you consider the attitudes more carefully and reevaluate the effects of your approaches to instruction, you will find other ways to help children develop a healthy attitude toward mathematics and an enthusiasm for the enjoyment it can offer. Each day as the teacher enters the classroom with plans for a learning experience, he might well ask himself: "What effect will *this* have on the attitudes of the students in my classes?"

EXERCISE SET 9

1. Select a text from the *Investigating School Mathematics* series and find at least five activities which could contribute to the child's development of a positive attitude toward mathematics.
2. Explain how you think some of the other types of learning might also contribute to better child attitude toward learning in general and mathematics specifically.
3. Complete the "Learning an Attitude" lesson on page I-21 of this text; then answer the following questions.
   A Was the lesson fun?
   B How did you feel when you had finished the lesson?
   C Did the lesson change any of your ideas about mathematics?

IV. Some Learning Experiences for the Teacher

In Section II you were introduced to an outline for a learning experience which involved preparation, investigation, discussion, utilization, and extension. In Section III the types of things children learn—concepts, skills, generalizations, facts, and attitudes—were categorized. In this section, we combine these ideas and use them in presenting five learning experiences designed especially for the teacher. That is, in order to gain a first-hand view of lessons which develop these types

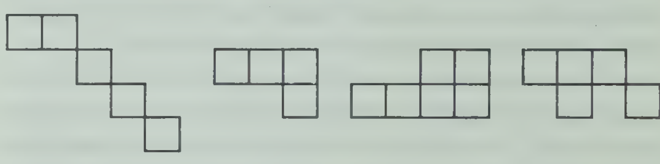
Lesson 1. Learning a Concept

What is a pentominoe?

Investigating the Ideas

Each of these is a **pentominoe.**

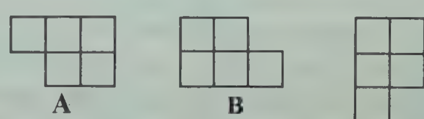
None of these is a **pentominoe.**



How many more pentominoes can you find and show on graph paper?

Discussing the Ideas

1. How many pentominoes did you find?
2. Can you give some characteristics of a pentominoe?
3. How would you "broadly classify" a pentominoe?
4. Can you define a pentominoe?
5. Are the pentominoes in Figures A, B, and C the same?



A **B** **C**

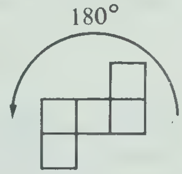
6. How could you convince someone that you have found all possible pentominoes?

of learning, the teacher will have experiences with each of these in the five lessons; and, in order to become more familiar with the suggested structure for a learning experience, each of these five lessons will involve an investigation, a discussion, a utilization, and an extension of the ideas.

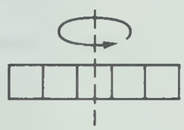
It might be valuable for the teacher, after he has become involved in each of these lessons and has completed the activities, to rethink and discuss his reactions to the various phases of the lesson structure and to the various types of learnings involved. In this way, he might gain a new insight into the way the children in his classes might react to these kinds of situations.

Using the Ideas

1. Which of the pentominoes can be folded to form a box with the "lid missing"?
2. Some pentominoes can be rotated about a point 180° and returned to their starting position. These pentominoes are said to have 180° rotational symmetry. Which pentominoes have 180° rotational symmetry?



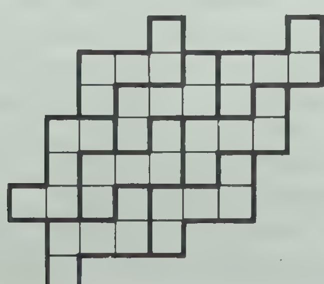
3. Some pentominoes can be flipped about a line and returned to their starting position. Such pentominoes are said to have reflectional symmetry. Which pentominoes have reflectional symmetry?



4. What do you think a hexominoe would be? How many hexominoes can you find?

Extension

Some pentominoes can be used to tessellate (fill without overlapping) the plane, as shown below. Can you find at least two more pentominoes that can be used to tessellate the plane? Show the tessellations on graph paper.



Lesson 2. Learning a Skill

Can you find the product of two 2-digit numbers "in your head"?

Investigating the Ideas

Follow these steps for writing the *answer only* for 74×36.

Step 1	Step 2	Step 3
Think $4 \times 6 = 24$	*Think* $4 \times 3 = 12$; $7 \times 6 = 42$; 54; Add 2: 2; 56	*Think* $7 \times 3 = 21$; Add 5: 5; 26
Write 4, Remember 2	Write 6, Remember 5	Write 26
36×74 → 4	36×74 → 64	36×74 → 2664

Can you use this method to write answers only for the products below? Check your answer using the "long" method.

$53 \times 48 \qquad 37 \times 62 \qquad 45 \times 23 \qquad 67 \times 32$

Discussing the Ideas

1. Explain this statement: In Step 1 you are finding the number of ones.
2. In Step 2 you are finding the number of ?.
3. The 2 you remembered is really 2 ?.
4. Explain what you are finding in Step 3.

Using the Ideas

Write answers only for each product.

1. 28×42
2. 46×33
3. 37×42
4. 82×56
5. 53×34
6. 64×27
7. 29×63
8. 48×35
9. 53×53
10. 27×64

Extension

1. Study the figures below for finding the product of two 3-digit numbers.

352×436	352×436	352×436	352×436	352×436
2	72	472	3472	153472

2. Use the method shown in exercise 1 to find each product.

$$\begin{array}{r} 125 \\ \times 365 \\ \hline \end{array} \qquad \begin{array}{r} 757 \\ \times 426 \\ \hline \end{array} \qquad \begin{array}{r} 841 \\ \times 215 \\ \hline \end{array} \qquad \begin{array}{r} 525 \\ \times 525 \\ \hline \end{array}$$

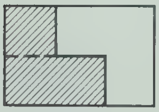
*3. Devise a rule for multiplying two 4-digit numbers.

Lesson 3. Learning a Generalization

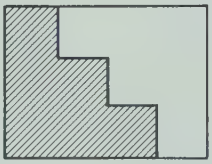
Can you find a pattern?

Investigating the Ideas

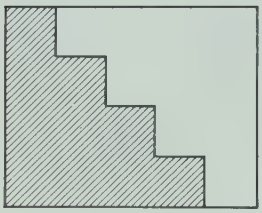
Use the small square in Figure A as the unit. Can you find the area of each shaded part in two different ways? For each part, write an equation to show that the two ways of calculating the area give the same result.



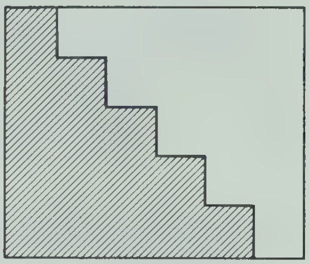
A



B



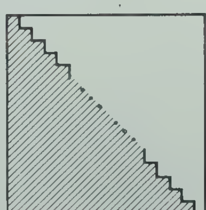
C



D

Discussing the Ideas

1. A Describe one way you found for finding area in the figures above.
   B Describe another way you found.
   C Did you find any other way?
2. Can you write an equation to show that these two methods give the same area?
3. A Suppose there are 50 vertical segments in the "stairsteps" of Figure E. What is the area of the shaded part?
   B Which of the two methods for finding the area would you use?
   C Can you write an equation about this?



4. Can you find the area of the shaded portion of Figure E if there are 100 vertical segments?
5. Can you use what you have learned so far to explain this generalization?

$$1 + 2 + 3 + 4 + 5 + \ldots + n = \frac{n \cdot (n+1)}{2}$$

Using the Ideas

1. Without adding each number, find the sum of the whole numbers through 25.
2. Find the sum of the first 75 whole numbers.
3. Find the sum of the first 200 whole numbers.
4. What is the sum of the first 1000 whole numbers?

Extension

1. What is this sum? $50 + 51 + 52 + 53 + \ldots + 99 + 100$
2. Can you find a short way to find the sum of
   A these even numbers? $0 + 2 + 4 + 6 + 8 + \ldots + 100$
   B these odd numbers? $1 + 3 + 5 + 7 + 9 + \ldots + 99$

*3. Can you state a rule for what you found in exercise 2 by using a variable?

Lesson 4. Learning Some Facts

Can you learn some "new" facts?

Investigating the Ideas

Many rapid "human Calculators" consider these products to be facts.

×	10	11	12	13	14	15
10						
11						
12						
13						
14						
15						

How many of these "facts" can you give without calculating?

(Record the facts you know and shade that portion of the table with a red pencil. Then fill in the remainder of the table by figuring out the remaining facts.)

Discussing the Ideas

1. Which facts in the table need not be memorized provided you know the others and also know the commutative principle? Shade these facts blue.
2. A How many facts altogether are in the table?
   B How many facts remain to be memorized?
3. A What is the "largest" fact?
   B Which facts are over 200?
   C Which facts are in the 190's?
   D Do you notice other patterns in the table that might help you remember certain facts?

Using the Ideas

1. Give these products as quickly as possible.

A 15×15	**E** 13×13	**I** 11×13
B 15×14	**F** 14×12	**J** 11×14
C 14×14	**G** 11×11	**K** 11×15
D 13×15	**H** 11×12	**L** 12×13

2. Make flash cards for the "facts" in exercise 1 that you do not know. Practice with a friend.
3. In exercise 1, start with part L and, following reverse order, give each of the products as quickly as possible.

*4. Make a large multiplication table with all numbers up to 20. Mark out the "facts" you know. How many of these "facts" are left to memorize?

*5. A person who knew the distributive principle and the facts in the table referred to in exercise 4 looked at the multiplication 143×15 and wrote 2145. How did he do it so quickly?

Extension

Study the facts for these powers of 2.

$$2^2 = 2 \times 2 = 4$$
$$2^3 = 2 \times 2 \times 2 = 8$$
$$2^4 = 2 \times 2 \times 2 \times 2 = 16$$
$$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$$

1. Give the next six powers of 2.

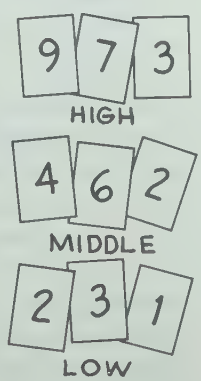
*2. Can you find some mnemonic aids to help you memorize the first ten powers of 2?

Lesson 5. Learning an Attitude

Let's try a place-value game.

Investigating the Ideas

Use 3 sets of 9 cards, each with the digits 1 through 9. Shuffle the 27 cards and deal 3 to each player. Each player then forms a 3-digit numeral, places his cards face down in order, and declares (starting to dealer's left and rotating clockwise) whether his number is high, middle, or low. Play the game in groups of three players.



Discussing the Ideas

1. One player arranged his cards like this and declared that he would try for the low hand. What was wrong with his strategy?

2. What is wrong with this arrangement for a middle hand?

3. If you were dealt these cards, would you try for a high or low? Why?

4. Suppose you are last to declare. Everyone else has declared either low or middle. What would you do with these cards?

Using the Ideas

1. Try playing this game with 2 or more other people.
2. Try the game with the rule that you can declare only high or low.
3. Make up rules for a game in which you turn up the cards one at a time starting with the ones' digit card.

Extension

1. Invent a place-value game in which 4 or 5 cards are dealt to each player.

*2. Find or invent another game or activity that strengthens understanding of the concept of place value.

V. Some Thoughts About Evaluation

The strategy of preparation, investigation, discussion, utilization, and extension is a flexible organizational plan that allows each teacher an opportunity to make a modest beginning toward an activity-oriented mathematics program. The lesson categorization of concept, skill, generalization, fact, and attitude provides a framework that allows each teacher an opportunity to apply the teaching strategy to various types of learning situations. Since there are different types of learning, it is reasonable to assume that there should be different types of evaluation used to measure these learnings.

When considering the facts and skills, for example, emphasis should be placed on child accountability. The teacher should determine the learning outcomes, consider performance objectives for these outcomes, and help the child attain these objectives. The evaluation of this attainment is most easily completed by use of fact and skill tests which determine the child's level of achievement. Since the child needs considerable practice in remembering facts and performing skills, the procedure for helping them is reasonably straightforward.

When evaluating concepts, generalizations, and attitudes, however, the desired performance objectives are often quite difficult to verbalize. We have mentioned earlier that concept learning often takes place

over a relatively long time span, that concepts are extended and broadened, and that concepts mature with each subsequent set of related experiences. Clearly, it is difficult to write a performance objective which specifies the exact level of concept maturity appropriate for a given child at a given time. Whenever possible, objectives for simple concepts should be written, and an attempt should be made to write test items which will show whether children understand these concepts. These items should involve requests for children to give examples of concepts, characteristics of a concept, and even, in some cases, a definition of the concept. For more difficult concepts, the evaluation of children's progress might be made through observation and recorded by means of a check-list which specifies certain levels of development for the given concept. The teacher should be alert for situations in which the child actually uses the concept correctly and should recognize also that understandings which are only partially developed indicate positive achievement. The teacher should also search for instances where the child has shown an ability to form concepts, for this is one of the desired learnings.

When evaluating the child's understanding of generalizations, the teacher should specify the simple generalizations which should be learned by all children. Specific performance objectives and the subsequent test items should be written to evaluate these generalizations. Beyond this, the teacher should again evaluate in greater depth through personal observations or interviews with the children. In the area of generalizations, the teacher should be ever aware that a child who is in the habit of looking for patterns or generalizations has learned a great deal. The teacher should also recognize that a child who can form a generalization from a sequence of specific examples has developed an understanding of a process that is extremely important. We would be remiss if we evaluated only the factual part of the learning of generalizations. As noted earlier, however, although these are important goals of mathematics learning, it is very difficult to write performance objectives for these goals. Whenever possible, objectives should be written which go deeper than facts and skills, but in the absence of objectives, the teacher should feel free to use other means of evaluation, including interviews to evaluate student learning.

While attitudes are not easy to measure in a conventional way, it is suggested that teachers frequently observe children and talk to them about their feelings about mathematics. It is important to realize that one's philosophy toward testing can also have a marked influence on the child's attitude toward mathematics. Testing should be reasonable and realistic, and the child should understand its purpose. The spirit of evaluation should be one of helpful assessment, rather than of critical evaluation. If children participate with teachers in understanding (if not in developing) the goals of instruction, the testing procedure can be a positive influence on the child's attitude and ability to improve.

It is hoped that the teacher will constantly take a broad view toward evaluating mathematics learning among his children. In the long run, evaluation of a child's learning should depend upon the interaction of that child and his teacher. For this interaction to be successful, it may be necessary for the teacher to reexamine his own beliefs about how children learn mathematics. As each teacher makes modest beginnings toward an activity-oriented approach to mathematics learning, he might ask himself the following questions:

1. Do I respect each child as an individual with unique interests, abilities, sensitivities, and significant thoughts?
2. Does the learning environment of my classroom provide a natural, free atmosphere in which children can explore, make decisions, be independent, and encounter exciting new experiences?
3. Does the learning experience also include a supportive, non-judgmental atmosphere in which children have enough routine activities to provide a comfortable threshold of security?
4. Is the child's need for earned success recognized in my classroom?
5. Do I recognize and treat mathematics as a dynamic, ever-growing discipline which offers limitless new vistas to be explored and an inexhaustible variety of new problems to be investigated and solved?
6. Do I view mathematics as a subject of beauty and a source of pleasurable fulfillment of intellectual curiosity?
7. Do I appreciate the significance of my role as a fellow-learner rather than merely a source of information?
8. Is my overall attitude toward mathematics one that encourages a basic freedom to learn through use of manipulative materials in an investigative environment, and through free discussion and exchange of ideas?

As a teacher evaluates the children in his class, he should also reevaluate his approach to mathematical learning. The goal of this short text has been to help in that reevaluation by encouraging the teacher to read, study, observe, experience, experiment, and reconsider. If that goal has been achieved, perhaps his resulting basic beliefs about children, mathematics, and evaluation methods will help him create a new climate of interaction that will spark more effective learning experiences in his classroom.

EXERCISE SET 10

1. Give a set of performance objectives for each lesson completed in Section IV.
2. Create an evaluation tool for each set of behavioral objectives given in exercise 1.

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INTRODUCING THE METRIC SYSTEM

Canada is committed to the metric system of measurement. You may be aware of this but may not have a clear idea of exactly what the metric decision means to you as a *teacher*. It is hoped that this section will serve three purposes—

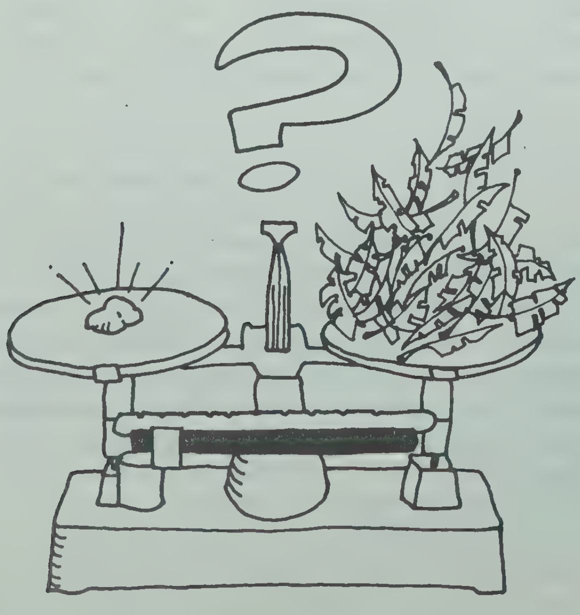
1. give you an idea of how the metric decision will affect you,
2. help you understand the metric system of measurement, and
3. give you some hints for teaching the metric system of measurement to your students.

History and Rationale

The English system of measurement developed from man's need to measure size and distances using units from the most readily available object—himself. He utilized his palm, span, finger, an ell, and a fathom for length; his foot, step, pace, an arrow's flight, and a day's journey for distance; and a handful, shellful, hornful, or gourdful for capacity.

There was little need for standardization until man began to travel and trade with other men. When "standard units" were developed, a new problem arose. Different countries used different definitions for the same unit. The foot was, at first, the length of any man's foot. In some countries, it was the length of the king's foot (since he was the "ruler") and this foot could change as the "rulers" changed. Later an effort was made to standardize some units; for example, England and Scotland decreed the foot to be 12 inches. Unfortunately, England and Scotland didn't use the same definition for the inch.

Today, in the age of technology, one still finds different units in those countries which are not yet metric. Canada and the United States are neighbouring countries, yet they use two different definitions for the gallon. A question at which people in metric countries must laugh is "Which is heavier, a pound of gold or a pound of feathers?" A pound of feathers is heavier since feathers are weighed by the avoirdupois pound (1 avoirdupois pound—7 000 grains) and gold is weighed by the troy pound (1 troy pound—5 760 grains). Which is heavier, an ounce of gold or an ounce of feathers? An ounce of gold is heavier. There are 12 ounces in the troy pound, so one ounce of gold weighs 480 grains; there are 16 ounces in the avoirdupois pound, so an ounce of feathers weighs 437.5 grains.



Out of such confusion there developed a need for a simple, standardized system of measurement. In 1670 Gabriel Mouton, a French abbé, developed a system of measurement organized according to the decimal system of numeration. It took over a hundred years for a system of measurement like the one Mouton put forth to get official sanction. In 1790 the French National Assembly appointed a committee to study the measurement situation and see if a rational system of measurement was possible. In 1795 France adopted a decimal system of measurement, defining the base unit of length to be the *metre* (from the Greek word *metron,* "a measure").

The metric system did not use parts of the human body as units. The metric system did not develop haphazardly adding more and more units as the need arose. The metre was defined as one ten-millionth of the distance from the North Pole to the equator, along the meridian passing near Dunkirk, Paris, and Barcelona. One can see that such a definition would be difficult to replicate in any one country. Also, the length of the metre changes as the position of the North Pole changes; at the time that the metre was defined, scientists were unaware that the position of the North Pole changed.

In 1870, because of the problem of replicating and comparing metric units from country to country, France called a meeting of the metric countries to develop a "unified metric system of measurement". In 1875, the *Treaty of the Metre* was signed to establish the General Conference on Weights and Measures which meets to determine the official definitions for the units used in the metric countries. In 1960 the Conference adopted the Système International des Unités (SI). It is this SI metric system that is most used throughout the world.

A Popular System

The popularity of the metric system stems from two characteristics—the high degree of standardization and its simplicity.

In the entire metric system there are only seven base units! They are **metre** (length), **kilogram** (mass), **second** (time), **ampere** (electric current), **degree kelvin** (thermodynamic temperature), **candela** (luminous intensity), and **mole** (amount of substance).

All units used in the metric system are related to these seven base units. The units you will be most concerned with (because they are the ones used in everyday living) appear in Table 1:

Table 1: Metric Units to be Studied

Quantity	**Unit**	**Symbol**
Length	metre	m
Mass	kilogram	kg
Capacity	litre	ℓ*
Temperature	degree Celsius	°C

*As a rule of thumb, the cursive letter (ℓ) is used as a symbol for the litre to avoid confusion with the numeral (1), however, in symbols such as ml (millilitre), kl (kilolitre) the cursive form is not used.

All other units to be discussed can be represented by the product of one of the units and a power of 10. For example, every possible unit of length can be developed by multiplying the number of metres by the appropriate power of 10.

Table 2: Metric Units of Length

Name (Symbol)	**Metres**
*kilometre (km)	10^3 m or 1000 m
hectometre (hm)	10^2 m or 100 m
decametre (dam)	10^1 m or 10 m
*metre (m)	10^0 m or 1 m
decimetre (dm)	10^{-1} m or $\frac{1}{10}$ m
*centimetre (cm)	10^{-2} m or $\frac{1}{100}$ m
*millimetre (mm)	10^{-3} m or $\frac{1}{1000}$ m

*preferred units

To make the system simpler the same prefixes are used with all units. For example, a millimetre (mm) is $\frac{1}{1000}$ of a metre, a millilitre (ml) is $\frac{1}{1000}$ of a litre, a milligram (mg) is $\frac{1}{1000}$ of a gram, etc.

According to the class, you may want to introduce the symbol "m" for metre, "cm" for centimetre, etc. The plurals, metres and centimetres, are also symbolized "m" and "cm", not "ms" or "cms." Remember, these are symbols and not abbreviations and no period is used after a symbol.

Countries which have been completely metric for several years find that some terms such as "decimetre" are not used in everyday living. People will talk of a book being 28 centimetres long rather than 2.8 decimetres long. You may wish to explain the term "decimetre," but it is not necessary.

Most people who feel that the metric system is complex are those who convert back and forth between the metric and English systems of measurement. When teaching the metric system, conversion to the English system is not necessary and should be avoided!

The metre is defined world-wide to be 1 650 763.73 wave lengths in a vacuum of the orange-red line of the spectrum of krypton 86. This is quite a definition! There are two reasons why such a complex definition was adopted —

1. the length never varies and

2. this measurement can be replicated in laboratories throughout the world.

From this brief history of the metric system it is hoped you will take three main thoughts —

1. The metric system resulted from concentrated effort to develop a rational system of measurement. It did not develop haphazardly.

2. The problem of standardization has been solved in the metric system.

3. The metric system is both popular and useful because of its simplicity.

Activities

Experience and activity are key words in the teaching of measurement. Measure things! The success of this material will depend upon the amount of experience each participant has with the activities. The limited number of activities that are presented should stimulate possibilities for many more. Although the content is approached through activities and measuring experiences, there is a need for exercises to further these experiences and to structure metric thinking. Two points should be emphasized —

1. It is *important* that *you* as well as your class do the activities in this section.

2. The activities will be more fun if done in a group situation.

Looking at Table 1 in the *History and Rationale* section, you will notice that you have to be concerned with only four base units. So, let's use the frontal attack, start right in on length, and begin inching our way down the metric road.

Length, Area, and Volume

In the groups where the metric system has been argued for years, there were two camps. One group wanted to use the centimetre, gram, and second for the core of the system and the other the metre, kilogram, and second. The latter group has prevailed.

It is strongly urged that first grade teachers **not** start with the metre. It is very difficult for first graders to handle a metre ruler. The same argument may be advanced for the kilogram and litre. Length will be approached as it should be covered with students, i.e., first measure with arbitrary units, then use the centimetre, next use the 10-centimetre (decimetre), and finally the metre. All measurement should be approached as a three step process –

1. Select a unit.

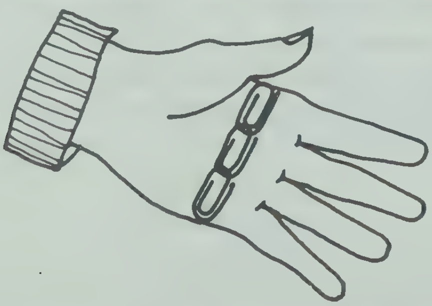
2. Partition the object to be measured into units.

3. Count the number of units used. That number is the measure of the object.

ACTIVITY 1

Measuring objects with an arbitrary unit. Students should do several activities of this type using arbitrary units such as their thumb, a paper clip, pencil, crayon, cutout of their shoe, width of their hand (a unit in the English system used for measuring the height of horses), cubit (another "English" unit, the length of the forearm from the elbow to the tip of the middle finger), or other selected units. For your experience measure the chalk eraser, the width of your hand, the width of this book, and the length of a pencil using a paper clip as the unit.

In the illustration, a "paper clip train" is being used to measure the width of a hand. Follow the three steps mentioned previously in the measurement process.



Record all answers. Then measure the object again using pieces of paper the length of a thumbnail. Repeat the process measuring other objects.

In class emphasize four points –

1. The first unit should be lined up with the "starting point" of the object.

2. The units should touch, but not overlap.

3. The "train" should be straight.

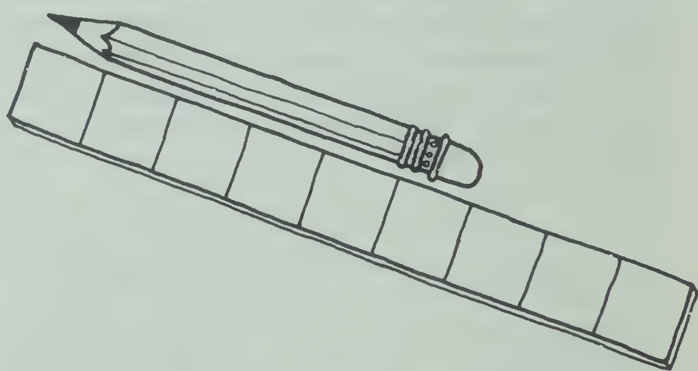
4. The units should be "rounded off" to the unit that has its right end nearest to the "finishing point" of the object.

In doing activities where arbitrary units are used, the need for standardized units becomes obvious. Ask several children to measure the same object, each with his own pencil. On the chalkboard, place their statements such as "The table (or whatever object you pick) is 5 pencils wide." "The table is 7 pencils wide." "The table is 8 pencils wide." Children will soon see that when pencils of differing lengths are used, different answers will result.

ACTIVITY 2

Developing the concept of a centimetre. Probably the first metric unit the children will make use of is the centimetre. You will need (and each student in the class will need) 9 centimetre strips – 9 pieces of paper or cardboard 1 cm by 1 cm square.

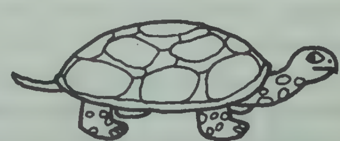
The children, especially the younger ones, should have the experience of measuring many objects using centimetre strips. (If at the time you present this activity your students have studied two-digit numbers, have them measure objects longer than 9 cm.)



Using the centimetre strips, measure the length of a paper clip, a piece of chalk, the Cuisenaire 6-rod, the width of a hand, and the width of a thumb to the nearest centimetre. In this initial activity, actually use centimetre strips and not a ruler marked in centimetres. An exercise the children can do at their desks is to measure the pictures of objects drawn on a duplicator master. The pictures can be of predetermined length. Measure the pictures below.

The arrow is about _____ centimetres long.

The snail is about _____ centimetres long.



The turtle is about _____ centimetres long.

In exercises like these, the length can be controlled. Some answers should require "rounding up," and some "rounding down." The word "about" is important in the sentence since a measurement is an approximation. As the children progress you can have them write not only the number but also the name of the unit.

ACTIVITY 3

Measuring with centimetre rulers. When the children have learned to use the centimetre strips in the measurement process, a ruler marked off in centimetres (not millimetres) should be introduced. It is strongly urged that the child construct his own 10-cm ruler during his first introduction to metric measure. He can do this by constructing a 10-cm train on a 10-cm long piece of paper, pasting the train on the paper, then numbering the cars from 1 to 10. Another approach is to construct a 10-cm ruler in front of the class. Then hand out 10-cm long pieces of paper already marked off in centimetres and have the children number the centimetres from 1 to 10.

The next few activities should involve the measuring of an object with a centimetre train, a 10-cm ruler, and finally with only a 10-cm ruler. When measuring an object with a 10-cm ruler work toward getting your students to "read the ruler" rather than counting the centimetres as they did with the trains.

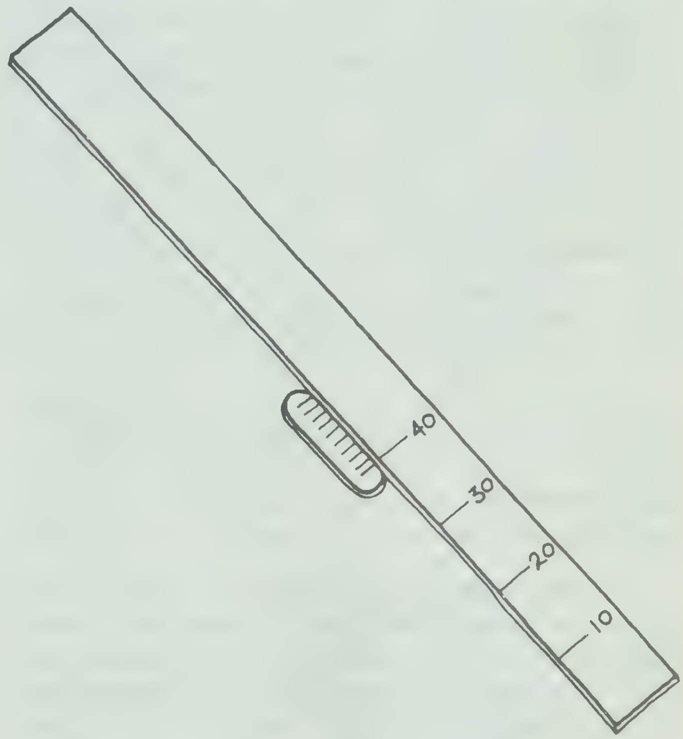
In the example illustrated the child should learn to round off to the nearest centimetre and then read the ruler, "8 centimetres," instead of counting "1, 2, 3, 4, 5, 6, 7, 8 centimetres."

After the children have become skilled in using a 10-cm ruler, they should be given activities requiring them to measure objects which are longer than 10 cm. When working with 5-and-6-year olds, be careful that the measure of the object is not a number the children haven't studied. In the activities concerning measurement it is the process that should be emphasized; the numbers themselves should never be a source of difficulty.

Now, using your 10-cm ruler, measure the length and width of this book and length of your forearm, the length of your foot, and length of your span (what is your span?).

ACTIVITY 4

The metre and notation. Initially, you may want to have your students measure objects with metre-long strips of unmarked cardboard. Then ask them to number the centimetres on the metre strip in groups of 10 using their 10-cm strips. Before proceeding



further, have the class subdivide these cardboard metre rulers into centimetres. It is important that you do the activities with the same type of ruler your students will use. If you have a classroom set of wooden metre rulers, use one of them. Ideally, the rulers used should be marked off in centimetres, but if the ruler is marked off in centimetres (cm) and millimetres (mm) no harm is done. Measure the length, width, and height of your desk rounding off to the nearest metre.

The measurements for a desk, accurate to the nearest metre, might be 2 m long, 1 m wide, and 1 m high. Such measurements would not be helpful. The metre is used for much longer measurements, such as the length and width of the classroom, the playground, the school, the block, etc. To measure the dimensions of objects such as desks, tables, bookshelves, and people, a metre ruler may be used and the results recorded in centimetres. For example, a desk may be 152 cm long, 76 cm wide, and 74 cm high.

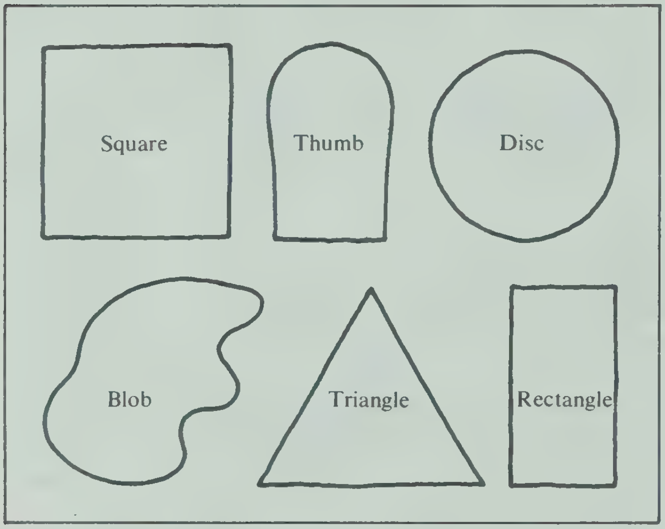
You might say: I am 178 cm tall; what is your height (in centimetres)?

Just as 153 cents is written as \$1.53, 153 centimetres is written as 1.53 metres. This can be interpreted as 1 metre and 53 centimetres which is read as "one point five three" metres. Do not dwell on the mathematical use of the notation—it is not necessary!

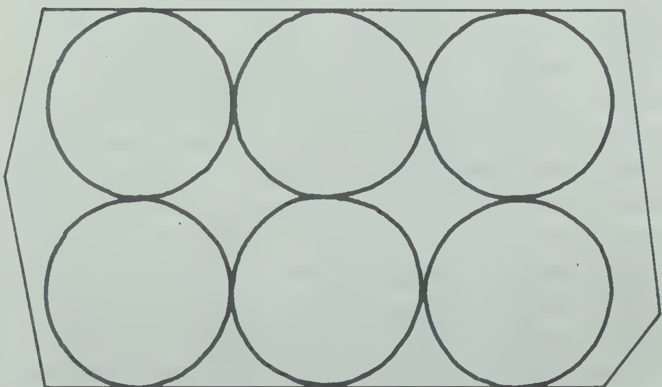
With your class, record the dimensions of your classroom, your desk, their desks, your height, and their heights in terms of centimetres, then in terms of metres using the decimal notation.

ACTIVITY 5

Area using arbitrary units. Here are some examples of area units:



Let the children give names to the units. Then follow the measurement process: select one of these units, match it against the area of some object, and count the number of units used. For example, the irregular figure below has an area of about 6 discs



(if disc is the name given to the unit used). Emphasize that you are trying to "cover" the object. The units should be "even with the edge" of the object, the units should touch, but not overlap, each other. Direct the children's attention to the parts of the object that are not "covered."

Make a cutout of some irregular area such as your thumb and make copies of it out of paper. Use your "thumb" to find the area of the top of a chalk eraser, of the irregular figure measured with the discs, of a cutout of your shoe, and of figure X.

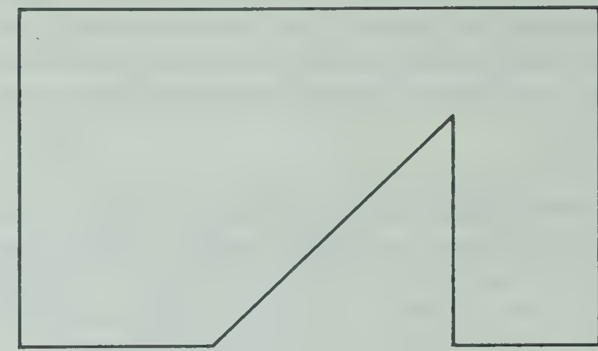


Figure X

Record the answers on the chalkboard in sentence form—

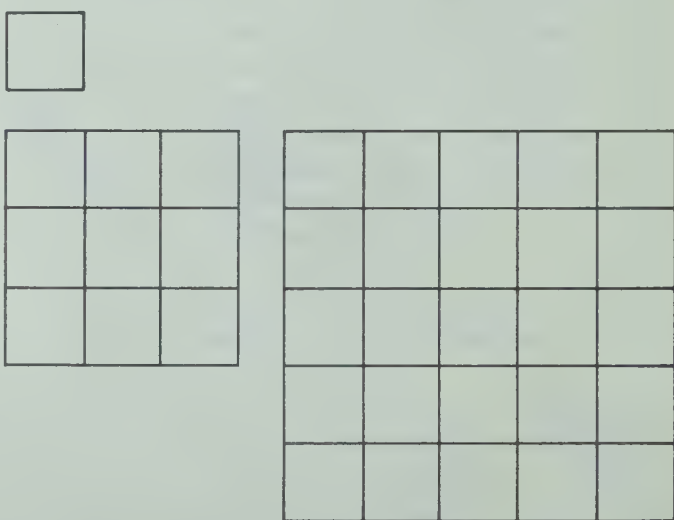
"The figure has an area of about ______ thumbs."

Have your class perform similar activities.

ACTIVITY 6

Area using the centimetre square (cm^2). Have the children make centimetre squares (or have them available for use). The children should have the experience of finding the area of many objects.

Make duplicator masters for some areas that the class can measure with their centimetre squares. The figures below are $1\ cm^2$, $9\ cm^2$, $25\ cm^2$, respectively.



You might point out that the square containing the $9\ cm^2$ has a side of 3 cm and the square containing the $25\ cm^2$ has a side of 5 cm.

Have the children use their centimetre squares to find the *area* of a stamp, a 10-cm ruler, the cutout of their thumb, the irregular figure which had an area of 6 discs, and figure X.

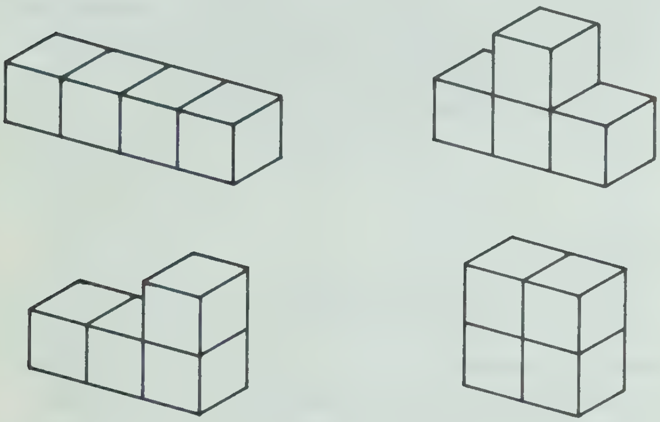
ACTIVITY 7

Volume, using the centimetre cube. In the initial development of the concept of volume, it is important

that children have the opportunity to construct several differently shaped objects each having the same number of volume units.

As with length and area, the study of volume should be introduced with activities making use of arbitrary units of volume, such as blocks, Cuisenaire rods, pencils, erasers, or even marbles.

Use 10 or 12 centimetre cubes in this activity. At first, let the children work on their own, constructing any objects they like. Encourage them to see that an object built of a specific number of cubes has a volume of the same number of cubes regardless of its shape. For example, the illustration shows 4 different constructions, each having a volume of 4 centimetre cubes (4 cm^3).



How many differently shaped objects can be constructed with a volume of 8 centimetre cubes? When those possibilities have been exhausted, try the activity with 10 cubes.

REVIEW: LENGTH, AREA, AND VOLUME

1. Have your class compare the length of their feet, spans, and cubits. Why are these units useless as standard units?
2. Complete these statements.

a.	128 cm = ____m	**e.**	1.06 m = ____cm
b.	108 cm = ____m	**f.**	10.01 m = ____cm
c.	15 cm = ____m	**g.**	23.86 m = ____cm
d.	1010 cm = ____m	**h.**	0.09 m = ____cm

3. What would be the length of the sides in a square containing:
   a. 36 cm^2 → ____ cm?
   b. 25 cm^2 → ____ cm?
   c. 4 cm^2 → ____ cm?
   d. 16 cm^2 → ____ cm?
4. How many different-shaped objects can you form with 6 centimetre cubes?

Capacity

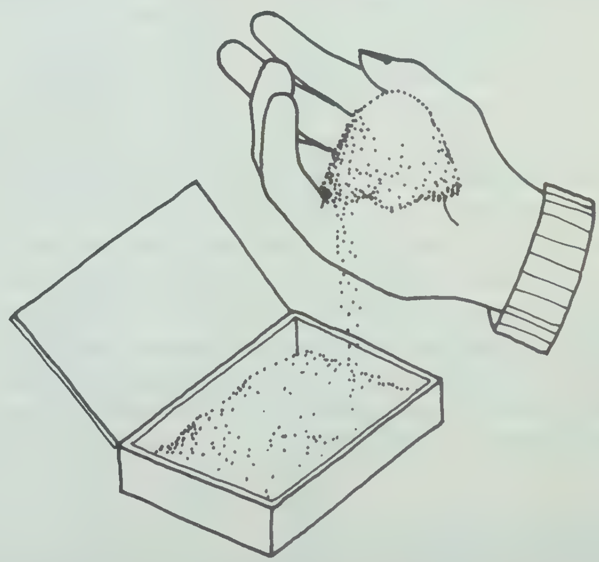
Capacity can be thought of as the amount of material a container will hold. Capacity is usually linked to liquid measure though you may have already had your classes measure capacity by using sand to avoid using liquids.

In the metric system of measurement, volume and capacity are directly related. A container with a volume of 1 cubic centimetre (1 cm^3) will hold 1 millilitre of water. One millilitre (1 ml) is one thousandth of a litre (0.001 ℓ).

The need for fractional names such as $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{4}$ etc. will diminish. The parts of the whole which need emphasis are – 0.1, 0.2, 0.3, . . . , 0.9. Of course, in measurement, fractions could disappear completely, since $\frac{3}{8}$ of a meter is 0.375 m or 375 mm. However, when working with the litre (the unit of capacity in the metric system) don't worry now about using $\frac{3}{4}\ell$, $\frac{2}{3}\ell$, etc. if it is the amount you want the children to see or work with. Since the metric system is based on 10 and since 1, 2, 5 and 10 are the only divisors of 10, we will probably talk about halves, fifths, and tenths of metric units. The decimal notation ($\frac{1}{2}$ is 0.5) will prevail eventually, even at the primary level.

ACTIVITY 8

Capacity and arbitrary units. The most obvious capacity units are handfuls. Give each child a container to fill with water or sand or other material you prefer to use. Have the children fill the container



(milk carton, ice cream carton, cigar box, etc.) with "handfuls" of material. Have them record their results on a piece of paper: "My carton holds ______ handfuls of ______." Compare the wide range of results. Re-emphasize the need for a standard unit to measure capacity. If further experience is necessary, you may want to repeat the project with cups brought from home (since there are so many different sized and shaped cups). Try the activity yourself or get several containers such as an ice cream carton, a milk carton, a wastebasket, a big cooking pan, and a litre container.

On a piece of paper write a pair of sentences for each container:

"The (name of container) holds about (guess) litres.

The (name of the container) actually holds (result) litres.

In the first blank "guestimate" the number of litres the container will hold. In the second, write in the results of measuring the object.

Don't forget the three step measuring process—

1. Select the unit—the litre.
2. Match the unit against the object—fill the object using the litre.
3. Count the number of units (litres) used.

When the container is full (it is best to have a "fill line" just below the top of the container) round off to the nearest whole litre according to whether more or less than half of the last litre was used.

ACTIVITY 9

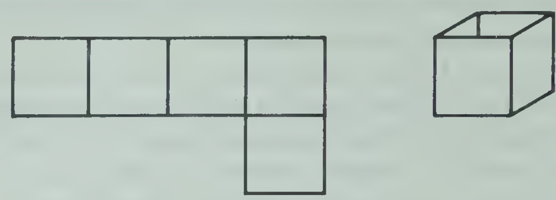
Working with the litre. Get a container that holds a litre of water(and, ideally, has submarkings for each 100 ml). When you are collecting containers for your classroom, try to get as many different shapes as you can. It is important, especially in early experiences, that the children see that litre containers can come in many different shapes. It is the quantity the container will hold, not its shape that determines a capacity of 1 litre.

Once you get a litre container you can make many more. Pour a litre of water into a container and mark a "fill line" for 1 litre on the outside with tape, or, if possible, cut the container so that it holds just 1 litre. Suggested existing containers which can be cut are quart, half-gallon, and gallon milk cartons, round quart, half-gallon, and gallon ice cream cartons. Containers that can be marked might be various shaped pans, cooking bowls, large tin cans, and bottles or jugs. Most activities for introducing the metric units should be accompanied by some estimation exercises. Have the students estimate and record how many litres a container will hold, then measure the container to see about how many litres it does hold. Compare records.

ACTIVITY 10

Introducing the millilitre. The litre is a unit for capacity that is used for milk, gasoline, paint, and other quantities of considerable size. The litre is not used to measure small quantities, such as toothpaste, soda pop, medicines, frozen orange juice, etc. The unit used for the smaller measures is the millilitre (ml). If your school is going to get a set of metric capacity containers, try to get them in these sizes—1 ℓ, 500 ml, 200 ml, 100 ml, 50 ml, 20 ml, and 10 ml. With such a set (whether bought, given, or constructed) one can do all the activities that are necessary.

Construct a container with a volume of 1 cubic centimetre (1 cm^3) to demonstrate the size of the millilitre (ml). Trace the figure below, then cut it out and tape it together along the edges. If you avoid spillage your cube will hold 1 ml of water.



The children need several activities measuring the capacity of objects and recording the results in millilitres. Have them first guess and then measure the capacity of a thimble, a match box, a tablespoon, and a teaspoon. Record the results in sentences like—

"I estimate that the thimble holds about ______ ml. It actually holds about ______ ml."

Mass

As the metric system becomes the predominant system of measurement you may hear talk about the difference between mass and weight. A lunar example may be the best way to show the difference. Now that we are in the space age, practically everyone knows that a man weighs less on the moon than he does on the earth. For example, a 300-kg man on earth would weigh about 50 kg on the moon, but he would have the same mass on the moon as he does on earth. Weight is dependent upon gravity, mass is not. Begin to stress the use of the correct metric term, mass.

The base unit of mass in the metric system is the kilogram (kg). For example, we say "I have a mass of 78 kg.

ACTIVITY 11

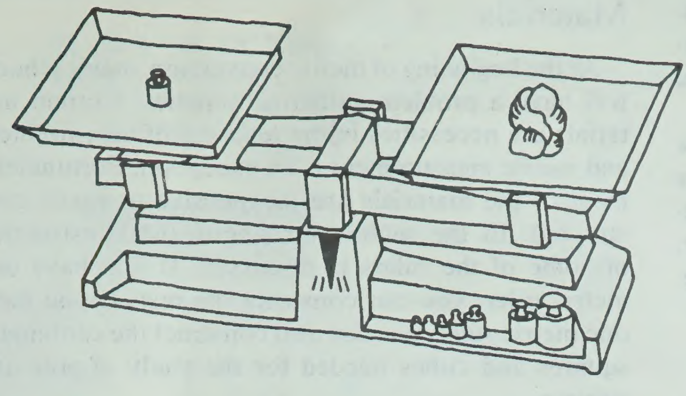
Arbitrary units of mass. To find the mass of an object you will need a balance and some arbitrary units such as paper clips, pencils, Cuisenaire rods, pennies, or other objects. Put a pencil on one side of the beam and then "balance the pencil" with pennies (or multiples of any other small unit). Record the results on paper in a sentence like:

"The pencil has a mass of about ______ pennies."

Repeat the activity with at least three other objects.

ACTIVITY 12

The unit used for small masses is the gram (g). This activity is very similar to the last. You will need gram masses. If you have a classroom set, that's great! If you don't, you can make one.



Put a gram mass on one side of the balance and balance it with a lump of clay or plasticine. Label your clay "1 g." In a similar manner make a set of clay or plasticine "masses" in multiples such as: 5 g, 10 g, 20 g, and 50 g. Use several small objects as test objects (a paper clip, a nickel, a penny, and a pencil). However, before you have the children put one of the test objects on the balance, ask them to estimate its mass in grams. Then find the mass of the object. Record both the guess and the result.

The quarter has a mass of about (guess) grams.
It actually has a mass of (result) grams.

Repeat the activity using other objects. Do you and the class get better at estimating mass?

ACTIVITY 13

Measuring mass using the kilogram. Hopefully, all schools will have metric scales available for finding the mass of children and other large objects using kilograms. For this activity, have each child find his own mass and then make and label a cutout of himself (perhaps using his projected shadow). Have him record his height and mass in metric units on the cutout.

Then you and your class might measure the mass of other objects, such as your own chairs, the textbooks used in the course of one day, litre of water (don't count the container—first find its mass when empty), a dictionary, and even the principal of the school (if he agrees). As mentioned earlier, there is a direct relationship between volume and capacity in the metric system of measurement. In fact, there is a direct relationship between volume, capacity, and mass. A container whose volume is 1 cubic cm (cm^3) holds 1 ml of water and the 1 ml of water has a mass of 1 g. A container whose volume is 1000 cubic cm (or 1 cubic decimetre) holds 1000 ml of water (or 1 litre), and the water has a mass of 1000 g (or 1 kilogram). What did you get for the mass of one litre of water?

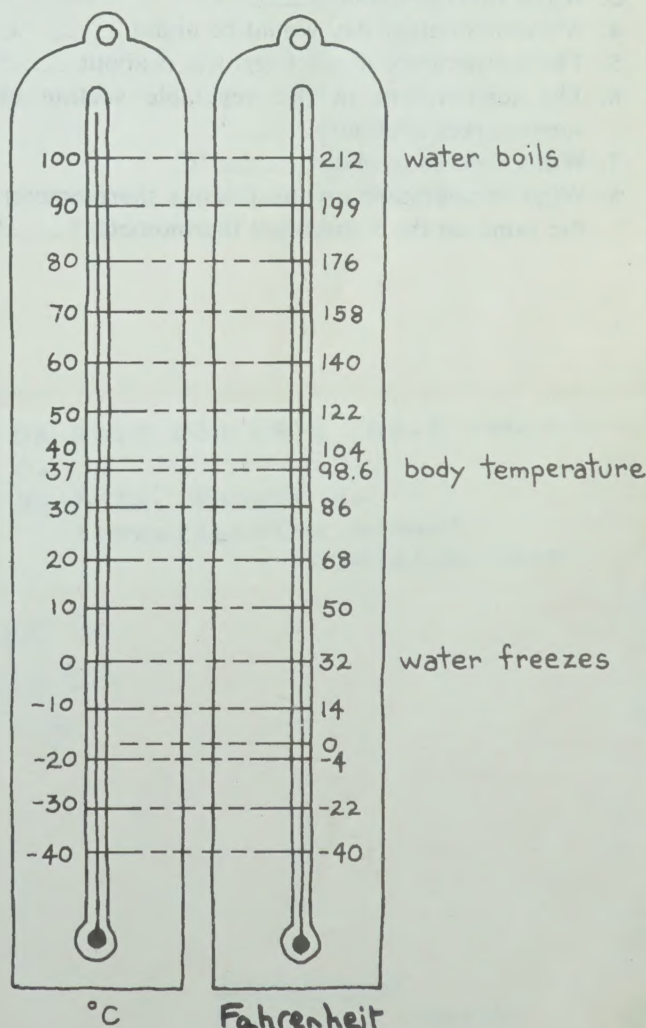
N.B. It is a good idea to label some of the objects in the room as you introduce each metric unit. For example, the aquarium may be 70 cm long, 40 cm wide, 35 cm high; have a water surface area of 2 800 cm^2, volume of 98 000 cm^3; a capacity of 98 ℓ of water and a mass of 12 kg. If the children label the objects as they study particular units, they will begin to think metric.

REVIEW: CAPACITY AND MASS

1. When finding the mass of something using a balance beam, how do you decide which unit to round off to?
2. Fill in the answers:
   a. 28 ml of water has a mass of about _____ grams.
   b. 170 ℓ is _____ ml.
   c. 3.12 kg is _____ g and 438 g or _____ kg.
   d. It would take _____ ml of water to balance 1 kg.
3. a. Will a car get a higher or a lower number of miles per litre than miles per gallon? (Is the litre larger or smaller than the gallon?)
   b. Will a car get a higher or a lower number of kilometres per gallon than miles per gallon? (Is the kilometre longer or shorter than the mile?)
   ★c. Gasoline consumption rates will be given in kilometres per litre. Will a car get a higher or a lower number of kilometres per litre than miles per gallon?

Temperature

This last section covers the introduction of a metric unit, the degree Celsius (°C), for which there is no physical model. On the Celsius scale for temperature, water boils at 100° C and freezes at 0° C. The unit is named after the Swedish scientist, Anders Celsius, who created the centigrade temperature scale. The



Celsius and centigrade scales are the same, but centigrade is no longer the proper term since the centigrade is a unit used to measure angles in the metric system.

The best way to get used to the Celsius temperature scale is to use it! It is almost a necessity that you have a Celsius thermometer. However, if you have a demonstration model of the Fahrenheit thermometer, you can rescale it using the nomograph shown here.

ACTIVITY 14

Graphing temperatures. Be sure to give the children lots of opportunities to read the temperature and record it in degrees Celsius (°C). Perhaps you could institute a morning weather report given by a different child each day to get the class to use Celsius thermometers and to give them a feeling for what the temperature is when expressed in degrees Celsius (° C). The previous day's high and low temperatures (taken from a newspaper account) could be recorded on a wall graph.

REVIEW: TEMPERATURE

1. My body temperature is about ______ °C.
2. Normal room temperature is about ______ °C.
3. Water boils at about ______ °C.
4. A warm summer day would be about ______ °C.
5. The temperature in a refrigerator is about ____ °C.
6. The temperature in the vegetable section of a supermarket is about ______ °C.
7. Water freezes at about ______ °C.
8. What temperature on the Celsius thermometer is the same on the Fahrenheit thermometer? ____ °C

Materials

At the beginning of metric conversion, many schools will have a problem gathering supplies. Certain materials are necessities in the teaching of measurement and metric measurement is no exception. Fortunately, most of the materials are inexpensive or easily constructed. In the section on length, the construction of some of the rulers is discussed. If you have one metric ruler, you can construct the rest. If you have one metric ruler, you can also construct the centimetre squares and cubes needed for the study of area and volume.

The construction of units of capacity and mass have also been discussed. When it comes to temperature you should have a thermometer available for classroom use. If it is a Fahrenheit thermometer, then you should rescale it to degree Celsius using the nomograph given earlier.

Following is a list of companies and government agencies that are currently producing materials or can give some assistance with this problem of teaching the metric system of measurement.

Addison-Wesley (Canada) Ltd. – Don Mills, Ontario
Buntin Gillies & Co. Ltd. – Ottawa, Ontario
Cameron Products – Bramalea, Ontario
Canadian Metric Association – (P.O. Box 35) – Fonthill, Ontario
Contrasts 20 – Calgary, Edmonton, Vancouver, Winnipeg, Regina (Nearest Barber-Ellis Office)
Kruger Pulp and Paper Ltd. – Moncton, Toronto, Hull, Montreal (Nearest Office)
Information Canada (Under Government of Canada) (Nearest Office)
Jack Hood School Supplies Co. Ltd. – Stratford, Ontario
Lufkin Rule Co. of Canada Ltd. – Don Mills, Ontario
Lily Cups Ltd. – Scarborough, Ontario
MacLean-Hunter Learning Materials Co. – Toronto 101, Ontario
Metric-Aids Ltd. – Toronto, Ontario
Moyer-Vico Ltd. – Moncton Weston, Winnipeg, Saskatoon, Edmonton, Vancouver and the Longueuil Co. in Chambly (Nearest Office)
The National Council of Teachers of Mathematics – 1906 Association Drive, Reston, Virginia 22091
Sargent-Welch Scientific Co. of Canada Ltd. – Weston, Ontario
Spectrum Education Ltd. – Toronto, Ontario
Spicars International Ltd. – Scarborough, Ontario
Toronto Dominion Bank (Nearest Office)

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